Lab 1a Input and Output Resistance

Goals of Experiment

Lab 1a is intended to improve your understanding of Thévenin equivalent circuits and the concepts of output resistance (impedance) and input resistance (impedance).

Necessary Equipment

1. Output box
2. Input box
3. Digital Multimeter (DMM)

Introduction

Figure 1(a) shows a complicated circuit with five batteries and ten resistors all in a box. It is an output box with two output terminals, red and black, to connect to the rest of the world. It is a wonderful fact that from the point of view of the rest of the world, the circuit in part (a) can be replaced by the Thévenin equivalent circuit in part (b). The equivalent circuit consists of one battery and one resistor, namely the Thévenin equivalent source and the Thévenin equivalent output resistance.

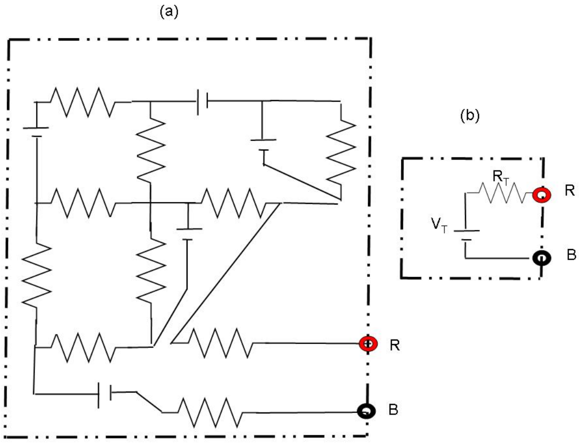

Fig. 1: (a) Complicated circuit. (b) Its Thévenin equivalent

OUTPUT BOX

The output box has a switch and two terminals. [The switch is normally in the OFF position (away from the output terminals) to avoid draining the battery inside the box.] The analysis of the box begins with somewhat lengthy definitions of *open-circuit voltage* and *short-circuit current.*

Open-circuit voltage: It is evident from Figure 1(b) that if you want to know the Thévenin equivalent voltage V_T you simply need to use a voltmeter to measure the voltage between the red and black terminals (the switch needs to be ON). That is because the voltmeter is expected to have a high input resistance compared to R_T.

Voltmeter: Voltage is measured *across* a circuit element. The voltmeter is in *parallel* with the element of interest. Therefore, the voltmeter needs to have a high input resistance so that it does not draw much current from the circuit being measured. In fact, an *ideal* voltmeter would have infinite input resistance. A practical voltmeter can be regarded as an ideal voltmeter in parallel with a large input resistor.

DO 1: Please draw the above description of the practical voltmeter in your lab notebook and label it. This model applies to any voltage measuring device such as an oscilloscope. Typically, such devices have input resistances of megohms. [2 p]

Short-circuit current: Figure 1(b) shows that if the red and black terminals are connected by a wire ("shorted") then the circuit is completed, and the current in the circuit will be equal to the V_T divided by the Thévenin equivalent resistance R_T. Therefore, to find the value of R_T you can measure the current (the short-circuit current) by connecting a milliammeter between the red and black terminals. Then R_T is given by dividing the open-circuit voltage by the short-circuit current.

Ammeter (milliammeter): Current is measured *through* a circuit element. The ammeter is in *series* with the element of interest. Therefore, the ammeter needs to have a low input resistance so that it does not drop much voltage within the circuit being measured. A practical ammeter can be regarded as an ideal ammeter, having zero resistance, in series with a very small resistor.

DO 2: Please draw this description of a practical ammeter in your notebook and label it. Current measurements are more difficult than voltage measurements. With a voltmeter, you can poke the probes here and there throughout a circuit to make voltage measurements without disrupting the circuit itself. With an ammeter, you need to break the circuit, at the point of interest, in order to insert the ammeter measuring the current. [2 p]

Warning: Because of their low input resistance, ammeters are something to worry about. Imagine that you connect an ammeter across a battery. With the usual idealizations, this is a circuit with zero resistance and therefore infinite current. In practice, the current could be high enough to destroy the ammeter. To avoid destruction, some ammeters have an internal fuse to limit the current. Then only the fuse is destroyed and it can be replaced. Other ammeters have a non-destructive internal protection and respond to excessive current with an "OVERLOAD" indication. Multimeters such as ours have a special connection for measuring current. This reminds the user to put the meter in series with the circuit and to be sure that the circuit offers adequate resistance in series with the ammeter.

Short-circuit current: The circuits in the output boxes are designed to tolerate a short-circuit current – no problem. Most circuits that you encounter in the real world do not function correctly with shorted outputs, and for them the concept of "short-circuit current" is mainly a theoretical one. To find the equivalent output resistance for such circuits requires a more "gentle" treatment than a short circuit. Measurements made with the input box later in this lab will be more gentle.

The Fluke 179 meter has a connection labeled "400 mA" and a switch position labeled "mA Hz." When the switch is moved to the "mA Hz" position, the meter is initially set to read alternating current (AC). To read direct current (DC) you need to change modes by pressing the yellow button.

DO 3: There are ten output boxes labeled "1" through "10", and no two are the same. There are ten input boxes labeled "A" through "K", and they too are all different. You *must indicate the number of your output box and the letter of your input box* in your lab notebook so the grader will know how to grade. Indicate this information clearly. [1 p]

DO 4: Use the multimeter to measure open-circuit voltage and short-circuit current on your output box. Calculate the Thévenin equivalent resistance R_T. You may find Figure 2 useful. [5 p]

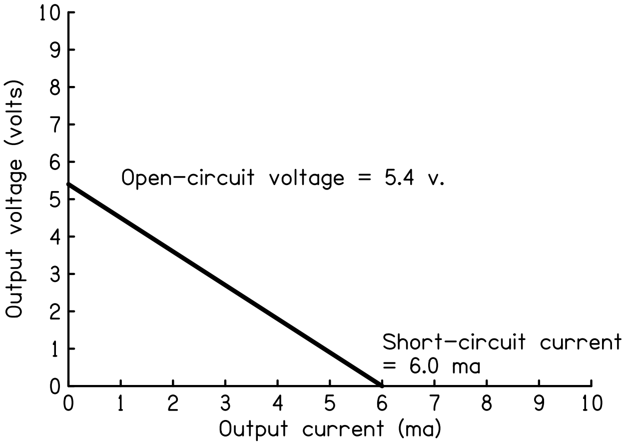

Fig. 2: Example of a determination of Thévenin parameters: For an open circuit there is no output current. For a short circuit there is no output voltage. That's two points on a straight line. The output resistance (impedance) is the slope 5.4/6.0 = 0.9 kohms or 900 ohms. You can draw a figure like this for your output box.

DO 5A: Open the output box and draw the circuit that you find there. Indicate (a) the resistances and (b) the battery voltage. [5 p]

(a) Resistances: You could choose to read the resistances from the resistor codes, but it's recommended that you measure them using the ohmmeter function of the multimeter. The resistances can be measured correctly while still connected in the circuit so long as the switch is in the OFF position.

(b) Battery voltage: Measure the battery voltage (it appears across the red and black leads from the battery holder). With this measurement the switch should be in the ON POSITION so that the battery is loaded by the circuit in the output box. It is important that you record this voltage clearly because the evaluation of other voltage and current measurements depends on it.

DO 5B: Use the information from your drawing (Part 5A) to calculate the expected Thévenin equivalent voltage and Thévenin equivalent resistance. Compare with your measurements. [5 p]

INPUT BOX

The input boxes are small and have no switch – only red and black terminals. The input box represents something that is connected to an output box. Ideally, the input box would not affect the operation of the output box. Ideally, the input box would have infinite input resistance to avoid "loading" the output box. Your input boxes have been constructed to be far from ideal.

DO 6: Use two wires to connect the input box to the output box. Record the voltages at the terminals of the output box before and after the connection. [2 p]

DO 7: To proceed further there is one critical thing to remember, and that is the operation of the standard voltage divider shown in Figure 3. Here, there is a source voltage V_1 and a divided voltage V_2. The two resistors form a voltage divider.

You should be able to prove the voltage divider equation, $V_2 = V_1 R_2/(R_1 + R_2)$. Put this proof in your notebook. [4 p]

The voltage divider equation applies to the output box connected to the input box. Voltage V_1 and resistance R_1 become the Thévenin equivalent voltage and resistance in the output box, V_T and R_T. Voltage V_2 becomes the voltage that you are measuring with the connected circuit. It is the output voltage under load, V_L. Resistance R_2 becomes the the input resistance of the input box R_i.

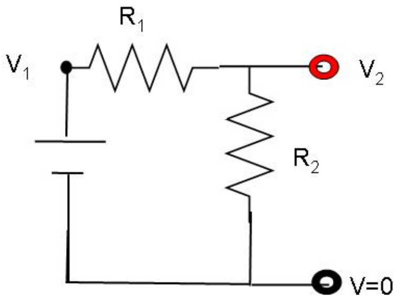

Fig. 3: The voltage divider

DO 8: Your goal is to use your measurements to compute the value of the input resistance, R_i. Do the algebra to find an expression to compute R_i based on the voltage divider equation, your measured voltage, V_L, and what you know about the Thévenin equivalent for the output box. [5 p]

DO 9: Calculate the input resistance for the input box from your equation. Then use the ohmmeter function of the multimeter to measure the input resistance across the (disconnected) terminals of the input box and compare. Include the input box identification letter "A" – "K" in your write up. [5 p]

DO 10: *Gently measuring the output resistance.* In this exercise, you reverse the order of the calculation steps from parts 8 and 9.

Suppose you did not know the output resistance of the output box but you did know the input resistance of the input box. Use the measured value of the input resistance of the input box and the measured voltages to determine the output resistance of the output box. Indicate the algebra steps involved. Compare with the previously measured output resistance (Thévenin resistance). [Note that this part does not require any new measurements.] [5 p]

DO 11: Note that in practical determinations of output resistance one searches through a supply of resistors to find a load resistor that will drop the output voltage by a factor of about 2. Explain mathematically why it is important to make a significant drop in output voltage in order to determine the output resistance accurately. [4 p]

Generalization: This lab involves only DC circuits powered by batteries. The impedances in the circuit have all been resistances, i.e. the impedances have been real quantities. The Thévenin equivalent concept and the concepts of output and input resistances can be generalized to AC sources and impedances that have complex values.

$$\text{Impedance} = \text{Resistance} + j\ \text{Reactance}$$

where j is the square-root of -1. The impedances also depend on the frequency (f) of the source because the reactance depends on frequency. Although the functions and numbers are complex for combinations of resistance and reactance, the AC system is still linear, and that is all that is required for Thévenin's theorem to work. The input impedance for a typical device can be represented by a resistance (R) in parallel with a capacitor (C).

You should (soon) be able to show that the input impedance of this device is

$$\frac{\mathrm{R}}{1 + j\omega \mathrm{RC}}, \tag{1}$$

where ω is the angular frequency $\omega = 2\pi f$.