LAB 3 RC and RL Circuits

Note: This is a long lab. You will probably not finish. You will probably not get all 45 points. You will get further along if you do the steps labeled *PREPARE before lab time.

RC Circuits

Within this part of the lab we study a simple circuit with a resistor and a capacitor following two perspectives: time and frequency. The time perspective relies on a differential equation. The differential equation demonstrates that the RC circuit acts as an approximate integrator or an approximate differentiator. The frequency perspective perceives the RC circuit as a filter, either low-pass or high-pass. For each experiment, starting with 2, make a copy of the screen showing sample input and output waveforms and place that copy in your lab notebook.

Experiment 1. Storage of charge by a capacitor

1A: Set up the circuit in Fig. 1 to charge the capacitor to 5 volts. Note that the scope is connected in parallel with the capacitor, with the scope ground connected to the circuit ground. Disconnect the power supply and watch the trace decay on the scope screen. In order to freeze a slowly varying non-periodic signal on the oscilloscope screen you can use the RUN/STOP button. Measure decay time τ within the exponential decay law: $V=V_o \exp(-t/\tau)$. [2 p]

For an RC combination, this decay time is equal to $\tau = RC$, where R is the resistance in ohms and C is the capacitance in farads. Relying on the latter result and on your measurement, calculate an approximate value for the effective resistance in parallel with the capacitor. (This resistance is the parallel combination of the intrinsic leakage resistance within the capacitor and of the input impedance of the scope. The decay time is expected to be of the order of 1 s.) [2 p]

`Hint:` Use an oscilloscope probe on the capacitors. That will increase the 'scope input resistance from 1 Mohm to 10 Mohm. It will also reduce the measured voltage by a factor of 10 unless the "X 10" feature on the 'scope is in effect. The brass-ring segments on the input BNC connector should do that automatically. (See Vertical Menu "Probe setup.")

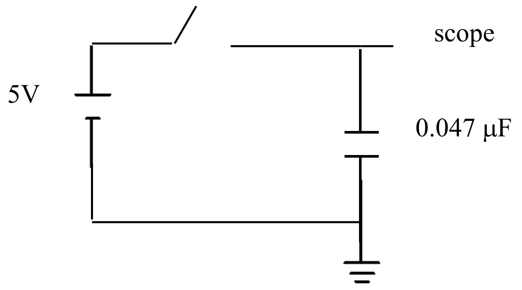

Figure 1: Capacitor charging circuit.

1B. Replace the 0.047 µF capacitor by a 1000µF electrolytic capacitor [footnote 1] and watch the voltage across the capacitor, after you disconnect the power supply. While you are waiting for something to happen, calculate the expected decay time and record it in your notebook. Come to a decision about whether you want to wait for something to happen. Act accordingly. [2 p]

*Note: Y*ou can't really calculate the expected decay time because you don't know the effective parallel resistance of this large capacitor. You can only assume that it is the same as in Experiment 1A.

(*Terminology note*: Added parallel resistance is sometimes called "a shunt resistance.")

Experiment 2. RC integrator in time

Consider the RC circuit in Figure 2 below:

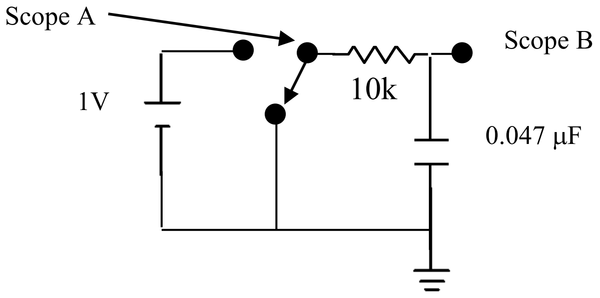

Figure 2: RC Circuit.

In lecture you learned that this circuit can be described by a differential equation for q(t) which is the charge on the capacitor at time t. Solution to that equation gives the voltage on the capacitor, $V_C = q(t)/C$, set to be consistent with no initial charge on the capacitor and equal then to $V_C = V_0(1-e^{-t/\tau})$, where $\tau = RC$ and V_0 represents the initial voltage.

*PREPARE: Calculate the expected rise time (RC) and 1-exp(-1) to copy into your notebook. [2 p]

Now build such a circuit, replacing the battery and the switch by a square wave generator. That is, the 10k resistor should be connected directly to the output of the generator. (Note: The square wave generator has positive and negative outputs, but this is the same as switching the battery with an added constant offset and a scale factor adopted for amplitude.) Please use a +/- 1-volt square wave, i.e. the difference between the two levels is 2 volts. Use two ‘scope channels, one for the input square wave (BNC input) and the other for the capacitor voltage (probe).

2A: Set the square wave frequency to 200 Hz, and observe the capacitor voltage.

Use the vertical cursors of your DPO to measure the time required for the voltage to rise from V_- by $(V_+ - V_-)(1-e^{-1})$. This rise time is expected to be equal to τ. Compare the measurement with the calculated value of τ. [2 p]

Note: The Measure Menu offers a program to measure rise time. Do not use it. [Footnote 2].

2B: Print the display. It should look something like Fig. 3 [PRINT 1] [2 P]

2C Set the frequency to 900 Hz. Observe the capacitor voltage and verify that the circuit indeed acts as an approximate integrator. Print the display [PRINT 2]. [2 p]
Does the 900-Hz input make the circuit look like a better integrator? [2 p]

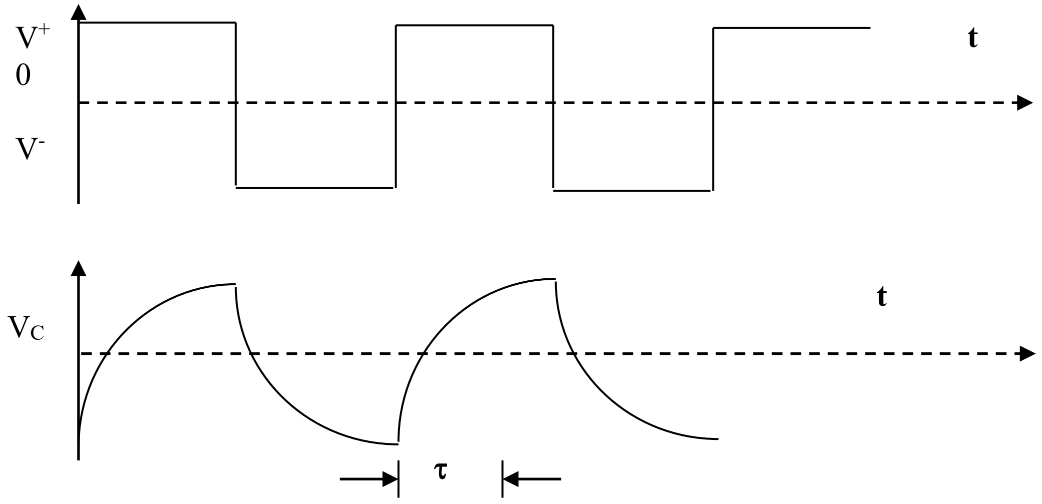

Figure 3: Square Wave and RC Integrator Output.

Experiment 3. RC low-pass filter

The low-pass filter is simply the integrator circuit above. We study its properties by replacing the square-wave by a sine wave so that we can measure circuit's response at a single frequency. (The sine wave is, in fact, the only waveform that represents a single frequency.) We define the transfer function for a filter as the complex output-to-input signal-voltage ratio, $H(\omega) = V_{out}/V_{in}$ (or equivalently the ratio of complex voltage amplitudes), at the angular frequency ω of the sinusoidal input voltage.

The transfer function in this case is given by:

$$H(\omega)=1/(1+j\omega\tau) ,$$

where j is the imaginary number $\sqrt{-1}$. You can explore the transfer function by varying the sine wave frequency.

*PREPARE Calculate the frequency, $f = \omega/(2\pi)$, of the so called “half-power point.” This is simply the frequency where the magnitude of output voltage amplitude is equal to the magnitude of input voltage amplitude divided by $\sqrt{2}$. Calculate the phase shift at this frequency, $\Phi = \tan^{-1}(\mathrm{Im}(H(\omega)/\mathrm{Re}(H(\omega))$. Start by proving that $\Phi = \tan^{-1}(-\omega\tau)$. [2 p].
Build the low-pass filter circuit and measure the frequency for half power. [2 p]
Use the “measure” utility of your DPO to find the phase shift at that frequency and compare that shift with calculations. (Note: The phase shift is related to the time, Δt, between associated zero-crossings for the input and output signals, through the formula

$$\Delta\Phi = 2\,\pi\, f\,\Delta\, t,$$

or

$$\Delta\Phi = 360\, f\,\Delta\, t,$$

for radians or degrees, respectively. We will use the convention that the phase shift is positive, if the output leads the input. Does the output lead the input for this filter?) [2 p]

Experiment 4. RC differentiator in time

Consider the RC circuit in Figure 4 below:

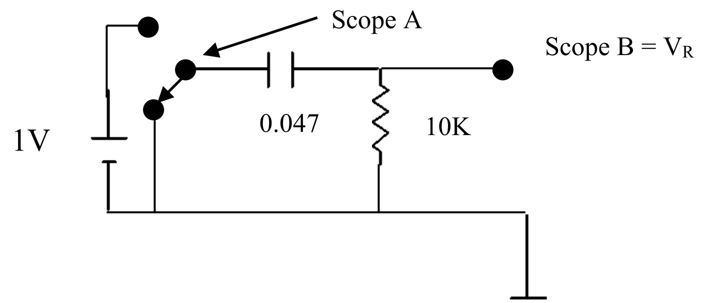

Figure 4: RC Differentiator.

The output is the voltage across the resistor, which is the current, or *dq/dt*, multiplied by the resistance R.

*PREPARE Show that the solution for this voltage, consistent with no initial charge on the capacitor, is $V_R = V_0 e^{-t/\tau}$, where τ =RC and V_0 = 1 volt. [2 p] [Footnote 3].

Now build the differentiator circuit, again replacing the battery and switch by a square wave generator rather than actually using a switch. Set the square wave frequency first to 20 Hz and then to 200 Hz, and observe the resistor voltage in each case.
Sketch the ideal derivative of a square wave. How does the output of the differentiator circuit compare to that derivative? Print the display showing the input and output of the differentiator. [PRINT 3]. The result should look something like Fig. 5. [3 p]

Experiment 5. RC high-pass filter.

The high-pass filter is simply the differentiator circuit above. We study it using a sine oscillator as a source to measure the response at a single frequency at a time.

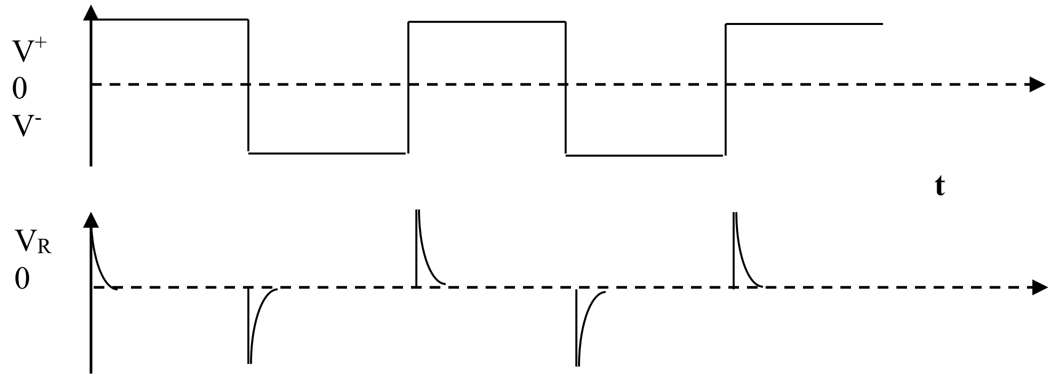

Figure 5: Input and Output of Differentiator.

*PREPARE

The transfer function is in this case

$$H(\omega) = j\omega\tau/(1+j\omega\tau).$$

Show that in the limit of high frequency H = 1.

Calculate the frequency, $f = \omega/(2\pi)$, of the "half-power point."

Calculate the phase shift at this frequency. [2 p]

Build the circuit and find the frequency for half power. [2 p]
Use the DPO to find the phase shift at that frequency and compare with calculations. [2 p]

RL Circuits

Within this part of the lab you will use a 27 mH inductor and resistors.

Experiment 6. Real inductors – the unfortunate truth

Use an ohmmeter to measure the DC resistance of the inductor. Write the answer in your lab notebook. [2 p]

Experiment 7. Real inductors – arcs and sparks

Set up the circuit in Fig. 6.

Once equilibrium is established, after the switch is closed, there remains a voltage across the inductor. Why should this be? [2 p]

Disconnect the power supply abruptly and carefully watch the voltage across the inductor. Connect, disconnect, connect, disconnect… You should observe voltage spikes that exceed the original supply voltage. How can this be? How can you get more voltage from the inductor than the power supply voltage? Is there a violation of a Kirchoff law? Is there a violation of conservation energy? [2 p]

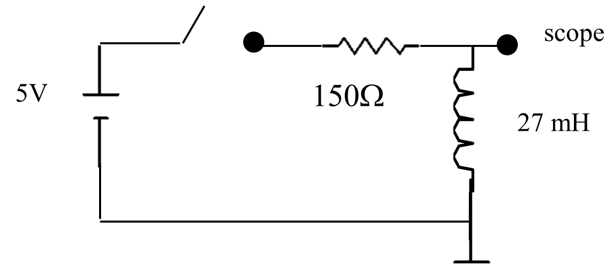

Figure 6: RL Circuit.

Experiment 8. RL differentiatior

Replace the power supply and switch above with a square-wave generator.

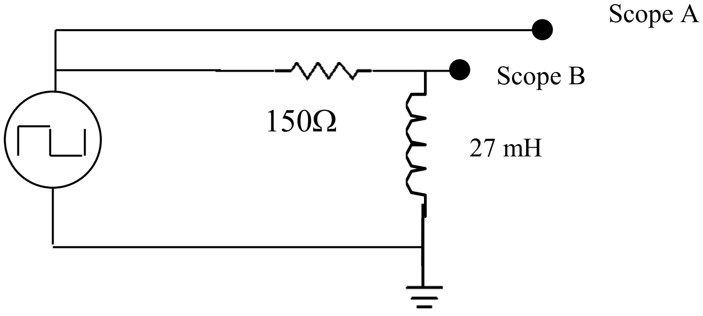

Figure 7: RL Differentiator.

Calculate time constant $\tau = L/R$. Within R remember to include the resistance intrinsic to the inductor as well as a 50Ω resistance contributed by the function generator. Measure the time constant on the DPO and compare with the calculated value. [2 p]
Print a copy of the display showing input and output [PRINT 4][2 p]

Footnote 1: Many electrolytic capacitors have a polarity requirement. One of the two leads must be at a positive potential with respect to the other. Our electrolytics are unpolarized. Polarity does not matter for them.

Footnote 2: There is an engineering definition of the rise time of a voltage step – the time for a voltage to rise from 10% of the step size to 90% of the step size. That is what the Measure Menu will give you for a rise time. This time is considerably longer than τ, which you are measuring.

Footnote 3: Contrary to the conditions of this exercise -- when the battery in Fig. 4 is replaced by a square-wave source there is ALWAYS an initial charge on the capacitor when the transition begins. Brain teaser: Can you explain why the spike in the output voltage has a maximum absolute value that is twice the positive (or negative) voltage of the square wave? (Assume that the square wave has equal and opposite max and min.