

LAB 4 RLC Circuits

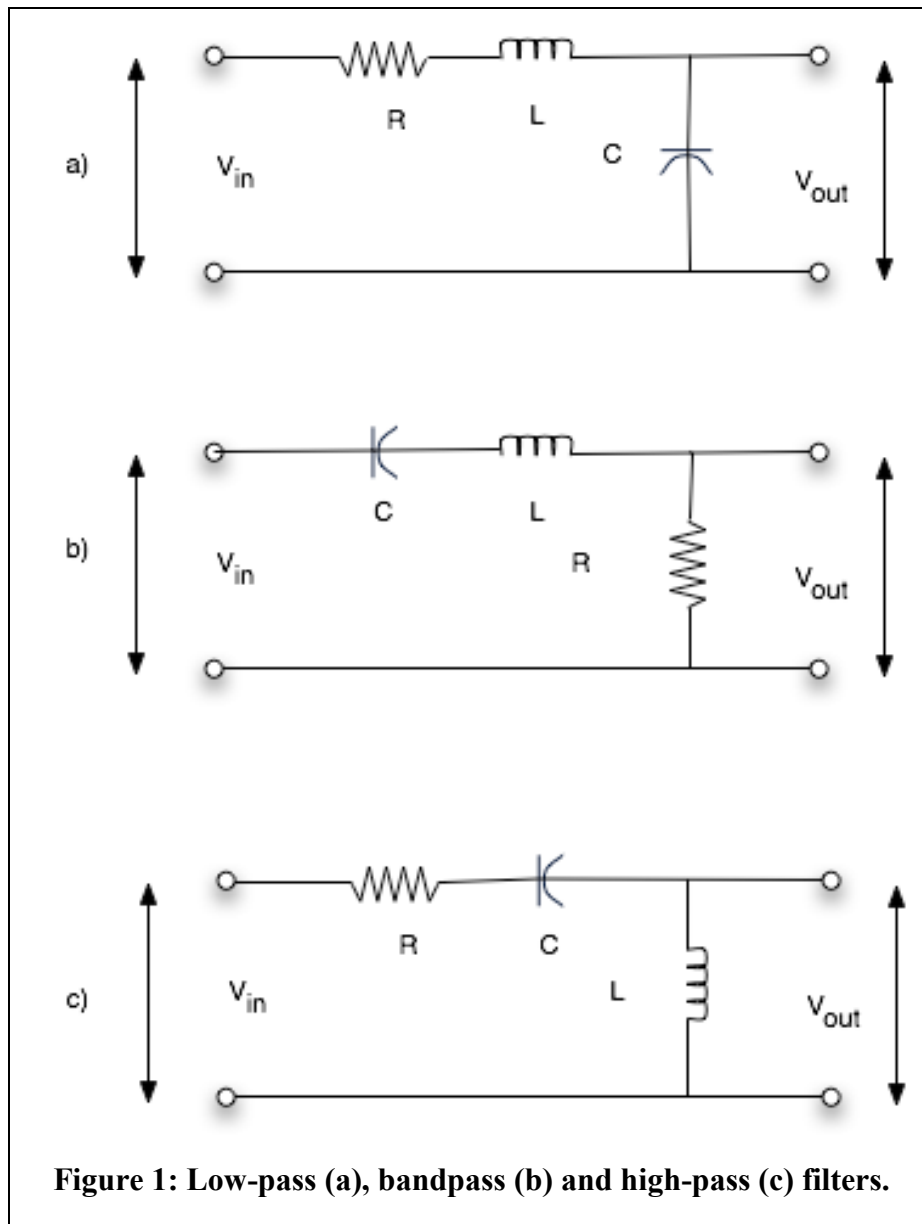
Note: Parts marked with * include calculations that you should do before coming to lab.

In this lab you will work with an inductor, a capacitor, and a resistor to demonstrate concepts of low-pass, bandpass, and notch filters, amplitude response (gain), phase response, power response, Bode plot, resonance and Q.

Series RLC Circuits

*1. Simple filters

Figures 1 (a), (b), and (c) show low-pass, bandpass, and high-pass filters. Write the transfer function $H(\omega)$ for *each* of these filters, showing the ratio V_{out}/V_{in} as a function of the



angular frequency ω of the input voltage. [2 p]

*2. *The low-pass filter power calculations*

(a) Show that the low-pass filter in Fig. 1 (a) above has a power response function:

$$|H(\omega)|^2 = \frac{W_0^4}{(\omega_0^2 - \omega^2)^2 + \omega^2(R/L)^2}, \quad \text{where } \omega_0 = 1/\sqrt{LC}. \quad [1 \text{ p}]$$

(b) Show that this is a low-pass filter by finding the limits at $\omega = 0$ and $\omega = \infty$. [1 p]

(c) Under what conditions is $|H|$ close to a maximum when $\omega = \omega_0$? [1 p]

The half-power points are the angular frequencies ω where the value of $|H(\omega)|^2$ is reduced to half the value at resonance.

(d) Show that the half-power points for this circuit are

$$\omega_1 = \omega_0 - \frac{R}{2L} \text{ and } \omega_2 = \omega_0 + \frac{R}{2L}.$$

Important note: You should use the approximation that the resonance peak width is small compared to the resonance frequency itself. [1 p]

The difference between half-power frequencies is the bandwidth of the resonance.

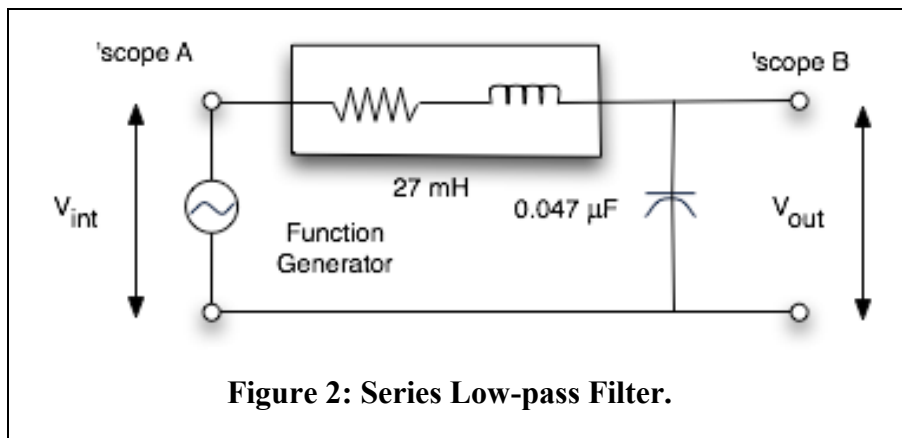
The Q of the resonance is equal to the resonance frequency divided by the bandwidth.

(e) Show that $Q = \omega_0 L/R$. [1 p]

3. *The low-pass filter experiment*

Set up the series low-pass filter shown below. An input voltage of 1 V peak-to-peak is good.

Notice that there is no discrete resistor. The resistor in this circuit represents the



resistance of the inductor plus the output resistance of the function generator (50 ohms).

(a) Measure the resistance of the inductor, add the value contributed by the function generator, and use this sum in your calculations below.

(b) *Calculate the resonance frequency and measure it by changing the function generator frequency. Use 'scope B, as shown in Fig. 2 to find the maximum voltage output. You can use an AC DVM on V_{out} to find a more precise maximum and resonance frequency. [2 p]

(c) *Calculate the phase shift at resonance and measure the phase shift by comparing 'scope A with 'scope B. You will get a more stable phase shift if you use the averaging capability in the Acquire menu. Print a screen showing input and output with phase shift. [2 p]

(d) *Calculate and measure the ratio of input and output voltages at resonance. [Important note: Measure the input voltage as it comes out of the function generator with the rest of the circuit DISCONNECTED.] [3 p]

(e) You should find that the output voltage is greater than the input! Explain how a passive circuit like this can give a voltage gain. Is this a violation of Kirchoff's Law? [1 p]

(f) Using the measured resonance frequency, L and R , *calculate* the Q. [1 p]

(g) *Using the C, L and R , *calculate* the Q. [1 p]

(h) Vary the frequency to find the half-power frequencies and determine the Q from the measurements on 'Scope B. (*Note:* By definition, at the half-power frequencies the output voltage is smaller than the output at resonance by a factor of $1/\sqrt{2}$.) (*Note:* Do not use the DVM to measure these voltages. Its frequency response is not flat enough.) [3 p]

(i) Measure the ratio of input and output voltages for very low frequency, about 1% of the resonance frequency. From the transfer function you expect them to be the same. Are they? What is the phase shift at very low frequency? [2 p]

4. *Reduced Q*

Reduce the Q of the filter by adding a $150\ \Omega$ resistor in series with the inductor. Measure the resonance frequency. Do you expect that frequency to be changed? Is it? Calculate and measure the Q for this circuit. [3 p]

5. *High-frequency limit*

Measure the output voltages when the input frequency is 20 kHz and when the input frequency is 40 kHz. Use these measured values to calculate the high-frequency response slope in dB per octave. Compare with the expected value of -12dB per octave, i.e. the output decreases by a factor of 4 when the frequency doubles (see the Appendix). [2 p]

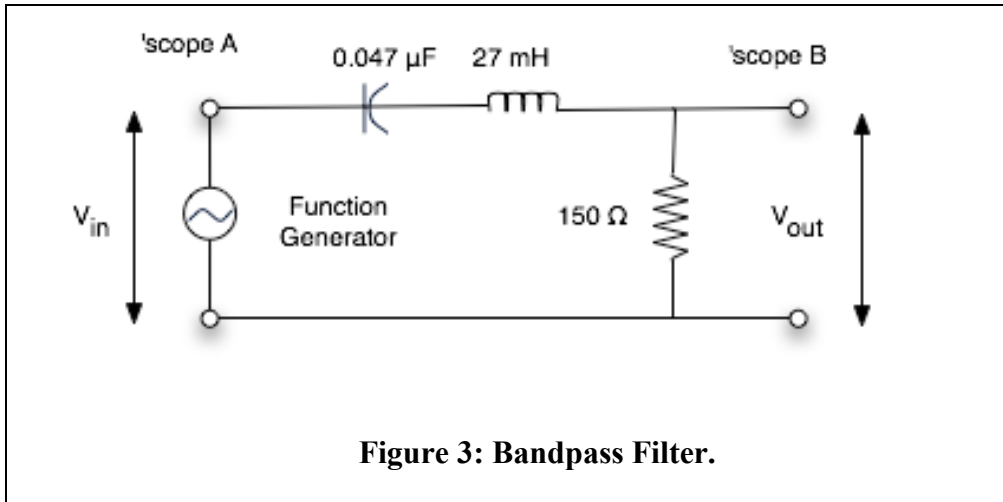
6. *Bandpass Filter*

(a) *Refer to your bandpass transfer function from Part 1. What is the resonance frequency? What is the gain at resonance? [2 p]

Connect the same components as in Figure 3.

(b) Use the function generator, choosing a convenient significant figure on the display as you change the frequency with the knob, to show that this is a bandpass filter. [1 p]

(c) Measure the resonance frequency, and measure the gain at resonance. You can use the DVM for relative voltage measurements at a single frequency. Compare with your expectations from part (a). [2 p]

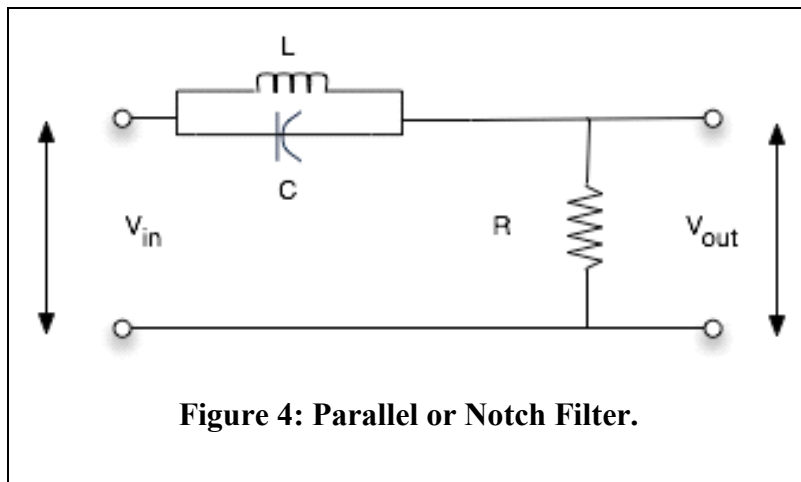


- (d) Measure the phase shift at resonance, *calculate an expectation, and compare. [2 p]
- (e) Measure the Q. Compare with the Q of the lowpass filter above. [2 p]
- (f) *Show from the transfer function that the amplitude response at high frequency is -6dB/octave , namely the output decreases by a factor of 2 when the frequency doubles. [1 p]
- (g) Measure the amplitude response at 20 kHz and 40 kHz to check -6 dB/octave . [2 p]

7. Parallel RLC Circuits

As an example of a parallel circuit, consider the filter Figure 4.

- (a) *Calculate its transfer function. [1 p]
- (b) *Explain why this is called a “notch” filter. What is the frequency of the notch? [1 p]
- (c) Use $L = 27\text{ mH}$, $C = 0.047\text{ }\mu\text{F}$ and $R = 150\text{ }\Omega$. Measure the depth of the notch by comparing the response at the bottom of the notch with the response at low or high frequency. Why doesn't the response actually go to zero at the bottom of the notch? [2 p]
- (d) Use the function generator knob to control a convenient significant frequency digit to demonstrate the notch to the instructor and your neighbor. [1 p]



Appendix: Asymptotic behavior

A filter can be described by its asymptotic frequency dependence. Although the transfer function may be a complicated, complex function of frequency, the asymptotic characteristic is simple. For example, consider a low-pass filter with a transfer function that is inversely proportional to frequency in the limit of high-frequency. We say $H(\omega) \sim \omega^{-1}$. This applies to the bandpass filter of Fig. 1(b).

In general, the asymptotic behavior of a low-pass filter is $H(\omega) \sim \omega^n$, where n is a negative number. Therefore, the filter has a gain characteristic of $n \cdot 20$ decibels per decade (dB/decade).

Proof: The gain characteristic in dB is

$$A_{dB} = 20 \text{ dB } \log_{10}|H(\omega)| .$$

If ω increases by a factor of 10 (one decade), then the change in gain is

$$\Delta A_{dB} = 20 \text{ dB } \log_{10}|H(10 \cdot \omega)/H(\omega)| ,$$

and this is just

$$\Delta A_{dB} = 20 \text{ dB } \log_{10}[10^n] = n \cdot 20 \text{ dB} .$$

Similarly, the asymptotic dependence can be given in dB/octave. Whereas a decade stands for a factor of 10 in frequency, an octave stands for a factor of 2 in frequency. If the asymptotic frequency dependence is, again, $H(\omega) \sim \omega^n$, then increasing the frequency by an octave leads to a gain change of

$$\Delta A_{dB} = 20 \text{ dB } \log_{10}[2^n] = n \cdot 6 \text{ dB} .$$

The asymptotic dependence describes the limiting slope of a Bode plot.