March 17

Induction and Inductance
Chapter 31
Review: Eddy currents

> Eddy currents will oppose the change that caused them – Lenz’s law
> Induced eddy currents will always produce a retarding force when plate enters or leaves $B$ field causing the plate to come to rest
> Cutting slots in metal plate will greatly reduce the eddy currents
Induction and eddy currents are used for braking systems on some subways and rapid transit cars. Moving vehicle has electromagnet (e.g. solenoid) which is positioned near steel rails. Current in electromagnet generates $B$ field. Relative motion of $B$ field to rails induces eddy currents in rails. Eddy currents produce a drag force on the moving vehicle. Eddy currents decrease steadily as car slows giving a smooth stop.
Eddy currents

- Eddy currents often undesirable since they **dissipate energy in form of heat**
- Moving conducting parts often laminated
  - Build up several thin layers separated by non-conducting material
  - Layered structure confines eddy currents to individual layers
- Used in transformers and motors to minimize eddy currents and improve efficiency
Inductance - Definition

- **Inductor** is a device used to produce and store a desired $B$ field (e.g. solenoid)
- A current, $i$, in an inductor with $N$ turns produces a magnetic flux, $\Phi_B$, in its central region
- **Inductance**, $L$ is defined as

\[
L = \frac{N\Phi_B}{i}
\]

- SI unit is henry, $H$

\[
H = T \cdot m^2 / A
\]
Inductance of a solenoid

> What is inductance of a solenoid?

> First find flux of single loop in solenoid

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} = BA = \mu_0 niA \]

> # of turns (N) per unit length (l)

\[ n = \frac{N}{l} \]

> Thus

\[ L = l\mu_0 n^2 A \]

or

\[ \frac{L}{l} = \mu_0 n^2 A \]

> Depends only on the physical properties of the solenoid
Inductance

- **Generators** – convert mechanical energy to electrical energy
- External agent rotates loop of wire in $B$ field
  - Hydroelectric plant
  - Coal burning plant
- Changing $\Phi_B$ induces an emf and current in an external circuit
> **Alternating current (ac) generator**

- Ends of wire loop are attached to slip rings which rotate with loop
- Stationary metal brushes are in contact with slip rings and connected to external circuit
- emf and current in circuit alternate in time
> Calculate emf for generator with N turns of area A and rotating with constant angular velocity, \( v \)

> Magnetic flux is

\[
\Phi_B = \int \vec{B} \cdot d\vec{A} = BA \cos \theta
\]

> Relate angular displacement to angular velocity

\[
\theta = \omega t
\]

> Flux through one loop is

\[
\Phi_B = BA \cos \omega t
\]
Faraday’s law says:

\[ \mathcal{E} = -N \frac{d\Phi_B}{dt} \]

Substitute

\[ \Phi_B = BA \cos \omega t \]

\[ \mathcal{E} = -NBA \frac{d}{dt} (\cos \omega t) \]

\[ \mathcal{E} = NBA \omega \sin \omega t \]

Maximum emf is when \( \omega t = 90 \) or \( 270 \) degrees

\[ \mathcal{E}_{\text{max}} = NBA \omega \]

Emf is 0 when \( \omega t = 0 \) or \( 180 \) degrees
Inductance

- **Direct current (dc) generator**
  - Ends of loop are connected to a single split ring
  - Metal brush contacts to split ring reverse their roles every half cycle
  - Polarity of induced emf reverses but polarity of split ring remains the same

- **Not suitable for most applications**
  - Can use to charge batteries

- **Commercial dc gen. use out of phase coils**
Motors

- Motors – converts electrical energy to mechanical energy
  - Generator run in reverse
  - Current is supplied to loop and the torque acting on the current-carrying loop causes it to rotate
  - Do mechanical work by using the rotating armature
  - As loop rotates, changing $B$ field induces an emf
  - Induced emf (back emf) reduces the current in the loop – remember Lenz’s law
  - Power requirements are greater for starting a motor and for running it under heavy loads
Review for Inductance

> **Inductor** is a device used to produce and store a desired $B$ field (e.g. solenoid)

> A current $i$ in an inductor with $N$ turns produces a magnetic flux, $\Phi_B$, in its central region

> **Inductance**, $L$, is defined as

$$L = \frac{N\Phi_B}{i}$$

> Inductance per unit length for a solenoid

- Depends only on geometry (like capacitor)

$$\frac{L}{l} = \mu_0 n^2 A$$
Inductance in a Circuit

- A changing current in a coil generates a self-induced emf, $\mathcal{E}_L$, in the coil.
- Process is called self-induction.
- Change current in coil using a variable resistor, $\mathcal{E}_L$ will appear in coil only while the current is changing.

\[
\mathcal{E}_L = -N \frac{d\Phi_B}{dt} = - \frac{d(N\Phi_B)}{dt} = - \frac{d(Li)}{dt} = -L \frac{di}{dt}
\]
Inductance

> Induced emf only depends on rate of change of current, not its magnitude

> Direction of $\mathcal{E}_L$ follows Lenz’s law and opposes the change in current

> Self-induced $V_L$ across inductor
  - Ideal inductor
  - Real inductor (like real battery) has some internal resistance

\[
V_L = \mathcal{E}_L - iR
\]
Inductance

> Checkpoint #4 – Have an induced emf in a coil. What can we tell about the current through the coil? Is it moving right or left and is it constant, decreasing or increasing?

Only get $\mathcal{E}_L$ if current changing

Decreasing and rightward

OR

Increasing and leftward
Inductance

> **Mutual induction** – current in one coil induces emf in other coil
> **Distinguish from** self-induction
> **Mutual inductance**, $M_{21}$ of coil 2 with respect to coil 1 is

\[ L = \frac{N\Phi}{i} \]

\[ M_{21} = \frac{N_2\Phi_{21}}{i_1} \]
Inductance

\[ M_{21} = \frac{N_2 \Phi_{21}}{i_1} \]

> Rearrange equation:

\[ M_{21} i_1 = N_2 \Phi_{21} \]

> Take time derivative:

\[ M_{21} \frac{di_1}{dt} = N_2 \frac{d\Phi_{21}}{dt} \]

> Faraday’s law

\[ \mathcal{E}_2 = -N_2 \frac{d\Phi_{21}}{dt} \]

> Induced emf in coil 2 due to \( i \) in coil 1 is

\[ \mathcal{E}_2 = -M_{21} \frac{di_1}{dt} \]

> Obeys Lenz’s law (minus sign)
Inductance

- Reverse roles of coils
- What is induced emf in coil 1 from a changing current in coil 2?
- Same game as before

\[ M_{12} = \frac{N_1 \Phi_{12}}{i_2} \]

\[ \mathcal{E}_1 = -M_{12} \frac{di_2}{dt} \]
Inductance

> The mutual inductance terms are equal

\[
M_{12} = \frac{N_1 \Phi_{12}}{i_2} \quad M_{21} = \frac{N_2 \Phi_{21}}{i_1} \quad M_{21} = M_{12} = M
\]

> Rewrite emfs as

\[
\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}
\]

> Notice same form as self-induced emf

\[
\mathcal{E}_L = -L \frac{di}{dt} \quad L = \frac{N \Phi_B}{i}
\]