Class 21

Moment of Inertia

Angular kinetic energy
Announcements
Concepts overview
1. Moment of Inertia
2. Angular kinetic energy
3. Angular momentum
Problem Solving
Overview

• Rolling objects
Moment of Inertia

- What is mass?
- Moment of inertia (I) is the *angular* equivalent of mass
- \( \mathbf{F} = m \mathbf{a} \)
- \( \tau = I \alpha \)
- Note: \( \tau \) is the NET torque
- \( I = \Sigma m r^2 \)  
  (\( r \) is the distance from the axis of rotation)
  - Depends on the mass of the object
  - Depends on the *distribution* of the mass

PHY 231
Fall 2004
Prof. S. Billinge
Some different moments of inertia:

<table>
<thead>
<tr>
<th>TABLE 8.1</th>
<th>Moments of Inertia for Various Rigid Objects of Uniform Composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoop or thin cylindrical shell</td>
<td>$I = MR^2$</td>
</tr>
<tr>
<td>Solid sphere</td>
<td>$I = \frac{2}{3} MR^2$</td>
</tr>
<tr>
<td>Solid cylinder or disk</td>
<td>$I = \frac{1}{2} MR^2$</td>
</tr>
<tr>
<td>Thin spherical shell</td>
<td>$I = \frac{2}{3} MR^2$</td>
</tr>
<tr>
<td>Long thin rod with rotation axis through center</td>
<td>$I = \frac{1}{12} ML^2$</td>
</tr>
<tr>
<td>Long thin rod with rotation axis through end</td>
<td>$I = \frac{1}{3} ML^2$</td>
</tr>
</tbody>
</table>

© 2003 Thomson - Brooks/Cole
\[ cg = \Sigma mr \]
\[ I = \Sigma mr^2 \]

Which of the following statements is true:

1. Both I and cg are intrinsic properties of the object (they only depend on the object)
2. Both I and cg are only defined if the object is rotating
3. Both I and cg only are defined with respect to (w.r.t.) a rotation axis.
4. I is only defined w.r.t. a rotation axis but cg is an intrinsic property of the object
5. cg is only defined w.r.t. a rotation axis but I is an intrinsic property of the object
A 1-kg rock is suspended by a massless string from one end of a 1-m measuring stick. What is the weight of the measuring stick if it is balanced by a support force at the 0.25-m mark?

1. 0.25 kg
2. 0.5 kg
3. 1 kg
4. 2 kg
5. 4 kg
Angular kinetic energy

- $K_{lin} = \frac{1}{2}mv^2$
- $K_{ang} = \frac{1}{2}I\omega^2$
- $= \frac{1}{2} I\nu^2/r^2$
- Conservation of total energy applies =>

$\Delta PE = \Delta K_{lin} + \Delta K_{ang}$
- (no friction, no other losses)
Some different moments of inertia:

<table>
<thead>
<tr>
<th>Moment of Inertia</th>
<th>Object Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = MR^2$</td>
<td>Hoop or thin cylindrical shell</td>
</tr>
<tr>
<td>$I = \frac{1}{2} MR^2$</td>
<td>Solid cylinder or disk</td>
</tr>
<tr>
<td>$I = \frac{1}{12} ML^2$</td>
<td>Long thin rod with rotation axis through center</td>
</tr>
<tr>
<td>$I = \frac{2}{3} MR^2$</td>
<td>Thin spherical shell</td>
</tr>
<tr>
<td>$I = \frac{3}{5} MR^2$</td>
<td>Solid sphere</td>
</tr>
<tr>
<td>$I = \frac{1}{3} ML^2$</td>
<td>Long thin rod with rotation axis through end</td>
</tr>
</tbody>
</table>
Two cylinders of the same size and mass roll down an incline. Cylinder A has most of its weight concentrated at the rim, while cylinder B has most of its weight concentrated at the center. Which reaches the bottom of the incline first?

1. A
2. B
3. Both reach the bottom at the same time.
A solid disk and a ring roll down an incline. The ring is slower than the disk if

1. $m_{ring} = m_{disk}$, where $m$ is the inertial mass.
2. $r_{ring} = r_{disk}$, where $r$ is the radius.
3. $m_{ring} = m_{disk}$ and $r_{ring} = r_{disk}$.
4. The ring is always slower regardless of the relative values of $m$ and $r$. 

PHY 231
Fall 2004
Prof. S. Billinge
Two wheels with fixed hubs, each having a mass of 1 kg, start from rest, and forces are applied as shown. Assume the hubs and spokes are massless, so that the rotational inertia is $I = mR^2$. In order to impart identical angular accelerations, how large must $F_2$ be?

1. 0.25 N
2. 0.5 N
3. 1 N
4. 2 N
5. 4 N
Two wheels initially at rest roll the same distance without slipping down identical inclined planes starting from rest. Wheel B has twice the radius but the same mass as wheel A. All the mass is concentrated in their rims, so that the rotational inertias are $I = mR^2$. Which has more translational kinetic energy when it gets to the bottom?

1. Wheel A
2. Wheel B
3. The kinetic energies are the same.
4. Need more information