Solid State Communications, Vol.35, pp.753-757.
Pergamon Press Ltd. 1980. Printed in Great Britain.

CYCLOTRON RESONANCE OF TWO-DIMENSIONAL ELECTRONS FORMING WIGNER CRYSTAL

M. I. Dykman

Institute of Semiconductors, Academy of Sciences of the Ukrainian SSR, Kiev, USSR

(Received 20 April 1980 by E.A.Kaner)

The cyclotron resonance peak shape is analysed for two-dimensional electrons interacting with surface vibrations of a medium. Explicit expressions for the broadening and shift of the peak are obtained at low and high electron densities.

Cyclotron resonance (CR) is widely used now for the investigation of two-dimensional (2D) electrons localized in the semiconductor surface inversion layers or above the liquid-helium surface. The CR spectrum was shown experimentally to depend strongly on the electron areal density N. Therefore in the theory of CR the many-body effects are to be taken into account. It is interesting that the electron-electron interaction may change the shape and position of the CR peak even at relatively small N when the system is far from degeneration, because this interaction influences strongly the scattering of electrons by impurities or by vibrations of a medium in quantizing transverse magnetic field. The corresponding change of the scattering was considered earlier[1] at low densities when $\omega_p \ll \omega_c$ ($\omega_p=(2\pi e^2N^{3/2}/m)^{1/2}$ is the characteristic frequency of 2D plasmons, ω_c is the cyclotron frequency). This allowed to explain[2] the interesting experimental results[3] on CR of 2D electrons above a helium surface in the corresponding density range.

In what follows the CR peak shape is investigated for a wide density range including $\omega_p \gg \omega_c$. To consider many-body effects the electrons are supposed to form a crystal. The 2D Wigner crystallization takes place when the Coulomb potential energy per particle $e^2\sqrt{N}$ exceeds substantially the characteristic kinetic energy T. It was observed recently[4] for electrons on helium surface at $e^2\sqrt{N}/T \gtrsim 10^2$ and H=0 (H is the transverse magnetic field strength). There was shown in Refs.[5-7] that magnetic field promotes the crystallization. Since the condition $e^2\sqrt{N}/T > 10^2$ is fulfilled frequently in experiments on CR of 2D electrons the theoretical analysis of CR for Wigner crystal seems to be important. Such analysis is carried out below supposing the relaxation of the total momentum of electrons to be due to the scattering by vibrations of a medium (phonons in semiconductors or ripplons, i.e. helium surface capillary waves).

I. General Expression for the CR Peak Shape

The energy spectrum of 2D Wigner crystal in magnetic field consists[8] of

two phonon branches $\alpha = \pm I$. At small wave number k the phonons of the branch $\alpha = I$ present magnetoplasma waves with the dispersion law $\omega_{k,I} = \omega_c + (\omega_p^2/2\omega_c)k/\sqrt{N}$. The existence of the branch $\alpha = -I$ is connected with the ordering of electrons, and at small k $\omega_{k,-I} \sim k^{3/2}$. Cyclotron resonance is caused by the excitation of the long-wavelength phonons of the branch $\alpha = I$ by an outer electric field with a frequency $\omega \sim \omega_c$. Therefore according to the linear-response theory the CR peak shape is determined by the correlator of the creation and annihilation operators $a^+_{\vec{k}\alpha}$, $a_{\vec{k}\alpha}$ for corresponding vibrations.

Using the explicit expressions[7] for the coefficients $\vec{A}_{\vec{k}\alpha}$ of the expansion of electron coordinates $\vec{r}_n$ in Bose operators $a_{\vec{k}\alpha}$, $a^+_{\vec{k}\alpha}$

$$\vec{r}_n = \vec{R}_n + \sum_{\vec{k}\alpha} (\vec{A}_{\vec{k}\alpha} \exp(i\vec{k}\vec{R}_n) a_{\vec{k}\alpha} + \text{H.c.}) \qquad (I)$$

($\vec{R}_n$ is the lattice vector) the following expression for the conductivity $\sigma_{xx}(\vec{k},\omega)$ of nondegenerate 2D electrons near the CR peak may be obtained

$$\sigma_{xx}(\vec{k},\omega) = \frac{e^2 N}{2m} \operatorname{Re} Q(\omega), \quad \omega \sim \omega_c, \quad k \to 0,$$
$$Q(\omega) = \int_0^\infty dt \, \exp(i\omega t) \langle [a_{\vec{k},I}(t), a^+_{\vec{k},I}(0)] \rangle \qquad (2)$$

where $\langle \ldots \rangle$ denotes the statistical averaging. The spatial dispersion of the conductivity is neglected below (it should be noted that Eq.(2) coincides with the expression for 2D plasma conductivity).

The interaction between 2D electrons and surface vibrations of a medium is given by a Hamiltonian

$$H_1 = \sum_n \sum_{\vec{q}} V_{\vec{q}} \exp(i\vec{q}\vec{r}_n)(b_{\vec{q}} + b^+_{-\vec{q}}) \qquad (3)$$

where $b_{\vec{q}}$ is the annihilation operator for a surface vibration with 2D wave vector $\vec{q}$ (the generalization on the case of coupling to volume vibrations of a medium is obvious, cf. Refs.[9,I]).

It follows from Eq.(I) that the interaction (3) is essentially nonlinear in the Wigner crystal phonons. Since the characteristic wave number q_s of the vibrations of a medium providing for relaxation is given by the inverse electron displacement over the time equalling to the duration of a collision (this definition of q_s is valid both for electron gas and solid), it is necessary to take into account in H_1 all terms in the expansion of $\exp(i\vec{q}\vec{r}_n)$ in $\vec{q}(\vec{r}_n - \vec{R}_n)$.

To calculate $Q(\omega)$ the method of double-time Green functions[10] may be used. Then to the second order in $V_{\vec{q}}$

$$\boxed{\begin{aligned}
&Q(\omega) = i[\omega - \omega_c - \Pi(\omega)]^{-I}; \quad \Pi(\omega) = \Pi_1(\omega) + \Pi_2(\omega), \quad \Pi_2(\omega) = -\Pi_I(\omega - \omega_c);\\
&\Pi_I(\omega) = -\tfrac{1}{2} i \sum_{\vec{q}} l^2 q^2 |V_{\vec{q}}|^2 \int_0^\infty dt \, \exp(i\omega t)\, \Pi_q(t), \quad \operatorname{Im}\omega \to +0; \quad l = \frac{I}{\sqrt{m\omega_c}}; \quad \hbar = I;\\
&\Pi_q(t) = \exp(W_q(t))\, \varphi_q(t) - \text{c.c.}; \quad \varphi_q(t) = (\bar{n}_q + I)\exp(-i\omega_q t) + \bar{n}_q \exp(i\omega_q t);\\
&W_q(t) = -\sum_{\vec{k}\alpha} |\vec{q}\vec{A}_{\vec{k}\alpha}|^2 \left\{ (\bar{n}_{\vec{k}\alpha} + I)\left[I - \exp(-i\omega_{\vec{k}\alpha} t)\right] + \bar{n}_{\vec{k}\alpha}\left[I - \exp(i\omega_{\vec{k}\alpha} t)\right] \right\};\\
&\bar{n}_q \equiv \bar{n}(\omega_q), \quad \bar{n}_{\vec{k}\alpha} \equiv \bar{n}(\omega_{\vec{k}\alpha}), \quad \bar{n}(\omega) = (\exp(\omega/T) - I)^{-I}.
\end{aligned}} \qquad (4)$$

The expression for the polarization operator $\Pi(\omega)$ in Eq.(4) contains the surface vibrations Green function $\varphi_q(t)$ weighted with a specific factor $\exp(W_q(t))$. The latter describes the influence of the Wigner crystal phonons (and hence of the electron-electron interaction) on the long-wavelength magnetoplasma mode scattering, i.e. on the broadening and shift of the CR peak. The

factor analogous to $\exp(W_q(t))$ is well-known in the Mössbauer effect theory.

Strictly speaking, according to Eq.(3) the expression for $\Pi(\omega)$ to the second order in H_i ought to present the double sum over the electron labels n, m. In Eq.(4) there are kept only diagonal terms (with n=m) in this sum. The nondiagonal ones contain an additional factor $\exp(i\vec{q}(\vec{R}_n-\vec{R}_m))$ which is fastoscillating in the range $q\sim q_s$ since $q_s\gg\sqrt{N}$ (the latter inequality follows from the fact that the electron vibrations amplitude is small as compared with the lattice constant). The correction to $\Pi(\omega)$ due to nondiagonal terms appears to be $\sim\exp(-\frac{I}{4}q_s^2(\vec{R}_n-\vec{R}_m)^2)\ll I$.

The terms of the higher order in $V_{\vec{q}}$ may be neglected in Eq.(4) if $|\Pi(\omega)|\ll\omega_c$, $|\partial\Pi(\omega)/\partial\omega|\ll 1$, $\omega=\omega_c+i0$. These conditions are fulfilled wittingly when the characteristic frequency of surface waves $\omega_s=\omega(q_s)$ exceeds $|\Pi(\omega)|$. Nevertheless in a number of cases (see below) the approximation (4) holds and the CR peak shape is Lorentzian even at $\omega_s\ll|\Pi(\omega)|$. As a rule[9,2] the frequency ω_s is very small and it is most interesting to investigate CR in the range

$$\omega_s\ll T,\ \omega_c. \tag{5}$$

We shall suppose also that $T\ll\omega_c$.

2. CR Peak Broadening at Low Densities

The expression for $\Pi(\omega)$ is substantially simplified in the density range where $\omega_p\ll\omega_c$ and $\omega_p^2/\omega_c\ll T$. The quantity ω_p^2/ω_c at $\omega_p\ll\omega_c$ determines the widths of the phonon branches and hence the maximum frequency of the branch $\alpha=-I$.

In the range $t\ll\omega_c/\omega_p^2$ one may expand $\exp(\pm i\omega_{\vec{k},-I}t)$ in $W_q(t)$ in a series

$$W_q(t)=-\frac{l^2q^2}{2NS}\sum_{\vec{k}}\left[I-\exp(-i\omega_{\vec{k},I}t)\right]-\frac{(lq)^2\pi}{4\tau_e^2}\left(\frac{it}{T}+t^2\right),\quad t\ll\frac{\omega_c}{\omega_p^2}; \tag{6}$$

$$\tau_e=l\sqrt{\pi/2T}\left(\sum_{\vec{k}}|\vec{A}_{\vec{k},-I}|^2\omega_{\vec{k},-I}\right)^{-I/2}$$

Here S is the area and the equality $|\vec{A}_{\vec{k},I}|^2\simeq l^2/NS$ valid at $\omega_p\ll\omega_c$ is used.

It is obvious from Eq.(4) that the range of integration over $\vec{q}$ and t in $\Pi(\omega)$ is limited by the condition $W_q(t)\sim I$. Then from Eq.(6) follows $q\lesssim q_s=l^{-I}$, $t\lesssim\tau_e\approx(T\omega_p^2/\omega_c)^{-I/2}\ll\omega_c/\omega_p^2$. The latter inequality justifies the expansion (6) (this expansion is "self-consistent").

Substituting Eq.(6) into Eq.(4) and integrating over time one obtains for the CR peak halfwidth the expression

$$\Gamma=-\mathrm{Im}\,\Pi(\omega_c+i0),\quad \Gamma=(T\tau_e/2)\times\sum_{\vec{q}}|V_{\vec{q}}|^2\omega_q^{-I}(lq)^3\exp\left(-\frac{l^2q^2}{2}-\frac{I}{\pi}(\omega_q\tau_e/ql)^2\right), \tag{7}$$

$$\omega_c>T\gg\omega_p^2/\omega_c.$$

With an accuracy to the second addend in the exponent Eq.(7) coincides with the results obtained earlier[I] at $\omega_s\tau_e\ll I$ (when this addend is negligible) by quite another method (in particular the electrons in Ref.[I] were not supposed to form a crystal). The expression for the CR peak shift following from Eqs.(4), (6) also coincides with the results of Ref.[I].

According to Eqs.(6),(7) at $\omega_c>T\gg\omega_p^2/\omega_c$ the CR broadening is due to the decay of the long-wavelength magnetoplasma mode to another (short-wavelength) mode of the same branch ($\alpha=I$) accompanied by birth or death of the great number ($\sim\omega_c/\omega_p^2\tau_e\gg I$) of short-wavelength ($k\sim\sqrt{N}$) low-frequency ($\alpha=-I$) phonons. Obviously the short-range order only is essential for such decays.

3. CR Peak Shape at High Densities

In a density range where $\omega_p\gg\omega_c$ the widths of the both phonon bands $\alpha=\pm I$ exceed substantially ω_c. Then the main contribution to the integral over time (4) determining $\Pi_I(\omega_c)$ comes from the range $t\lesssim\omega_c^{-I}$ where the function $\exp(i\omega t)$ under the integral is not fastoscillating. In this time range to zeroth order in T/ω_c

$$W_q(t) \simeq -w_q^o + w_q(t), \quad t \ll T^{-I},$$
$$w_q^o = w_q(0), \quad w_q(t) = \sum_{\vec{k}\alpha} |\vec{q}\vec{A}_{\vec{k}\alpha}|^2 \exp(-i\omega_{\vec{k}\alpha} t). \tag{8}$$

Thus the Wigner crystal thermal vibrations do not affect Π_I (with an accuracy to $T^3/\omega_c^2\omega_p$).

To calculate $\Pi_I(\omega_c+i0)$ one is to expand $\exp(W_q(t))$ in $w_q(t)$. At $\omega_p \gg \omega_c$ it is sufficient to keep only linear term in this expansion. Then the imaginary part of Π_I is given by

$$\Gamma = \pi T \sum_{\vec{q}} l^2 q^2 |V_{\vec{q}}|^2 \omega_q^{-I} \exp(-w_q^o) f_q,$$
$$f_q = \sum_{\vec{k}\alpha} |\vec{q}\vec{A}_{\vec{k}\alpha}|^2 \delta(\omega_c - \omega_{\vec{k}\alpha}) \simeq q^2/8\pi m c_t^2 N, \tag{9}$$

where c_t denotes the transverse sound velocity at $H = 0$.

According to Eq.(9) the CR peak broadening at $\omega_p \gg \omega_c$ is due to decays of long-wavelength magnetoplasma modes ($\alpha = -I$) to the phonons with $\alpha = -I, k \approx \omega_c/c_t$. These decays are induced by vibrations of a medium[II].

The addend $\Pi_2(\omega)$ in Eq.(4) does not contribute to the CR peak decay broadening, but it is essential when the shift $P = \mathrm{Re}\,\Pi(\omega_c+i0)$ is calculated. To evaluate $\Pi_2(\omega)$ "slow" and "fast" processes are to be distinguished. The latter are those processes where phonons with frequencies $\omega_{\vec{k}\alpha} \gg T$ participate. Therefore to find the "fast" processes contribution P_f to the CR peak shift Eq.(8) for $W_q(t)$ may be used. This gives

$$P_f = 2T \sum_{\vec{q}} l^2 q^2 |V_{\vec{q}}|^2 \omega_q^{-I} \exp(-w_q^o) \tilde{f}_q, \tag{I0}$$
$$\tilde{f}_q \simeq \text{v.p.} \sum_{\vec{k}\alpha} |\vec{q}\vec{A}_{\vec{k}\alpha}|^2 \left(\frac{\omega_{\vec{k}\alpha}}{\omega_c^2 - \omega_{\vec{k}\alpha}^2} + \frac{\Theta(\omega_{\vec{k}\alpha} - \bar{\omega})}{\omega_{\vec{k}\alpha}} \right) \simeq$$
$$\simeq q^2 \ln(\omega_c/\bar{\omega})/4\pi m c_t^2 N.$$

In $\tilde{f}_q$ the corrections $\sim \omega_c^2/\omega_p^2$ are omitted. It is seen from Eq.(I0) that P_f weakly depend on the frequency cutoff $\bar{\omega}$.

The "slow" processes contribution to $\Pi_2(\omega)$ depends on the ratio of the contributions of thermal and zero-point fluctuations to the electron displacement over the time $t_o = \max(\omega_s^{-I}, \Gamma^{-I}, P^{-I})$. This ratio is characterized by a parameter

$$r = 2 \sum_{\vec{k}\alpha} \Theta(\omega_{\vec{k}\alpha} t_o - I) |\vec{A}_{\vec{k}\alpha}|^2 \bar{n}_{\vec{k}\alpha} \times$$
$$\times \left(\sum_{\vec{k}\alpha} |\vec{A}_{\vec{k}\alpha}|^2 \right)^{-I} \simeq (T\omega_p / 2\pi c_t^2 N) \ln(T t_o) \tag{II}$$

(t_o determines the range of integration over time in $\Pi_2(\omega)$, the "slow" processes only having been taken into account).

If $T \gg \omega_s \gg \Gamma, P_f$ the most essential "slow" processes contributing to $\Pi_2(\omega)$ are the virtual processes of creation and annihilation of surface vibrations at fixed occupation numbers of the electron vibrations, and

$$P_{sl} = \sum_{\vec{q}} l^2 q^2 |V_{\vec{q}}|^2 \omega_q^{-I} \exp[-w_q^o] \tag{I2}$$

Correct expression is given in J. Phys. C 15, 7397 (1982)

It follows from the results of the present paper that the electron relaxation at $T \gg \omega_p^2/\omega_c$ is determined by a short-range order in the electron system. At higher densities ($\omega_p > \omega_c, T$) the correlation over large distance becomes essential (e.g. to obtain Eq.(9) the correlation radius was supposed to exceed $2\pi c_t/\omega_c$). Hence CR may be used to detect the 2D Wigner crystallization. It should be noted in the conclusion that our theory allows to describe quantitatively the experimental dependences[3] of the CR peak broadening and shift on concentration, magnetic field strength, temperature and coupling parameter for 2D electrons above the helium surface.

Acknowledgement - The author is grateful to M.A.Krivoglaz for stimulating discussions.

References

I. M.I.Dykman and L.S.Khazan, Journal eksperim.teor.phys. 77, I488 (I979).

2. M.I.Dykman, Physica nizkikh temperatur, 6, 560 (I980).

3. V.S.Edel'man, Journal eksperim. teor. phys. 77, 673 (I979).

4. C.C.Grimes and G.Adams, Phys. Rev. Lett. 42, 795 (I979).

5. Yu.E.Lozovik and V.I.Yudson, JETP, Pis'ma, 22, 26 (I975).

6. H.Fukuyama, Solid State Communications, I7, I323 (I975); I9, 55I (I976).

7. F.R.Ulinich and N.A.Usov, Journal eksperim. teor. phys. 76, 288 (I979).

8. A.V.Chaplik, Journal eksperim. teor. phys. 62, 746 (I972).

9. M.I.Dykman, phys. stat. sol.(b) 88, 463 (I978).

IO. D.N.Zubarev, Soviet Physics - Uspekhi, 3, 320 (I960).

II. Wigner crystal thermal fluctuations do not contribute to the decay broadening (9). They may be shown in general to influence weakly the damping of high-frequency ($\omega_{\vec{k}\alpha}>T$) modes at $\omega_p \gg T$. R.Mehrotra and A.J.Dahm(Phys. Rev. Lett., 43, 467 (I979)) obtained a different result (the case H=0 was considered) since they used the expansion of $\exp(i\vec{q}\vec{r}_n)$ in off-diagonal (in the occupation number representation) part of the operator $\vec{q}(\vec{r}_n-\vec{R}_n)$.