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## Toward the theory of cyclotron resonance of two-dimensional electrons on a liquid helium surface

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It is demonstrated that in describing quantum cyclotron resonance of two-dimensional electrons on a helium surface in the one-electron approximation, the interaction with ripplons cannot be considered weak. The cyclotron resonance peak in this approximation is formed because of transitions between Landau levels with generation or annihilation of large numbers of ripplons. The method of moments is used to calculate the peak half-width. General expressions for the cyclotron resonance peak half-width and shift, as obtained in Refs. 1, 2 in the one-electron approximation (for weak and strong coupling to oscillations of the medium), and in Ref. 3 in many electron theory (for the relatively weak concentration range), are compared. An explanation is offered for the qualitative peculiarities of the peak half-width and shift which appeared in the experiments of Ref. 4 at low concentration levels.

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Cyclotron resonance is now widely used to study the dynamics of electrons located in a thin surface layer of semiconductors or near the phase boundary between liquid and gaseous helium. The two-dimensional electron systems in crystals and on liquid helium surfaces differ markedly. On a crystal surface, the majority of experiments the electron gas is degenerate, and scattering is produced by impurities. On the other hand, on a liquid helium surface the electron gas is far from degeneration, and at sufficiently low temperatures ( $T < 1^\circ\text{K}$ ) scattering is produced by surface oscillations (ripplons). An interesting peculiarity of this system is the possibility of changing the ripplon interaction constant by varying the value of the electric field  $E_\perp$  which presses the electrons to the surface.<sup>5</sup> A dependence of relaxation parameters upon  $E_\perp$  was observed experimentally,<sup>4,6,7</sup> with Refs. 4 and 7 using cyclotron resonance.

In a transverse magnetic field the energy spectrum of the two-dimensional electrons is converted into a set of equidistant Landau levels. Single-electron scattering is then possible only on ripplons, the frequency of which  $\omega(q)$  is an integral multiple of the cyclotron frequency  $\omega_c = eH/mc$ . Inasmuch as to observe cyclotron resonance it is necessary that  $\omega_c \gg \Gamma$  (where  $\Gamma$  is the half-width of the cyclotron resonance peak), and in the system under consideration  $\Gamma > 10^8 \text{ sec}^{-1}$  (see Refs. 4-7), independent of the ratio of the quantity  $kT$  to  $\hbar\omega_c$  for  $T \leq 0.5^\circ\text{K}$ , single ripplon decays produce a small contribution to the peak width. In fact, it is evident from the explicit expression for the transition probability matrix elements<sup>1</sup>

that the characteristic momentum of a ripplon, upon which an electron with energy  $\epsilon \gg \hbar\omega_c$  may scatter, is limited by the same inequality as in the absence of a magnetic field:  $q \leq q_1 = \hbar^{-1} \sqrt{8me}$ , and if  $\epsilon \sim \hbar\omega_c$  then  $q \leq l^{-1}$ , where  $l = (\hbar/m\omega_c)^{1/2}$ . At  $T \leq 0.5^\circ\text{K}$  for capillary waves on a thick helium film  $\omega(q_{KT}) < 10^8 \text{ sec}^{-1}$ , and even for  $\hbar\omega_c \ll kT$  the chain of inequalities  $\omega(q_g) > \omega_c \gg \Gamma$  is fulfilled for only a small portion of the electrons, while for  $kT \leq \hbar\omega_c$  and  $\omega_c \gg 10^8 \text{ sec}^{-1}$  the ripplon frequency  $\omega(l^{-1}) \ll \omega_c$ , and the concept of single-ripplon scattering in the one-electron approximation is inapplicable.

A theoretical approach to the determination of the form of the cyclotron resonance peak depends significantly on the relationship of several characteristic system parameters:  $kT$ ,  $\hbar\omega_c$ ,  $\hbar\omega_g$ ,  $e^2N^{1/2}$ , and  $C$ , where  $\omega_g = \omega(l^{-1})$ ,  $N$  is the two-dimensional electron concentration, and  $C$  is the characteristic ripplon coupling constant. The most detailed experimental data presently available are contained in Refs. 4, 7, for  $T \approx 0.4^\circ\text{K}$ ,  $\omega_c \sim 10^{11} \text{ sec}^{-1}$  ( $\omega_g \sim 3 \cdot 10^8 \text{ sec}^{-1}$ ). It is thus of interest to study the form of the cyclotron resonance peak at

$$\hbar\omega_c \ll \hbar\omega_g, kT; \quad C \ll \hbar\omega_c \quad (1)$$

(the second inequality is necessary for observation of cyclotron resonance). Below we will consider the range of relatively low electron concentrations, in which  $\omega_p \ll \omega_c$  ( $\omega_p = \left(\frac{2\pi e^2}{m} N^{3/2}\right)^{1/2}$  the characteristic plasma frequency). However, it happens that even at such concentrations in-

terelectron interactions can lead to qualitatively new effects. one of which was apparently observed in Ref. 4.

# 1. ONE-ELECTRON THEORY OF THE FORM OF THE CYCLOTRON RESONANCE PEAK FOR WEAK AND STRONG COUPLING

The frequency dependence of the conductivity of two-dimensional electrons interacting with surface and volume oscillations was found for  $N \rightarrow 0$  in Refs. 1, 2, 8, and has been analyzed for semiconductors. It has been shown that when Eq. (1) is satisfied the form of the resonance peak is determined by the ratio of the interaction constant  $C$  and  $\hbar\omega_s$ . For  $C \ll \hbar\omega_s$  the peak appears as a narrow zero-phonon line, somewhat shifted relative to  $\omega_s$ . At a distance  $\sim \omega_s$  from this line wings are distributed, corresponding to optical transfers between Landau levels with creation or destruction of a phonon (ripplon) and with relative intensity  $\sim kTC^2/\hbar^3\omega_s^3$ . The basic contribution to the width of this zero-phonon line is produced by a term which is fourth order in  $C$ . It is produced not by electron scattering on riplons, but on the contrary, by ripplon scattering on electrons, and is defined by Eq. (10) of Ref. 8. We note that in the case of weak coupling an important contribution to the broadening of the cyclotron resonance peak because of such scattering can be produced by two-ripplon interaction,<sup>2,8</sup> the parameters of which were determined in Ref. 9.

However, for electrons on a helium surface weak coupling is not realized over a wide range of temperature and magnetic field, as will be shown below. If we write the single-ripplon interaction Hamiltonian in the

standard form

$$H_i = \sum_q V_q \exp(iqr) (b_q + b_{-q}^+), \quad (2)$$

where  $b_q, b_q^+$  are the ripplon creation and annihilation operators, using  $V_q^1$  from Ref. 5, the expression for the characteristic coupling constant  $C$  in a sufficiently strong clamping field  $E_{\perp}$  can be represented in the following form:

$$C = S^{1/2} q |V_q| \Big|_{q \rightarrow l^{-1}} = eE_{\perp} (\hbar/2\rho\omega_s l^2)^{1/2}, \quad (3)$$

$$l^2 = ch/eH, \quad \omega_s = \omega(q = l^{-1}).$$

Here  $\rho$  is the density of the liquid He and  $S$  is the area of the system. For  $E_{\perp} \approx 0.5$  cgs units [in weaker fields Eq. (3) is inapplicable] and for  $\omega_s \approx 10^{11} \text{ sec}^{-1}$ , we have  $C/\hbar \approx 1.5 \cdot 10^8 \text{ sec}^{-1}$ , whence  $C/\hbar\omega_s \approx 0.6 \sim 1$ .

A single-electron theory of the cyclotron resonance spectrum for intermediate values of  $C/\hbar\omega_s \gtrsim 1$  can be constructed due to the inequality  $kT \gg \hbar\omega_s$  specific to electrons on the helium surface in the region

$$\hbar\omega_c \gg C \sqrt{kT/\hbar\omega_s} \gg C, \quad \hbar\omega_s, \quad (4)$$

using the method of moments developed in Ref. 8. The method is based on separation from ripplon interaction operator of the "slow" (adiabatic) component, which does not produce transitions between Landau levels. The remaining component of the interaction can be considered by perturbation theory; it leads to a peak shift  $P_T \sim (C^2/\hbar^2\omega_s)(kT/\hbar\omega_c)$  (see Refs. 2, 8). The explicit form of

TABLE I. Expressions of Half-width  $\exp(\hbar\omega_c/kT) \gg 1$  and Shift  $P$  of the Cyclotron Resonance Peak for  $kT \gg \hbar\omega_s$

Model	$\Gamma$	$P$
One-electron theory (weak coupling)	$\frac{\pi}{2\hbar^2} \sum_q  V_q ^2 l^2 q^2 \times e^{-l^2 q^2/2} \delta(\omega_c - \omega_q) \approx 0$	$\sum_q \frac{ V_q ^2}{\hbar^2 \omega_q} l^2 q^2 \left(1 - \frac{1}{4} l^2 q^2\right) \times \exp\left(-\frac{1}{2} l^2 q^2\right) + P_T$ $\approx \frac{e^2 E_{\perp}^2}{8\pi\alpha\hbar} \left(1 + \frac{2kT}{\hbar\omega_c}\right)$
One-electron theory (strong coupling)	$\left[\frac{kT}{2\hbar^2} \sum_q \frac{ V_q ^2}{\omega_q} l^2 q^2 \times e^{-l^2 q^2/2}\right]^{1/2} \approx \frac{eE_{\perp}}{\hbar} (kT/4\pi\alpha)^{1/2}$	$\sum_q \frac{ V_q ^2}{\hbar^2 \omega_q} l^2 q^2 \exp\left(-\frac{1}{2} l^2 q^2\right) + P_T \approx \frac{e^2 E_{\perp}^2}{4\pi\alpha\hbar} \left(1 + \frac{kT}{\hbar\omega_c}\right)$
Many-electron theory (quasi-elastic scattering)	$\frac{kT\tau_c}{2\hbar^2} \sum_q \frac{ V_q ^2}{\omega_q} l^2 q^2 \times e^{-l^2 q^2/2} \approx \frac{e^2 E_{\perp}^2 kT\tau_c}{8\sqrt{2\pi}\alpha\hbar^2}$	$P_T \approx \frac{e^2 E_{\perp}^2}{4\pi\alpha\hbar} \frac{kT}{\hbar\omega_c}$
Shift due to virtual transitions between different Landau levels	$P_T = \frac{2kT}{\hbar^2 \omega_c} \sum_q \frac{ V_q ^2}{\omega_q} \exp\left(-\frac{1}{2} l^2 q^2\right) \left\{ \frac{3}{2} \left(\frac{1}{2} l^2 q^2\right)^2 - 2 \sum_{m=1}^{\infty} \left(\frac{1}{2} l^2 q^2\right)^m [(m-1)(m-2)m]^{-1} \right\}$ $\approx \frac{e^2 E_{\perp}^2}{4\pi\alpha\hbar} \frac{kT}{\hbar\omega_c}$	

the expression for  $P_T$  for  $\exp(\hbar\omega_c/kT) \gg 1$  is presented in the last column of Table I, for which the electron concentration  $N$  is assumed low,  $N \ll (m\omega_c^2/2\pi e^2)^{1/2}$ , and the electron-rippion interaction is taken in the form of Ref. 5. The explicit form of the integrals is presented for the case of strong pressing fields  $E_\perp$ ;  $\alpha$  is the surface tension coefficient.

In order to separate the adiabatic (diagonal) component  $M_0(q)$  of the operator  $\exp(iqr)$ , it is convenient to express the two-dimensional vector  $r = (x, y)$  in terms of the operators  $p_\alpha$ :

$$r_\alpha = -i\alpha l \sqrt{2} p_\alpha + R_\alpha, \quad r_\alpha = x - iay, \\ R_\alpha = X - i\alpha Y, \quad \alpha = \pm 1, \quad (5)$$

where

$$p_\alpha = \frac{i}{\sqrt{2}} \left[ -i \frac{\partial}{\partial x} + \frac{e}{\hbar c} A_x(r) - i\alpha \left( -i \frac{\partial}{\partial y} + \frac{e}{\hbar c} A_y(r) \right) \right], \quad \alpha = \pm 1, \\ [p_{-1}, p_1] = 1, \quad [p_\alpha, R_\alpha] = 0. \quad (6)$$

Here  $A(r)$  is the vector potential of the transverse magnetic field,  $H = (\text{curl} A)_z$ . The commutation of the operators  $R_\alpha$  and  $p_\alpha$  follows from their definition, Eq. (5).

Since the operator  $p_1(p_{-1})$  acting on an electron wave function increases (decreases) the number of the Landau level by unity, it follows from Eq. (6) that the operator  $R_\alpha$  is diagonal in level number. Substituting  $r$  from Eq. (5) in  $\exp(iqr)$ , we obtain with consideration of Eq. (6)

$$M_0(q) = \langle \exp(iqr) \rangle_d \\ = \exp(-1/4 l^2 q^2) \exp(iqR) \sum_{n=0}^{\infty} \frac{(-1)^n}{(n!)^2} \left( \frac{1}{2} l^2 q^2 \right)^n p_1^n p_{-1}^n. \quad (7)$$

In the frequency range  $\omega \sim \omega_c$  the form of the two-dimensional electron absorption peak is defined by the function (see Refs. 1, 2, 8)

$$Q(\omega) = [\bar{n}(\omega) + 1]^{-1} \text{Re} \int_0^\infty dt e^{i\omega t} \langle p_{-1}(t) p_1(0) \rangle, \\ \bar{n}(\omega) = \langle \exp(\hbar\omega/kT) - 1 \rangle^{-1}, \quad (8)$$

where the angular brackets denote statistical averaging. On considering only the slow component of  $H_1$  the second moment of the distribution of Eq. (8) is equal to

$$\gamma^2 = \int_{-\infty}^{\infty} d\omega Q(\omega) (\omega - \omega_c)^2 \left[ \int_{-\infty}^{\infty} d\omega Q(\omega) \right]^{-1} \\ \approx 2kT \sum_q \frac{|V_q|^2}{\hbar^2 \omega_q} \langle [p_{-1}, M_0(q)], [p_1, M_0(-q)] \rangle \\ = \frac{2kT}{\hbar^2} [\bar{n}(\omega_c) + 1]^{-2} \sum_q \frac{|V_q|^2}{\omega_q} e^{-1/2 l^2 q^2} \\ \times \sum_{m=0}^{\infty} e^{-\hbar m \omega_c / kT} (m+1) \left[ \sum_{n=1}^{m+1} \binom{m}{n-1} \frac{(-1)^n}{n!} \left( \frac{1}{2} l^2 q^2 \right)^n \right]^2. \quad (9)$$

The quantity  $\gamma$  to an accuracy of a factor of  $\sim 1$  is equal to the width of the cyclotron resonance peak. From analysis of higher moments of  $Q(\omega)$  it is evident that the form of the peak is close to Gaussian; the asymmetry is proportional to  $(C/\hbar\omega_c) (kT/\hbar\omega_c)^{-1/2} \ll 1$ . In fact, the distribution  $Q(\omega)$  upon fulfillment of Eq. (4) represents the envelope of a large number of absorption lines, corresponding to transition of an electron to the next higher Landau level with simultaneous generation or annihilation of several ripples. The characteristic number of ripples participating in peak formation is equal to  $\gamma/\hbar\omega_c \sim (C/\hbar\omega_c) \cdot (kT/\hbar\omega_c)^{1/2} \gg 1$ . Thus, even for  $C/\hbar\omega_c \sim 1$  strong coupling

is realized for two-dimensional electrons on a helium surface in the one-electron approximation.

Explicit expressions for the characteristic half-width and shift of the cyclotron resonance peak on fulfillment of Eq. (4) and for  $\exp(\hbar\omega_c/kT) \gg 1$  are presented in the second column of Table I. For comparison, the first column presents the expression for the peak shift in the weak coupling case (which, however, is very difficult to realize in practice). The first term in this expression coincides with the results of Refs. 10, 11, which were actually obtained with the assumption that interaction was weak.

## 2. MANY-ELECTRON EFFECTS AT LOW CONCENTRATIONS

In Ref. 3 it was shown that consideration of interaction of nondegenerate two-dimensional electrons can qualitatively change the character of their scattering even at  $\omega_p \ll \omega_c$ . The cause of this is drift in the electric field, produced by fluctuations in the electron concentration. In order of magnitude, for  $e^2 N^{3/4} \ll kT < \hbar\omega$ , such a field is equal to  $N^{3/4} (kT)^{1/2}$ . The characteristic electron drift velocity  $cN^{3/4} (kT)^{1/2} / H$  proves to be significantly higher than the ripplon phase velocity  $l\omega_s$  over a wide range of magnetic field, concentration, and temperature ( $kT \ll \hbar\omega_s$ ). Therefore the electron emits or absorbs a Cerenkov ripplon, i.e., normal quasielastic scattering occurs. The corresponding expressions for the half-width and peak shift are given in column three of Table I. The parameter  $\tau_e$  defines the characteristic duration of the scattering act (the "transit time" for a magnetic length  $l$  due to drift) and in order of magnitude is

$$\tau_e \approx \hbar [e l N^{3/4} (kT)^{1/2}]^{-1}, \quad (kT)^{1/2} \gg (\hbar\omega_p^2/\omega_c)^{1/2}, \\ \omega_p^2 = 2\pi e^2 N^{3/2} / m. \quad (10)$$

Together with Eq. (10), a condition for applicability of the theory of Ref. 3 is the inequality  $\Gamma_{m,el} \tau_e \ll 1$  (where  $\Gamma_{m,el}$  is the peak half-width in the many-electron theory). This means that the duration of the scattering act is a small value in comparison to the reciprocal to the scattering probability. Since  $\Gamma_{m,el} \sim \Gamma_{sb}^2 \tau_e$  (where  $\Gamma_{sb}$  is the peak half-width in the strong coupling model), the theory of Ref. 3 is applicable only in the case where  $\Gamma_{m,el} \ll \Gamma_{sb}$ . From Table I (see also Ref. 8) it is evident that  $\Gamma_{sb} \propto E_\perp$  in strong pressing fields; for a concentration independent of  $E_\perp$ ,  $\Gamma_{m,el} \propto E_\perp^2$ , while for a maximally charged surface ( $E_\perp = 2\pi eN$ ) with consideration of Eq. (10)  $\Gamma_{m,el} \propto E_\perp^{5/4}$ . Therefore, in sufficiently strong fields the inequality  $\Gamma_{m,el} \ll \Gamma_{sb}$  is destroyed, and the form of the cyclotron resonance peak becomes significantly non-Lorentzian and is described by strong coupling theory. Since  $\tau_e$  increases with increase in  $N$ , in the region of very small  $N$ , as would be expected, one-electron theory is also applicable ( $\Gamma_{sb} \rightarrow \text{const}$  for  $E_\perp \rightarrow 0$ ).

We note that upon transition with change in  $E_\perp$  to the region of applicability of the strong coupling theory, as is evident from Table I, not only the half-width, but the expression for the shift of the peak change significantly. In order to reach the region of applicability of strong coupling theory for high  $E_\perp$  in experiment, it is necessary to increase  $E_\perp$  with a sufficiently low fixed electron concentration  $N$ . In the region of high concentrations,

where  $\omega_p > \omega_c$ , and consequently, the criterion of Eq. (10) is not fulfilled, the mechanism for cyclotron resonance peak broadening is significantly multi-electronic (and not one-electronic, as in strong coupling theory). However, the expressions for the half-width and shift in this region differ significantly from the results of Ref. 3 presented in the third column of Table I, while, as will be demonstrated in a separate study,  $\Gamma \sim TE_1^2/\alpha\omega_c$ , which corresponds to the experimental data of Ref. 4.

Under the experimental conditions of Ref. 4 ( $E_1 = 2\pi eN$ ;  $T \approx 0.4^\circ\text{K}$ ;  $\omega_c \approx 1.2 \cdot 10^{11} \text{ sec}^{-1}$ ) the most severe limitation on the region of applicability of the many-electron theory of Ref. 3 is imposed by inequality (10):  $E_1 \ll (kT/l^2\sqrt{e})^{2/3} \approx 0.8 \text{ cgs units}$ . Analysis reveals that it must be satisfied with a large reserve. In corresponding pressing fields it is necessary to consider the polarization interaction with ripplons (change in image forces upon surface curvature). If we use the approximation of Ref. 5, then

$$\Gamma_{m,e1} \approx \frac{kT\tau_e e^2}{8\pi\hbar^2\sqrt{2\pi}} \left\{ E_1^2 + E_1 \frac{\hbar\omega_c\gamma_0}{2e} [6(\ln(4\gamma_0 l) - 1) - 4.2] + \left( \frac{\hbar\omega_c\gamma_0}{2e} \right)^2 [15(\ln(4\gamma_0 l) - 1)^2 - 27(\ln(4\gamma_0 l) - 1) + 13] \right\}, \quad (11)$$

where  $\alpha$  is the surface tension coefficient of the helium;  $\gamma_0$  is the reciprocal of the electron localization length in the direction perpendicular to the surface, and  $\gamma_0 \approx 1.3 \cdot 10^6 \text{ cm}^{-1}$ .

Inasmuch as according to Eq. (10),  $\tau_e \sim N^{-3/4} \sim E_1^{-3/4}$ , from Eq. (11) there follows the qualitative result:  $\Gamma_{m,e1}$  as a function of  $E_1$  has a minimum, and for  $E_1 < E_{1\text{min}}$  the half-width  $\Gamma_{m,e1}$  increases with decrease in  $E_1$ . A similar minimum and increase in  $\Gamma_{m,e1}(E_1)$  was actually observed in Ref. 4. Calculation with Eqs. (10), (11) reveals that the position of the minimum depends on the magnetic field, and  $E_{1\text{min}} \approx 0.4 \text{ cgs units}$  for  $\omega_c \approx 1.2 \cdot 10^{11} \text{ sec}^{-1}$ . However such a field evidently corresponds to a concentration at which the theory of Ref. 3 is still inapplicable, and in actuality the minimum should be ob-

served in weaker fields. In experiment the minimum was observed at  $E_1 \sim 0.2 \text{ cgs units}$ . Estimates of  $\tau_e$  based on the data of Ref. 4 and use of Eq. (11) coincide with Eq. (10). Equations (10), (11) properly describe the character of the  $E_1$  dependence of  $\Gamma$  at low  $E_1$ . The field dependence of the cyclotron resonance peak observed in Ref. 4 agrees within the limits of experimental error for low  $E_1$  with the results of Ref. 3, as presented in Table I.

Thus, the general theory of cyclotron resonance of two-dimensional electrons at low concentration levels developed in Refs. 1, 2, 3, and 8 when applied to two-dimensional electrons on a helium surface, explains the qualitative features observed in experiment, and also permits a description of the dependence of the form of the cyclotron resonance peak on electron concentration, pressing field, and temperature.

## NOTATION

Here  $\omega_c$  is the cyclotron resonance frequency;  $\Gamma$ , cyclotron resonance peak width;  $P$ , CR peak shift;  $N$ , electron concentration;  $l$ , quantum magnetic length;  $\omega_s$ , characteristic frequency of ripplons coupled to electrons;  $C$ , coupling parameter;  $E_1$ , electric field clamping electrons to helium surface;  $\alpha$ , helium surface tension.

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