

Dichroic optical bistability in a nonlinear Fabry-Perot interferometer

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It is shown that a new type of optical bistability may occur in a Fabry-Perot resonator containing a nongyrotropic cubic crystal. This instability is due to an anisotropy of the nonlinear resonance absorption in the crystal. The explicit conditions for the establishment of dichroic optical bistability are determined for cases of two-photon absorption and resonance absorption by reorientable impurity centers.

One of the important optical properties of cubic crystals is an anisotropy of the nonlinear absorption. As a result of self-induced dichroism, the direction of polarization of high-intensity resonance radiation is rotated during its propagation in the crystal.¹ It is clear from the symmetry concepts that for radiation propagating parallel to the third- or fourth-order symmetry axis in a nongyrotropic crystal, there are several limiting directions to which the direction of polarization can be rotated (depending on the orientation on entry). The directions dividing the ranges of "attraction" to different limiting directions are unstable in the case of polarization: small deviations of the direction of polarization from these directions increase with the crystal thickness.¹ In particular, it is clear that for radiation propagating parallel to the C_4 axis (or S_4), the C_{2n} axes and the planes of symmetry either correspond to limiting or unstable orientations of the direction of polarization.

It was shown in Ref. 2 that symmetric unstable directions of polarization observed when a crystal is inserted in a ring resonator may result in a new type of optical bistability—dichroic optical bistability (DOB). In this case, in a certain range of angles between the direction of polarization of the incident radiation and the unstable direction of polarization, the field at the exit from the resonator has three orientations and the intensity has three values (one of these states being unstable).

In experimental studies of optical bistability, Fabry-Perot resonators are used much more widely than ring resonators (see, for example, Ref. 3). The present paper reports an investigation of the conditions for the establishment of DOB in a nonlinear Fabry-Perot interferometer. Bearing in mind the complex field structure (as a result of absorption in a crystal) in the interferometer, an analysis is made of DOB for two specific mechanisms of nonlinear resonance absorption.

1. CRITERION FOR THE ESTABLISHMENT OF DICHROIC OPTICAL BISTABILITY IN A FABRY-PEROT RESONATOR

In physical terms, DOB is established when, in the presence of feedback, the fluctuation-induced deviations of the direction of polarization of the incident radiation from the unstable direction of polarization may increase with time, so that under steady-state conditions, the direction of polarization at the exit deviates by a finite angle in a specific direction. The instability criterion is deduced directly from the equation for the field $E^{(r)}(0)$ incident in the resonator on the front (specifically, the left-hand) face of the crystal

$$E^{(r)}(0) = \vec{E} + R^{(l)} \exp(i\varphi^{(l)}) E^{(l)}(0), \quad (1)$$

where \vec{E} is the external field incident in the resonator; $E^{(l)}(0)$ is the radiation field emerging from the front face of the crystal toward the left; $R^{(l)}$ is the coefficient of reflection of the left-hand mirror (we shall neglect reflection from the crystal faces).

The field $E^{(l)}$ is formed as a result of propagation of the radiation through the crystal and reflection from the rear (right-hand) mirror. It is therefore found that $E^{(l)} \equiv E^{(l)}[E^{(r)}(0)]$. The condition for amplification of the fluctuations of the orientation of the direction of polarization at the front face of the crystal in the most favorable case, when $E^{(r)}(0)$ [and \vec{E}] are polarized parallel to the symmetric unstable direction X , takes the form

$$R^{(l)} |\partial E_y^{(l)}(0) / \partial E_y^{(r)}(0)| > 1 \quad (|E_y^{(r)}(0)| \rightarrow 0). \quad (2)$$

Using the symmetry properties (see Ref. 2), it can be shown that if condition (2) is satisfied and exact resonance is achieved

$$\varphi^{(l)} + \arg E^{(l)}(0) - \arg E^{(r)}(0) = 2\pi n, \quad (3)$$

DOB is established for linearly polarized radiation in a Fabry-Perot resonator. The critical field $\mathcal{E}_c \equiv \mathcal{E}_b$ for which DOB is established is determined from Eq. (1) and from the condition

$$R^{(l)} |\partial E_y^{(l)}(0) / \partial E_y^{(r)}(0)| = 1 \quad (|E_y^{(r)}(0)| \rightarrow 0). \quad (4)$$

As for DOB in a ring resonator, the dependence of the field intensity \mathcal{E}_b at which optical bistability occurs on the angle Ψ between the direction of polarization of the incident radiation and the unstable directions of polarization near the threshold takes the form $\mathcal{E}_b(\Psi) - \mathcal{E}_b(0) \propto \Psi^{2/3}$. The Ψ dependence of the orientation of the direction of polarization of the output radiation is S-shaped in the DOB range and for $\mathcal{E} = \mathcal{E}_c$ it has an inflection at the point $\Psi = 0$.

The relationship between $E^{(l)}(0)$ and $E^{(r)}(0)$ is determined by solving the Maxwell equations for an optically nonlinear crystal. Assuming that the resonant nonlinear polarization $P(z) \exp(-i\omega t) + \text{c.c.}$ is small, the equations for the envelopes $E^{(r,l)}(z)$ of the waves propagating to the right and the left can be expressed in the form

$$\frac{dE^{(r)}}{dz} = i\xi \langle P(z) e^{-ikz} \rangle, \quad \frac{dE^{(l)}}{dz} = -i\xi \langle P(z) e^{ikz} \rangle, \quad (5)$$

$\xi = 2\pi\omega/\text{nc}$, $k = \omega n/c$ ($E_0 = E e^{-i\omega t} + \text{c.c.}$; $E = E^{(r)} e^{ikz} + E^{(l)} e^{-ikz}$). Here, the angular brackets denote averaging over the rapidly varying phase $\varphi_z = kz$; E_0 is the total field in the crystal. At the exit (right-hand) crystal face ($z = d$), we have

$$E^{(l)}(d) = R^{(r)} \exp(i\varphi^{(r)}) E^{(r)}(d) \quad (6)$$

($R^{(r)}$)² is the coefficient of reflection of the right-hand mirror). As a result of the criterion for the establishment of DOB (4), we can solve the system of equations (5) confining ourselves to the case of small deviations of the direction of polarization from the symmetric directions, and we can linearize Eq. (5) with respect to these deviations (with respect to E_y). In view of the symmetry, we find $P_y \propto E_y$, so that $E_y^{(l)} \propto E_y^{(r)}$ and from Eqs. (5) and (6) for linearly polarized radiation, a system of three equations can be derived for $|E_x^{(r,l)}(z)|$ and $\mu \equiv \mu(z) = |E_y^{(l)}(z)/E_y^{(r)}(z)|$.

2. ANALYSIS OF DICHROIC OPTICAL BISTABILITY FOR TWO-PHOTON ABSORPTION AND ONE-PHOTON ABSORPTION BY REORIENTABLE IMPURITY CENTERS

We shall first analyze DOB for resonance two-photon absorption

$$i\epsilon P_x = -\alpha E_x - (\gamma_1 E_x |E|^2 + \gamma_2 E_x^* E^2 + \gamma_3 E_x |E_x|^2),$$

$$x = x, y, \quad (7a)$$

$\alpha > 0$, $\gamma_1 + 1/2\gamma_3 > 0$, $\gamma_1 + \gamma_3 + 1/2\gamma_3 > 0$, $\gamma_1 + \gamma_2 + \gamma_3 > 0$ (7b) (in this case, the X , Y , and Z axes are directed along the $\langle 100 \rangle$ axes; the inequalities correspond to the absence of amplification in the crystal). For $\gamma_3 > 0$ the X and Y axes are unstable directions of polarization (the case $\gamma_3 < 0$ is reduced to the case $\gamma_3 > 0$ by rotating through $\pi/4$, see Ref. 2).

If $|E_y| \ll |E_x|$, for linearly polarized radiation we obtain from Eqs. (5)–(7)

$$\frac{dv_r}{dz} = -v_r(1 + v_r^2 + 2v_l^2), \quad \frac{dv_l}{dz} = v_l(1 + 2v_r^2 + v_l^2), \quad (8a)$$

$$\frac{d\mu}{dz} = \frac{\gamma_1 + 2\gamma_3}{\gamma_1 + \gamma_3 + \gamma_3} v_r v_l (1 + \mu^2) + \mu \left[2 + \frac{2\gamma_1 + \gamma_3}{\gamma_1 + \gamma_3 + \gamma_3} (v_r^2 + v_l^2) \right], \quad (8b)$$

$$v_{r,l} = \alpha^{-1/2} (\gamma_1 + \gamma_2 + \gamma_3)^{1/2} |E_x^{(r,l)}|, \quad \mu = |E_y^{(l)}|/|E_y^{(r)}|, \quad z' = \alpha z, \quad (8c)$$

$$v_l(d') = R^{(r)} v_r(d'), \quad \mu(d') = R^{(r)}. \quad (8d)$$

Equations (1) and (8) together with the DOB criterion (4) [$R^{(l)}\mu(0) = 1$] form a closed system that can be used to find the critical field \mathcal{E}_c . In some limiting cases, this system can be solved analytically and the explicit dependence on the resonator and crystal parameters can be determined. In particular, for a high- Q resonator and weak linear absorption in the crystal, we find

$$\mathcal{E}_c = \frac{\gamma_3}{(-\gamma_1 - \gamma_3)^{1/2}} (6d)^{-1/2} [1 - R^{(l)}R^{(r)} + 2\alpha d]^{1/2} \times (1 - R^{(l)}R^{(r)} \ll 1; \alpha d \ll 1). \quad (9)$$

In this case, DOB is only found for $\gamma_1 + \gamma_2 < 0$. It is significant that the dependence of the field \mathcal{E}_c on the "weak-field" optical thickness αd is nonmonotonic: \mathcal{E}_c has a minimum ($\mathcal{E}_c = \mathcal{E}_c^{\min}$) for $\alpha d = (1 - R^{(l)}R^{(r)})/4$; for $2\alpha d \gg 1 - R^{(l)}R^{(r)}$, the field is $\mathcal{E}_c \propto \alpha d$.

In the case of strong linear absorption, we find

$$\mathcal{E}_c = [\alpha/(\gamma_1 + \gamma_2 + \gamma_3)]^{1/2} \left(\frac{3\gamma_3}{\gamma_1 + 2\gamma_3 + 3\gamma_3} R^{(r)}R^{(l)} \right)^{-1/2} \times \exp(2\nu\alpha d), \quad (10a)$$

$$\nu = -(2\gamma_1 + \gamma_2)/(\gamma_1 + \gamma_2 + \gamma_3) > 0; \exp(2\alpha d) \gg 1. \quad (10b)$$

Dichroic optical bistability is established for $2\gamma_1 + \gamma_2 < 0$;

\mathcal{E}_c increases exponentially with increasing optical thickness of the crystal [when deriving the system (10), Eq. (8) for $\mu(z')$ was linearized taking into account the condition $\exp(2\alpha d) \gg 1$ and it was assumed that $1 + v_r^2 \gg v_l^2$].

It can be seen from Eqs. (9) and (10) that, depending on the optical thickness of the crystal, DOB occurs in the Fabry–Perot resonator subject to various relationships between the two-photon absorption coefficients $\gamma_{1,2,3}$ [unlike the case of a ring resonator, where the condition $\gamma_1 + \gamma_2 < 0$ must be satisfied for DOB (Ref. 2)]. The dependence of \mathcal{E}_c on the thickness obtained by a numerical solution of Eq. (8) for $\gamma_1 = 0$, $\gamma_2 = -\gamma_3/2$ (this relationship is satisfied for example, for two-photon absorption due to the $A_{1g} \rightarrow T_{1u}$ transition with T_d symmetry, see Ref. 4) is shown in Fig. 1. It can be seen that the most favorable conditions for DOB are found in a high- Q resonator with an optimal optical thickness of the crystal, the nonlinear absorption in this case being appreciably weaker than the linear absorption [it follows from Eq. (9) that $\gamma \mathcal{E}_c^{\min 2} \propto \alpha(1 - R^{(l)}R^{(r)})^2 \ll \alpha$].

An important class of systems in which a strong anisotropy of the resonance absorption is observed in comparatively weak fields comprise crystals with reorientable centers.¹ In particular, in KCl crystals containing F_A (Li) centers, self-induced rotation of the direction of polarization of resonance radiation by some tens of degrees is observed in non-laser fields.⁵ This system is therefore of particular interest from the point of view of obtaining DOB.

Using the model of an F_A (Li) center,^{6,5} the resonance polarization of a doped crystal may be expressed in the form

$$i\epsilon P_x = -(\alpha + \beta) E_x - 1/2\beta(1 - \chi) [2\chi E_x |E|^2 - (1 - \chi) E_x^* E^2 + 2(1 - \chi) E_x |E_x|^2] |E|^2 [\chi(2 + \chi) |E|^4 + 1/4(1 - \chi)^2 |E_x^2 - E_y^2|^2]^{-1}, \quad (11a)$$

$$x = x, y. \quad (11b)$$

Here, the X and Y axes are directed parallel to the $\langle 110 \rangle$ axes (light propagates parallel to the $[001]$ axis); α is the coefficient of absorption of the main crystal; the parameter β is proportional to the concentration of the centers, χ is the ratio of the absorption cross sections for light polarized perpendicular and parallel to the axis of the center; the F_A (Li) centers are oriented parallel to the $\langle 100 \rangle$ axes.⁶ The X and Y axes are unstable directions of polarizations.

It can be seen from Eq. (11) that $P_x \propto E_x$ is found for $|E_y| \ll |E_x|$, so that Eqs. (5), (6), and (11) for the strong field component E_x can be solved directly, and after some

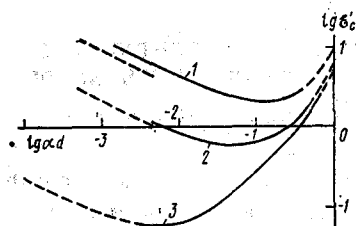


FIG. 1. Dependences of the critical field $\mathcal{E}_c' = (\gamma/2\alpha)^{1/2} \mathcal{E}_c$ on the reduced crystal thickness αd for two-photon absorption ($\gamma_1 = 0$, $\gamma_2 = -\gamma/2$, $\gamma_3 = \gamma$) in a symmetric resonator with $R^{(l)} = R^{(r)} = R = 0.7$ (1), 0.9 (2), and 0.99 (3). The dashed curves show the asymptotic forms of Eqs. (9) and (10) for small d' , $1 - R^2$, and large d' [curve 3 is almost completely described by Eqs. (9) and (10)].

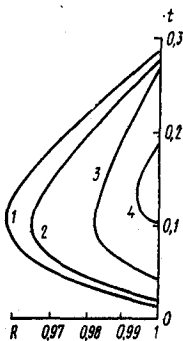


FIG. 2. Dependences of the effective crystal transmission $t = \exp(-\alpha_s d)$ for which DOB is observed on the coefficient of reflection and the mirrors $R = R^{(l)} = R^{(r)}$ for KCl crystals containing F_A (Li) centers at the frequency of an He-Ne laser ($\chi = 0.04$) for $\alpha/\beta = 0$ (1), 0.0015 (2), 0.005 (3), and 0.01 (4).

cumbersome averaging with respect to $\exp(\pm ikz)$ in the system of equations (5) for E_y , the definition of the critical parameters of the system for DOB is reduced to the solution of the equation

$$\frac{d\mu}{dz} = -\lambda p_r (1 + \mu^2) + \mu [2\alpha\alpha_s^{-1} + 12\lambda\chi(1-\chi)^{-2} + \lambda p_r^2],$$

$$z'' = \alpha_s z, \quad (12a)$$

$$\alpha_s = 2\alpha + 6\beta(1+\chi)/(1+5\chi), \quad \lambda = 2\beta(1-\chi)^2/\alpha_s(1+\chi)(1+5\chi), \quad (12b)$$

$$p_r = p_r(z'') = R^{(r)} \exp(z'' - d''), \quad (12c)$$

$$\mu(d'') = R^{(r)}, \quad \mu(0) = 1/R^{(l)} \quad (12d)$$

(α_s is the reciprocal of the absorption length of the component E_x).

It is significant that Eq. (12) does not contain the amplitude of the strong field and therefore only determines the relationship between the resonator and crystal parameters for which DOB may be observed. Dichroic optical bistability has no threshold with respect to the field; the field intensity at liquid nitrogen temperature and below determines only the kinetics of the reorientation of F_A (Li) centers in KCl (Ref. 7).

It is readily shown that since $\mu(0) > 1 > \mu(d'')$, a solution of Eq. (12) can only exist if

$$k_0 = 2\beta(1 - 14\chi + \chi^2)[(1+\chi)(1+5\chi)]^{-1} - 2\alpha > 0 \quad (13)$$

[for real values of $\chi = 0.04$ (Ref. 7) and $\chi \gg 10^2$ corresponding to the maxima of the red and green bands of the F_A (Li)

center, the condition (13) is satisfied in the absence of absorption by the main crystal].

If the coefficients of reflection of the mirrors are high, in accordance with Eq. (12), DOB occurs if the crystal thickness is $d > k_0^{-1}(1 - R^{(l)}R^{(r)})$. It is significant that DOB is only found for fairly large $R^{(l)}$, $R^{(r)}$ and is observed in a range of thicknesses having upper and lower limits (the upper limit on the crystal thickness is specific to a Fabry-Perot resonator and is due to absorption of the component $E_y^{(l)}$ for large thicknesses; for a ring resonator there is no such limit). For a symmetric resonator ($R^{(l)} = R^{(r)} = R$) the dependence of the range of thicknesses d on R for $\chi = 0.04$ and various ratios β/α is shown in Fig. 2. It can be seen that the range of existence of DOB both with respect to R and the crystal thickness, is reduced greatly by an increase in the nonresonance absorption coefficient α .

CONCLUSIONS

It is deduced from these results that anisotropic nonlinear absorption in cubic crystals may result in a new type of optical bistability in a Fabry-Perot resonator, which is dichroic optical bistability. This bistability is observed in the intensity and in the orientation of the direction of polarization of the output radiation, and the characteristic orientation dependence can be used to identify uniquely this type of optical bistability.

There are various mechanisms of optical nonlinearity resulting in DOB. The simple mechanisms studied—two-photon absorption and light-induced reorientation of impurity centers—typically have a low (almost zero for the specific centers analyzed) intensity threshold. The optimal range of crystal and interferometer parameters ensuring the most favorable conditions for the observation of DOB is indicated.

¹M. I. Dykman and G. G. Tarasov, Zh. Eksp. Teor. Fiz. 72, 2246 (1977); 74, 1061 (1978) [Sov. Phys. JETP 45, 1181 (1977); 47, 557 (1978)].

²M. I. Dykman and G. G. Tarasov, Zh. Eksp. Teor. Fiz. 86, 2030 (1984) [Sov. Phys. JETP 59, 1180 (1984)].

³E. Abraham and S. D. Smith, Rep. Prog. Phys. 45, 815 (1982).

⁴M. Inoue and Y. Toyozawa, J. Phys. Soc. Jpn. 20, 363 (1965).

⁵M. Ya. Valakh, M. I. Dykman, M. P. Lisitsa, G. Yu. Rudko, and G. G. Tarasov, Solid State Commun. 30, 133 (1979).

⁶F. Luty, in: *Physics of Color Centers* (ed. by W. B. Fowler), Academic Press, New York (1968), pp. 182-242.

⁷M. Ya. Valakh, M. I. Dykman, M. P. Lisitsa, E. P. Pidlisnyĭ, G. Yu. Rud'ko, and G. G. Tarasov, Fiz. Tverd. Tela (Leningrad) 23, 418 (1981) [Sov. Phys. Solid State 23, 236 (1981)].

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