Theory of the line profile of the cyclotron resonance of nondegenerate two-dimensional electrons interacting with phonons

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The energy spectrum of two-dimensional electrons (holes) in a transverse quantizing magnetic field represents a set of discrete Landau levels. The line profile of the cyclotron resonance in such a system is studied in the absence of degeneracy when the broadening is small and is due to the inelastic scattering by phonons. In addition to the broadening, the nonequidistant distribution of the Landau levels is also considered. If the deviation from an equidistant distribution, which is due to a nonparabolic dispersion and due to a renormalization of the electron spectrum caused by the electron–phonon interaction, is much larger than the reciprocal of the relaxation time, the cyclotron resonance peak may exhibit a fine structure in a well-defined range of temperatures. The individual lines of such a structure correspond to the transitions between different neighboring levels. In general, when the ratio of the deviation from an equidistant distribution to the line width is arbitrary, the cyclotron resonance peak has a complex asymmetric profile which can be obtained by solving a system of microscopically derived linear equations.

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Size quantization of the motion of carriers (for example, electrons) can take place in thin films[1,2] or thin surface layers in semiconductors.[3,4] The separation of the neighboring two-dimensional subbands in such systems may be quite large ($\gtrsim 0.1$ eV). Therefore, at low temperatures and for moderate carrier densities, only the lowest subband is occupied, all the other bands can be neglected, and it is sufficient to consider a two-dimensional gas of carriers.

The energy spectrum of a two-dimensional gas in a transverse magnetic field transforms to a set of discrete Landau levels. The properties of the system under study depend strongly on the ratio of the cyclotron frequency ω_c to the electron relaxation time τ. We shall consider the cyclotron resonance in a two-dimensional gas with a weak damping $\omega_c\tau \gg 1$ when the discreteness of the Landau levels manifests itself most clearly and the cyclotron resonance peak is narrow.

The cyclotron resonance in two-dimensional systems was first observed in ref. 5 (for electrons in the vicinity of a gas–liquid helium boundary) and in refs. 6-8 for inversion p-type Si layers. A theoretical discussion of this effect based on a model in which electrons are scattered elastically from impurities was given in ref. 9. However, such a model cannot describe the energy relaxation of two-dimensional electrons.

We shall study the spectrum of the cyclotron resonance of nondegenerate two-dimensional electrons interacting with phonons. Since the electron energy levels are discrete, the scattering of phonons is strongly inelastic. When an electron undergoes a transition between levels accompanied by the emission or absorption of a phonon, the energy is conserved since a part of the phonon momentum is absorbed by the "barriers" containing two-dimensional electrons. Such a scattering differs qualitatively from the standard three-dimensional inelastic scattering by optical phonons. We shall assume that ω_c and its higher harmonics do not correspond to singularities in the phonon density of states.

In the investigation of the cyclotron resonance it is necessary to consider not only a finite width of the Landau levels but also their nonequidistant distribution. The nonequidistant distribution is due to a nonparabolic dispersion of electrons and also due to a weak electron–phonon interaction. The contribution of the electron–phonon interaction is related to unequal shifts of different Landau levels (a two-dimensional weakly coupled magnetic polaron).

The cyclotron resonance spectrum at temperatures comparable with $\hbar\omega_c$ (when several Landau levels are occupied by electrons) depends strongly on the ratio of the deviation from an equidistant distribution of the Landau levels $\Delta\omega_c$ to their width τ^{-1} ($\Delta\omega_c$, $\tau^{-1} \ll \omega_c$). For $\Delta\omega_c \gg \tau^{-1}$, the spectrum in the neighborhood of ω_c represents a set of fine-structure lines. Such lines correspond to the transitions between neighboring Landau levels. The separation of the levels is $\sim \Delta\omega_c$ and their width $\sim \tau^{-1}$. When $\Delta\omega_c/\tau^{-1}$ decreases, the lines begin to overlap and, for $\Delta\omega_c \ll \tau^{-1}$, the spectrum transforms into a continuous band (but asymmetric for $\Delta\omega_c \neq 0$).

For $\Delta\omega_c \sim \tau^{-1}$, the frequencies of the transitions between different neighboring levels overlap and the electron–phonon interaction leads to a mixing of levels (which will be discussed later), which complicates the calculation of the cyclotron resonance spectrum considerably. We shall carry out such a calculation for arbitrary $\Delta\omega_c/\tau^{-1}$ in a wide range of temperatures (in the absence of degeneracy) using a quantum–statistical method similar to that used in refs. 10 and 11 (the profile of a line representing the absorption of light by a nonlinear oscillator whose energy spectrum is analogous to a set of nonequidistant Landau levels was studied in refs. 10 and 11).

We shall consider a two-dimensional isotropic system (our results can be easily extended to anisotropic systems). The momentum $\mathbf{p}$ and position $\mathbf{r}$ vectors and also the vector potential $\mathbf{A} = 1/2[\mathcal{H} \times \mathbf{r}]$ [$\mathcal{H}$ is the strength of an applied transverse magnetic field ($\mathcal{H}\mathbf{r}$) = 0] are assumed to be two-dimensional. Phonons will be labeled by their two-dimen-

sional wave vector q and by an index j, which determines the type of phonon, its polarization, and the quantum number governing the transverse motion.

The Hamiltonian of the electron–phonon system is given by

$$\left.\begin{aligned} H=H_0+H_i;\quad H_0=\frac{1}{2m}P^2+\frac{V}{8m^2\omega_c^2}P^4+\sum_{qj}\omega_{qj}a_{qj}^+a_{qj};\\ P^2=\left(\mathbf{p}-\frac{e}{c}\mathbf{A}\right)^2;\quad \hbar=1;\\ H_i=\sum_{qj}\varepsilon_{qj}\exp\left(i\mathbf{qr}\right)a_{qj}^+ + \text{H.c.} \end{aligned}\right\} \tag{1}$$

Here, m is the effective electron mass at the bottom of the band, $\omega_c = |e|\mathcal{H}/mc$, V is the nonparabolicity parameter (for the Kane model, we obtain $V = -2\omega_c^2/\varepsilon_g$), a_{qj}^+, a_{qj}, and ω_{qj} are the phonon creation and annihilation operators and the phonon frequency, and ε_{qj} is the parameter governing a weak ($\omega_c\tau \gg 1$) electron–phonon interaction.

Apart from an unimportant constant, the energy of an electron at the n-th Landau level corresponding to the Hamiltonian H_0 is given by $\omega_c n + (1/2)V n(n+1)$. For $|g\mu_B\mathcal{H}-\omega_c| \gg \tau^{-1}, |V|$, the electron spin can be neglected and we shall consider only the contribution to the cyclotron resonance peak which is due to carriers with a given projection of the spin (different ω_c correspond to different spin projections).

We shall not discuss the method of calculation of the correlation functions, which is similar to that used in refs. 10 and 11 and will be described in a separate publication. Using this method, we can write the conductivity $\sigma(\omega)$ in a direction parallel to a layer of nondegenerate two-dimensional electrons in the following form, which applies to frequencies close to the cyclotron resonance:

$$\sigma(\omega)=\frac{e^2N}{2m}\,\mathrm{Re}\sum_{n=0}^{\infty}\varphi(n,\omega),\quad \omega\sim\omega_c, \tag{2}$$

where N is the density of electrons with a given projection of the spin.

Equation (2) has a simple physical interpretation. The function Re $\varphi(n,\omega)$ corresponds to the absorption line due to transitions between the n-th and (n+1)-th Landau levels and the total absorption spectrum $\sigma(\omega)$ represents the superposition of such lines. The electron–phonon interaction leads to a mixing of the wave functions corresponding to different Landau levels. Such effect gives rise to a coupling of functions $\varphi(n,\omega)$ with different n. It can be shown that the equation for $\varphi(n,\omega)$ has the form[1)]

$$\begin{aligned} &\varphi(n,\omega)\{-i(\omega-\omega_c-P(n))+\Gamma(n)\}\\ &-2(n+1)\sum_k\Gamma(n,k)(\bar n_k+1)\\ &\times\varphi(n+k,\omega)-2(n+1)\sum_k\Gamma(n-k,k)\,\bar n_k\varphi(n-k,\omega)\\ &=(\bar n_1+1)^{-2}(n+1)\exp(-n\omega_c/T);\\ &k=1,2,\ldots,k_{max};\quad \bar n_k=[\exp(k\omega_c/T)-1]^{-1};\\ &\Gamma(n,k)\equiv 0,\quad n<0. \end{aligned} \tag{3}$$

The parameters $\Gamma(n)$ and $\Gamma(n,k)$ in Eq. (3) govern the decay effects:

$$\begin{aligned} &\Gamma(n)=\sum_{qk}\Gamma_k(\mathbf{q})\{n_k[L_q^2(n+1,k)+L_q^2(n,k)]+(\bar n_k+1)\\ &\times[L_q^2(n+1-k,k)+L_q^2(n-k,k)]\};\\ &\Gamma(n,k)=\sum_q\Gamma_k(\mathbf{q})L_q(n+1,k)L_q(n,k)[(n+k+1)(n+1)]^{-1/2};\\ &\Gamma_k(\mathbf{q})=\pi\sum_j|\varepsilon_{qj}|^2\left(\frac{1}{2}lq\right)^{2k}\exp\left(-\frac{1}{2}l^2q^2\right)\delta(k\omega_c-\omega_{qj});\\ &L_q(n,k)=[2^k n!(n+k)!]^{1/2}L_n^k\left(\frac{1}{2}l^2q^2\right);\\ &l=\sqrt{\frac{\hbar}{m\omega_c}};\quad L_q(n,k)\equiv 0,\ n<0, \end{aligned} \tag{4}$$

where L_n^k are the Laguerre polynomials.

The terms in Eqs. (3) and (4) with different k are due to electron transitions across k levels. Each such transition is accompanied by the emission or absorption of a resonant phonon, which is reflected in the delta functions in Eq. (4).

The quantity P(n) in Eq. (3) represents the shift of the transition frequency $n \to n+1$,

$$P(n)=V(n+1)+P_{n+1}-P_n, \tag{5}$$

where P_n is the shift of the n-th level due to a weak polaron effect, which can be calculated for the interaction defined by Eq. (1) from the standard perturbation theory.

The system (3) describes the interference of the lines $\varphi(n,\omega)$ for an arbitrary ratio of the line shift P(n) to the width parameters $\Gamma(n)$, $\Gamma(n,k)$. It follows from Eqs. (3) and (4) that the number of lines k_{max} which are coupled directly to $\varphi(n,\omega)$ is determined by the maximum separation of the levels which are connected by a one-phonon transition. For large ω_c ($> 10^{12}$ sec^{-1}) used in the cyclotron resonance experiments on two-dimensional systems,[6-8] k_{max} is small and can be often set equal to unity, $k_{max} = 1$. For the interaction with acoustic phonons, this is due to a restriction on the momentum of the resonant phonons. In fact, it follows from Eq. (4) that $\Gamma_k(q)$ is exponentially small for $ql \gg 1$. It is also obvious that the transverse phonon wave number should not be much greater than the reciprocal of the thickness of the electron layer ($\sim 10^6$ cm^{-1}). At low temperatures $\omega_c/T \gg 1$, the transitions with k = 1 lead to a finite width $\sigma(\omega) = (e^2N/2m)\,\mathrm{Re}\,\varphi(0,\omega)$.

The system of linear equations (3) can be easily solved on a computer for arbitrary electron–phonon interaction and nonparabolicity [the infinite chain (3) can be truncated since the function $\varphi(n,\omega)$ is exponentially small for $n \gg \bar n_1$]. The case of a highly nonequidistant distribution of the levels

$$|P(n)-P(n+k)| \gtrsim \Gamma(n),\ |\Gamma(n,k)|, \tag{6}$$

which can be studied analytically, corresponds to the cyclotron resonance peak, consisting of a set of virtually nonoverlapping fine-structure lines. The profile of the n-th fine-structure line is given by

$$\begin{aligned} &\mathrm{Re}\,\varphi(n,\omega)=(\bar n_1+1)^{-2}\Phi(n,\omega)\\ &\times\left[1+2\exp\left(\frac{n\omega_c}{T}\right)\sum_{f=\pm1}\sum_k\Gamma\left(n-\frac{k}{2}+\frac{kf}{2},k\right)\right.\\ &\left.\times\left(\bar n_k+\frac{1}{2}+\frac{f}{2}\right)\Phi(n+kf,\omega)\right.\times \end{aligned}$$

$$\times \frac{\Gamma(n)\Gamma(n+kf)-\Omega(n)\Omega(n+kf)}{\Gamma(n)\Gamma(n+kf)}\Bigg];$$

$$\Phi(n,\omega)=(n+1)\exp\left(\frac{-n\omega_c}{T}\right)\frac{\Gamma(n)}{\Gamma^2(n)+\Omega^2(n)};$$

$$\Omega(n)=\omega-\omega_c-P(n). \qquad (7)$$

The first term on the right-hand side of Eq. (7) describes a Lorentzian curve with a half-width $2\Gamma(n)$ equal to the sum of the widths of the n-th and (n + 1)-th levels (between which the transition takes place) calculated from the standard formula of Weisskopf and Wigner. The second term describes an asymmetric correction, which arises since the frequencies $\Omega(n)$ lie close to one another and the lines interfere.

The widths $\Gamma(n)$ increase when the temperature is raised, the lines overlap, the condition (6) ceases to be satisfied, and the fine structure disappears. Since only a single line $\sigma(0, \omega)$ is excited at low temperatures $\omega_c/T \gg 1$, the fine structure can exist only in a narrow range of temperatures.

It follows from Eqs. (2)-(4) that the general cyclotron resonance peak has a complex asymmetric profile. The total intensity of the peak is independent of the interaction with phonons. The wings, where $|\omega-\omega_c| \gg \Gamma(n)$, $|P(n)|$, and $n \sim \bar{n}_1$, are Lorentzian. Using the iteration method, we obtain from Eqs. (2)-(4)

$$\sigma(\omega) \propto \frac{e^2V}{2m}\frac{\gamma}{(\omega-\omega_c)^2}, \quad |\omega-\omega_c| \gg \gamma; \quad \gamma=(\bar{n}_1+1)^{-2}$$

$$\times \sum_{n=0}^{\infty}(n+1)\exp\left(\frac{-n\omega_c}{T}\right)\left[\Gamma(n)-4\sum_k \Gamma(n,k)\bar{n}_k(n+k+1)\right]. \qquad (8)$$

The interaction of an electron with two phonons is not included in the Hamiltonian (1). If this interaction is taken into account in the second order of the perturbation theory, Eqs. (2) and (3) remain unchanged and the interaction leads only to corrections to $\Gamma(n)$, $\Gamma(n, k)$, and P_n in Eq. (3). Such corrections are especially important for $\omega_c > \omega_m$ (ω_m is the maximum phonon frequency) when the one-phonon transitions are forbidden because of the law of conservation of energy.

Our discussion indicates that a study of the cyclotron resonance of two-dimensional electrons can not only give the spectrum of the electron energies but also yield information about the interaction of two-dimensional electrons with phonons. It will be shown in our forthcoming publications that non-Lorentzian lines (even for equidistant levels) are very sensitive to the type of phonon participating in the inelastic scattering. The line profile can yield not only the parameters governing the electron–phonon interaction but also the form of the phonon spectrum (when the phonon spectrum splits in size-quantized two-dimensional bands). Most accurate information can be obtained when the cyclotron resonance spectrum exhibits a fine structure. When the strength of the magnetic field is increased, the distribution of the levels becomes more nonequidistant and the fine structure should manifest itself in strong fields (for InSb and $\mathcal{H}$ = 20 kOe, the nonparabolicity leads to a separation of the fine-structure peaks $V \approx 14\ cm^{-1} \approx 2.4$ kOe). It should be noted that a fine structure should appear at $T \sim \omega_c$ even in the spectrum of the cyclotron resonance of degenerate two-dimensional electrons.

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[1)] In the derivation of Eq. (3), we have made use of the fact that, for all relevant $n \sim \bar{n}_1$, the corrections $\Gamma(n)/\omega_c|d\Gamma(n)/d\omega_c|$, $|P(n)|/\omega_c \ll 1$, and also $\Gamma(n)/T$, $|P(n)|/T \ll 1$ are negligible.

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