Self-induced change in the polarization of electromagnetic waves in cubic crystals

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It is shown that electromagnetic waves traveling in an optically nonlinear cubic crystal can be accompanied by spatial oscillations of the polarization. Such oscillations are studied for electromagnetic waves traveling in the direction of a fourfold axis. The oscillations can occur when the intensity of electromagnetic waves is varied. Both the angle of inclination of the polarization ellipse axis and the degree of polarization oscillate. The polarization ellipse in crystals exhibiting nonlinear absorption rotates toward one of the symmetry axes as the electromagnetic waves propagate and the waves become either linearly or circularly polarized.

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One of the important properties of optically anisotropic crystals is the change in the polarization of the transmitted electromagnetic waves (see, for example, Ref. 1). If the intensity of an electromagnetic wave is high enough, such a change can become self-induced. For transparent isotropic media, the self-induced anisotropy has the effect that the polarization ellipse of a strong electromagnetic wave rotates.[2] The change in the polarization which occurs in crystals is more complex. In particular, it has been shown that in transparent nongyrotropic crystals the polarization ellipse rotates only for certain orientations of the ellipse relative to the crystallographic axes and also only when the incident electromagnetic wave is strong enough; in the opposite case, the polarization ellipse oscillates about certain special directions.

The self-effect of the field $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp(-i\omega t)$ + c.c. for moderate intensities of the field is determined by the third-order nonlinear polarization of the crystal $\mathbf{P}^{(3)}(\mathbf{r}) \exp(-i\omega t)$ + c.c. Neglecting the spatial dispersion, we find that the frequency-dependent susceptibility tensor $\chi_{ijkl}(\omega, \omega, -\omega, -\omega)$ relating $\mathbf{P}^{(3)}(\mathbf{r})$ to $\mathbf{E}(\mathbf{r})$ has three independent components in a cubic crystal and the corresponding expression for $\mathbf{P}^{(3)}(\mathbf{r})$ assumes the form

$$P_{\varkappa}^{(3)}(r) = A_1 E_{\varkappa} |\mathbf{E}|^2 + A_2 E_{\varkappa}^* \mathbf{E}^2 + A_3 E_{\varkappa} |E_{\varkappa}|^2, \quad E_{\varkappa} \equiv E_{\varkappa}(\mathbf{r}). \tag{1}$$

Here, $\varkappa$ labels the projections of the vectors $\mathbf{P}^{(3)}$ and $\mathbf{E}$ on the coordinate axes which are chosen to be parallel to the fourfold crystal axes. In the absence of frequency dispersion, we obtain $A_2 = (1/2)A_1$; the term proportional to A_3 in Eq. (1) determines the behavior of the nonlinear optical effects in cubic crystals compared with the isotropic medium where $A_3 = 0$.

In general, the parameters $A_{1,2,3}$ in Eq. (1) are complex. Their real parts correspond to a nonlinear renormalization of the refractive index; such a renormalization is clearly anisotropic, i.e., a nonlinear crystal exhibits birefringence.[3] The imaginary parts of $A_{1,2,3}$ describe the nonlinear absorption; it follows from Eq. (1) that such absorption exhibits the dichroism effect. The self-induced birefringence and optical dichroism of cubic crystals were first observed in the microwave frequency range in Refs. 4 and 5. These effects have been observed in many-valley semiconductors and interpreted as the manifestation of an analog of the Sasaki effect at microwave frequencies.

It was shown in Ref. 6 that a large self-induced rotation of the plane of polarization can occur in the impurity absorption region of crystals. This effect was observed at optical frequencies in Ref. 7 and rotation angles greater than 10° were obtained for nonlaser fields, i.e., the optical nonlinearity can be termed "giant." It is clear that the real part of A_i is greater than the imaginary part when the frequency detuning from a resonance is greater than the absorption line width. At the same time, the value of $|A_i|$ is large (although, naturally, smaller than at a resonance).

It is, therefore, of interest to use the general phenomenological expression (1) to study the change in the polarization of electromagnetic waves under strongly nonlinear conditions when $(l/\lambda)|A_i E^2| \gtrsim 1$ is satisfied, where l is the thickness of a crystal and λ is the wavelength of electromagnetic waves (the nonlinear terms in $\mathbf{P}$ are then small compared with the linear terms, $|A_i E^2| \ll 1$, and the terms higher than cubic in $\mathbf{E}$ can be neglected in the polarization). Since the change in the polarization due to a strong self-induced dichroism was already studied in Ref. 6, we shall concentrate on nearly transparent crystals. For simplicity, we shall neglect the natural and self-induced[8] gyrotropy, i.e., we shall treat centrosymmetric crystals as crystals where the spatial dispersion in the frequency range considered is weak.

1. EQUATIONS DESCRIBING THE CHANGE IN THE POLARIZATION

It is most convenient to study experimentally and also easy to treat theoretically the self-induced change in the polarization of electromagnetic waves traveling in the direction of one of the fourfold symmetry axes (to be specific, we shall consider the z axis). For a weak damping and $|A_i E^2| \ll 1$, the equations for the envelopes of the field $\tilde{E}_{\varkappa}(z)$ slowly varying over the wavelength λ have the form

$$\frac{\partial \tilde{E}_{\varkappa}}{\partial z} = -\frac{1}{2} k_0 \tilde{E}_{\varkappa} + \frac{2\pi i \omega}{c n_0} \tilde{P}_{\varkappa}^{(3)}, \quad \varkappa = x, y, \quad \tilde{E}_{\varkappa} = E_{\varkappa} \exp\left(-i \frac{\omega n_0}{c} z\right). \tag{2}$$

Here, n_0 and k_0 are the refractive index and the absorption coefficient (in the limit $|\mathbf{E}| \to 0$) and the expression for $\tilde{P}_{\varkappa}^{(3)}$ can be obtained from Eq. (1) for $P_{\varkappa}^{(3)}$ by the replacement of $E_{\varkappa}$ by $\tilde{E}_{\varkappa}$.

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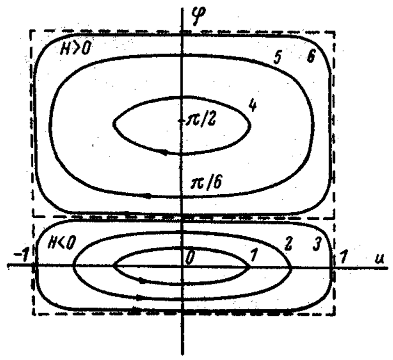

FIG. 1. Phase trajectories for $|f| < 1$. Curves 1-6 correspond to $\mathscr{H} = -0.2, -0.1, -0.01, 0.6, 0.2$, and 0.01 for $f = 0.5$. The dashed lines denote the separatrices corresponding to $\mathscr{H} = 0$.

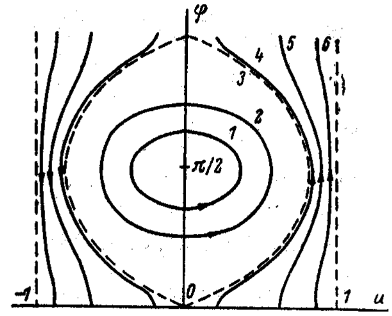

FIG. 2. Phase trajectories for $|f| < 1$. Curves 1-6 correspond to $\mathscr{H} = 1.3, 1, 0.5, 0.48, 0.3$, and 0.1 for $f = 2$. The dashed curves denote the separatrices corresponding to $\mathscr{H} = \frac{1}{2}(f-1)$, and the dashed lines denote the asymptotes $u = \pm 1$ corresponding to $\mathscr{H} = 0$.

The polarization of electromagnetic waves is determined by the angle α between the major axis of the polarization ellipse and the x axis ([100] axis) and by a parameter $p = |(I_{\parallel} - I_{\perp})/(I_{\parallel} + I_{\perp})|$ describing the degree of polarization ($I_{\parallel}$ and $I_{\perp}$ are the maximum and minimum values of the square of the electromagnetic field strength). The quantities α and p can be easily expressed in terms of the relative difference u between the squares of the amplitudes of the field components E_y and E_x and in terms of the difference between the phases of these components φ which can be more easily evaluated

$$u = \frac{|E_y(z)|^2 - |E_x(z)|^2}{|E(z)|^2}, \quad \varphi = \operatorname{Arg}[E_y(z)/E_x(z)]. \tag{3}$$

The expressions for α and p in terms of u and φ have the form

$$\left.\begin{aligned} &\alpha = -\frac{1}{2}\operatorname{arc\,tg}\left[\frac{(1-u^2)^{1/2}}{u}\cos\varphi\right] + \frac{\pi}{2}\theta(u) + n\pi, \\ &p = (u^2\sin^2\varphi + \cos^2\varphi)^{1/2}, \quad \theta(u) = \begin{cases} 1, & u > 0, \\ 0, & u < 0, \end{cases} \quad n = 0, \pm 1, \ldots \end{aligned}\right\} \tag{4}$$

(the values of α which differ by π are equivalent as regards the polarization of electromagnetic waves).

The equations of motion for u and φ obtained from Eqs. (1) and (2) are given by

$$\frac{du}{d\tau} = \frac{\partial\mathscr{H}}{\partial\varphi} + u(1-u^2)(\varepsilon_1\cos 2\varphi - \varepsilon_2), \quad \frac{d\varphi}{d\tau} = -\frac{\partial\mathscr{H}}{\partial u} + \varepsilon_1\sin 2\varphi,$$

$$\mathscr{H} = \frac{1}{2}(1-u^2)(f - \cos 2\varphi),$$

$$f = 1 + (A_3'/A_2'), \quad \varepsilon_1 = A_2''/A_2', \quad \varepsilon_2 = (A_2'' + A_3'')/A_2',$$

$$\tau = \frac{2\pi\omega}{cn_0} A_2' \int_0^z |E(z')|^2 dz'; \tag{5}$$

it is assumed that $A_2 \neq 0$.

It follows from Eq. (5) that the polarization regarded as a function of τ depends on three dimensionless parameters: f, ε_1, and ε_2, i.e., on the ratio of the complex quantities A_2 and A_3. The isotropic part of the nonlinear polarizability which is proportional to A_1 does not influence directly the change in the polarization; A_1'' appears only in the expression for $|E(z)|^2$. For a transparent crystal, we obtain $|E|^2 = \text{const}$ and τ is directly proportional to the path z traveled by the electromagnetic wave. For an isotropic medium, we obtain $\varepsilon_1 = \varepsilon_2$, and $f = 1$.

Equation (5) can be easily solved in the limit when the nonlinear absorption is much greater than the nonlinear refraction $|\varepsilon_{1,2}| \gg 1$,

$$\operatorname{tg}\varphi(\tau) = e^{2\varepsilon_1\tau}\operatorname{tg}\varphi(0), \quad \operatorname{tg}2\alpha(\tau) = e^{(\varepsilon_2-\varepsilon_1)\tau}\operatorname{tg}2\alpha(0), \quad |\varepsilon_1| + |\varepsilon_2| \gg 1; \tag{6}$$

it can be seen from Eq. (5) that $\varepsilon_1\tau$ and $\varepsilon_2\tau$ are independent of A_2'. It follows from Eq. (6) that the phase shift φ tends monotonically to $n\pi$ when electromagnetic waves travel in a nonlinearly absorbing crystal (for $A_2'' < 0$) or the phase shift tends to $(2n + 1/2)\pi$ (for $A_2'' > 0$) and α tends to $m\pi/2$ (for $A_3'' < 0$) or to $(2m + 1/4)\pi$; and n and m (n, m = 0, 1, ...) should be chosen so that the limiting values of φ and α are closest to $\varphi(0)$ and $\alpha(0)$. For $A_2'' < 0$, we thus find that the polarization of electromagnetic waves due to the self-induced dichroism tends to a linear polarization (monotonically for $A_3'' < 0$). For $A_3'' > 0$ and $A_3'' + 2A_2'' > 0$, electromagnetic waves acquire a circular polarization; for $A_2'' > 0$, but $A_3'' + 2A_2'' < 0$, we find that, although $\varphi \to \pm\pi/2$, the polarization in the limit $z \to \infty$ becomes linear since the amplitude of one of the components vanishes. The orientation and rate of rotation of the major axis of the polarization ellipse toward the nearest axis of the $\langle 100\rangle$ or $\langle 110\rangle$ type, which corresponds to $\alpha \to n\pi/2$ or $\alpha \to (2n+1)\pi/4$ are determined only by the value of A_3''.

2. SELF-INDUCED CHANGE IN THE POLARIZATION OCCURRING IN A TRANSPARENT MEDIUM FOR $|f| < 1$

In the absence of nonlinear dissipation ($\varepsilon_1 = \varepsilon_2 = 0$), Eq. (5) represents a system of Hamilton equations of motion in the variables u and φ (in fact, this is the reason why such variables are convenient). With an accuracy up to a constant, the Hamiltonian $\mathscr{H}$ is equal to the ratio of the energy of electromagnetic waves in the medium (related to the nonlinear polarization $P^{(3)}$) to $A_2'|E|^4$. For $\varepsilon_1 = \varepsilon_2 = 0$ and a weak nonlinearity when Eq. (2) holds, the constant value of $\mathscr{H}$ is a consequence of the law of conservation of energy. The solution of the system (5) can be expressed in terms of elliptic Jacobi functions. Since the resulting expressions are rather complex, we shall quote only the result of our analysis of the final expressions. The phase diagrams of the system are shown in Figs. 1 and 2. It follows from the system (5) that the equations of motion do not change when φ is replaced by $\varphi + n\pi$ and, therefore, only the portion of the trajectory corresponding to the interval of φ of length π is shown in Figs. 1 and 2; the complete phase diagram represents the periodic extension of the curves shown in Figs. 1 and 2. For $|f| < 1$, it follows from Fig. 1 that all the trajectories are symmetric convex closed curves with centers at the point $\varphi = n\pi$, $u = 0$ for $\mathscr{H} < 0$ or $\varphi = (2n + 1/2)$, $u = 0$ for $\mathscr{H} > 0$. The quantity $\mathscr{H}$ is bounded by the inequalities

$$\mathscr{H}_- \leqslant \mathscr{H} \leqslant \mathscr{H}_+, \quad \mathscr{H}_\pm = \frac{1}{2}(f \pm 1) \ (|f| < 1).$$

For $|\mathscr{H} - \mathscr{H}_\pm| \ll 1$ the corresponding trajectory is close to an elliptic trajectory and, for $\mathscr{H} \to 0$, it approaches a rectangle. The amplitudes of the quantities u and φ, i.e., u_0 and φ_0, and also the period T of the motion over the trajectory are given by

$$\begin{gathered} u_0 = \left[1 - \frac{2\mathscr{H}^2}{|\mathscr{H}| + f\mathscr{H}}\right]^{1/2}, \quad \varphi_0 = -\frac{\mathscr{H}}{2|\mathscr{H}|} \arccos(f - 2\mathscr{H}) + \frac{\pi}{2}\theta(\mathscr{H}), \\ T = |1 - f^2|^{-1/2} 4ku_0^{-1} K(k), \\ k = \left[\frac{|\mathscr{H}| - f\mathscr{H}}{|\mathscr{H}| + f\mathscr{H}} \frac{|\mathscr{H}| + f\mathscr{H} - 2\mathscr{H}^2}{|\mathscr{H}| - f\mathscr{H} + 2\mathscr{H}^2}\right]^{1/2}, \quad |f| < 1, \end{gathered} \tag{7}$$

where K(k) is a complete elliptic integral of first kind and k is its modulus.[9]

Equation (7) exhibits simple asymptotic behavior. For $\mathscr{H} \to \mathscr{H}_\pm$, the amplitudes u_0 and φ_0 tend to zero as $|\mathscr{H} - \mathscr{H}_\pm|^{1/2}$ and $T \approx T_\pm = \pi\sqrt{2}\,[1 \pm f]^{-1/2}$. The quantities T_+ and T_- determine, respectively, the smallest turning period for each f corresponding to $\mathscr{H} > 0$ and $\mathscr{H} < 0$. The period and amplitude increase with decreasing $|\mathscr{H}|$ For $|\mathscr{H}| \ll 1$, we obtain

$$\begin{gathered} \varphi_0 \approx -\frac{\mathscr{H}}{2|\mathscr{H}|} \arccos f + \frac{\pi}{2}\theta(\mathscr{H}), \\ T \approx 2(1 - f^2)^{-1/2} \ln[4(1 - f^2)/|\mathscr{H}|], \quad u_0 \approx 1. \end{gathered}$$

For small $|\mathscr{H}|$, the period then depends logarithmically on $|\mathscr{H}|$.

It follows from Eq. (4) that the phase trajectories shown in Fig. 1 for $|f| < 1$, $\mathscr{H} < 0$, correspond to periodic oscillations of the polarization such that the angle of inclination of the ellipse α varies between $(\pi/4) - \alpha_0$ and $(\pi/4) + \alpha_0$ and the degree of polarization varies from 1 to p_0 (the period of oscillations of p is T/2), where

$$\left.\begin{gathered} p_0 = \left[\frac{1}{2}(1 + f - 2\mathscr{H})\right]^{1/2}, \quad \alpha_0 = \frac{\pi}{4} - \frac{1}{2}\operatorname{arctg}\left(-\frac{2\mathscr{H}}{1 - f + 2\mathscr{H}}\right)^{1/2}, \\ \mathscr{H} < 0, \quad |f| \leqslant 1, \quad (2\mathscr{H} > f - 1). \end{gathered}\right\} \tag{8}$$

For $\alpha = \pi/4 + \alpha_0$, the polarization of electromagnetic waves is linear, but as α approaches $\pi/4$, the ellipticity increases, p approaches p_0 and then increases up to unity when α passes through $\pi/4$. The trajectories in Fig. 1 with centers at $\varphi = u = 0$ correspond to oscillations of the major axis of the polarization ellipse about the axis [110]; the trajectories with centers at $u = 0$, $\varphi = \pi$ (which are obtained by the shift of the aforementioned trajectories through π in the direction of the axis φ) correspond to oscillations of the polarization ellipse about the axis $[1\bar{1}0]$.

The trajectories in Fig. 1 with centers at $u = 0$, $\varphi = [(2n + 1)/2]\pi$ ($\mathscr{H} > 0$) correspond to the rotation of the polarization ellipse with a period T defined by Eq. (7) which is accompanied (in contrast to isotropic media) by periodic oscillations (with a period T/2) of the degree of polarization p between the minimum value p_{min} and the maximum value p_{max} given by

$$p_{min} = \left[\frac{1}{2}(1 + f - 2\mathscr{H})\right]^{1/2}, \quad p_{max} = \left(1 - \frac{2\mathscr{H}}{1 + f}\right)^{1/2}, \quad \mathscr{H} > 0, \quad |f| < 1. \tag{9}$$

For $|f| < 1$, p reaches its maximum when the axis of the ellipse lies in the direction of one of the axes $\langle 100 \rangle$.

3. SELF-INDUCED CHANGE IN THE POLARIZATION OF ELECTROMAGNETIC WAVES OCCURRING IN A TRANSPARENT MEDIUM FOR $|f| > 1$

For $|f| > 1$, it follows from Fig. 2 that the phase trajectories can be closed provided $|\mathscr{H}| > |f| - 1$, or open when this inequality is not satisfied. For $f > 1$, the closed trajectories have a center at a point $u = 0$, $\varphi = (2n + 1)\pi/2$; for $f < -1$, the center lies at $u = 0$, $\varphi = n\pi$. The trajectories corresponding to $f < -1$ can be obtained from those shown in Fig. 2 by a shift by $\pi/2$ in the direction of the φ-axis and by reversal of the direction in which they are circumvented; this property follows from the symmetry of the equations of motion (5) under the substitutions $f \to -f$, $\mathscr{H} \to -\mathscr{H}$, $\varphi \to \varphi + \pi/2$, and $\tau \to -\tau$. For $|f| > 1$, the signs of f and $\mathscr{H}$ coincide and the condition $|\mathscr{H}| \to 1 + |f|$, is satisfied. For $2|\mathscr{H}| \to 1 + |f|$, the trajectories are nearly elliptic and, for $|\mathscr{H}| \to 0$, the trajectories approach a straight line and tend to one of the asymptotes $u = \pm 1$.

The expressions for the amplitudes u_0 and φ_0 for a closed trajectory corresponding to $|f| > 1$ are identical with the expressions defined by Eq. (7) and the period T can be obtained from Eq. (7) provided the quantity k is renormalized as follows:

$$k = \frac{1}{2}u_0\left(\frac{f^2 - 1}{|\mathscr{H}|}\right)^{1/2}, \quad |f| > 1, \quad 2|\mathscr{H}| > |f| - 1. \tag{7a}$$

For $2|\mathscr{H}| \to |f| + 1$, it follows from Eqs. (7) and (7a) that the amplitudes u_0 and φ_0 tend to zero as $(1 + |f| - 2|\mathscr{H}|)^{1/2}$, and $T \approx \pi\sqrt{2}\,(|f| + 1)^{-1/2}$. The amplitudes u_0, φ_0 and the period T grow with increasing $|\mathscr{H}|$ and

$$\begin{gathered} u_0 \approx [2/(1 + |f|)]^{1/2}, \quad \varphi_0 \approx \pi/2 - \xi, \quad T \approx [2/(|f| - 1)]^{1/2} \ln\left[\frac{16(|f| - 1)}{|f| + 1}\xi^{-2}\right], \\ \xi = \left[|\mathscr{H}| - \frac{|f| - 1}{2}\right]^{1/2}, \quad \xi \ll 1. \end{gathered} \tag{10}$$

For open trajectories, i.e., for $2|\mathscr{H}| < |f| - 1$, the quantity $|u|$ varies from $(1 - k^2)^{1/2}u_0$ to u_0, where u_0 is defined by Eq. (7). The expressions for k and for the period T corresponding to a change in φ by 2π are given by

$$\left.\begin{gathered} T = 4u_0^{-1}(f^2 - 1)^{-1/2} K(k), \quad k = 2u_0^{-1}\left|\frac{\mathscr{H}}{f^2 - 1}\right|^{1/2}, \\ |f| > 1, \quad 2|\mathscr{H}| < |f| - 1. \end{gathered}\right\} \tag{11}$$

It can be easily shown that Eq. (11) for T reduces to Eq. (10) in the limit $2|\mathscr{H}| \to |f| - 1$ provided the sign under the square root in the expression for ξ is changed. For $|\mathscr{H}| \to 0$, the quantity $|u|$ tends to unity and the period to $T \approx 2\pi(f^2 - 1)^{-1/2}$.

For $|f| > 1$, the motion over closed trajectories leads to a polarization which changes as in the case of $|f| < 1$ and is described by Eq. (8) for $f < -1$ and by Eq. (9) for $f > 1$. The motion over open trajectories corresponds to oscillations of the polarization ellipse about an axis of the

⟨100⟩ type. The amplitude of the oscillations of the angle of inclination of the ellipse α and the minimum value of the polarization p are given by

$$\left.\begin{array}{c}\alpha_0=\frac{1}{2}\operatorname{arc\,tg}\,[2\mathscr{K}/(f-1-2\mathscr{K})]^{1/2}, \quad p_0=\left[1-\frac{2\mathscr{K}}{f+1}\right]^{1/2}, \\ |f|>1, \quad 2|\mathscr{K}|<|f|-1,\end{array}\right\} \tag{12}$$

where p varies from unity to p_0.

To conclude this section, we note that the polarization can be described in an adiabatic approximation for a weak nonlinear dissipation $|\varepsilon_{1,2}| \ll 1$ and far from the bifurcation points ($\mathscr{H}=0$ for $|f| < 1$ and $2|\mathscr{H}|=|f|-1$ for $|f| > 1$). In fact, we may assume that $\mathscr{H}$ is a slowly varying function during the period T and we can average $d\mathscr{H}/d\tau$ over the period. As a result, we arrive at a first-order differential equation for $\mathscr{H}(\tau)$ which has several stationary solutions. In particular, we obtain stationary solutions $\mathscr{H}=\frac{1}{2}\left(f+\frac{\mathscr{K}}{|\mathscr{K}|}\right)$, corresponding to a circularly polarized wave for $\mathscr{H}>0$ or to a wave polarized linearly in the direction of axes of the ⟨110⟩ type for $\mathscr{H}<0$. The circular polarization is stable for $\varepsilon_2+3\varepsilon_1>0$ and the linear polarization in the direction of one of the axes ⟨110⟩ is stable for $\varepsilon_2-3\varepsilon_1>0$.

It follows from our results that rotation of the polarization ellipse in a transparent nongyrotropic nonlinear crystal cannot occur for $f \leq -1$; if $f > -1$, the rotation takes place only for waves with a high degree of ellipticity. Self-induced oscillations of the polarization are a special feature of the nonlinear crystal optics. The axis of the polarization ellipse of electromagnetic waves traveling in the direction [001] oscillates about one of the crystal axes of the ⟨100⟩ or ⟨110⟩ type. For certain limiting deviations, electromagnetic waves are polarized linearly. Since the effective nonlinear optical "thickness" of a crystal $|\mathbf{E}(0)|^2 l$ depends on the intensity of light, we may increase or decrease the ellipticity of the transmitted waves by varying the intensity of light (this is in contrast to crystals with linear birefringence).

The self-induced change in the polarization which occurs in cubic crystals has also the following property that can be used to identify this particular mechanism: the angle of rotation of the polarization of waves traveling in the direction of the [001] axis changes its sign when a crystal is rotated by 90° about its axis [001] [see Eqs. (3)-(5)]. This effect has been observed[7,10] (but it has been attributed only to the self-induced dichroism).

Since the crystal axis about which the polarization ellipse performs its oscillations and also the amplitude and the spatial period of the oscillations are determined by the relative magnitudes of the nonlinearity parameters A_2 and A_3 in Eq. (1), the effect studied in the present paper can be employed to determine these parameters. The ratio of Im A_2 and Im A_3 in the impurity absorption band[6] (the nonlinearity due to the saturation of the absorption or the optical orientation of impurity centers) and also in the two-photon band–band absorption[11] is of the order of unity. It can be easily shown that we also obtain $A_3/A_2 \sim 1$ in the wings of the corresponding absorption bands where the condition $|\mathrm{Re}\, A_i| \gg |\mathrm{Im}\, A_i|$ is satisfied in the case of two-level (two-band) models employed in Refs. 6 and 11, i.e., the anisotropy of the nonlinear refraction determined by the parameter A_3 is essential.

Finally, we would like to point out that there are both "stable" directions of rotation of the polarization ellipse (axes of the type ⟨110⟩ or ⟨100⟩) and "unstable" directions (axes ⟨100⟩ or ⟨110⟩) separating regions in which the polarization ellipse turns toward one or the other axis. If such a nonlinear crystal is placed in a resonator and the incident waves are polarized in the direction of one of the unstable axes, a crystal may support a domain structure with domains in which the directions of the rotation of the polarization are different.

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