defined by Eq. (23); the vector representations C_6 (n = 6), C_4 (n = 4), C_3 (n = 3), and the nonvector representations C_6 (n = 3) when the free energy depends on the invariants defined by Eqs. (23) and (24).

Our results for n = 4 corresponding to Eq. (23) hold also for other n. It is only necessary to modify the parameter Δ defined by Eq. (8) as follows: $\Delta \equiv (\beta_n/\alpha_0) \cdot [(\alpha_0 - \alpha)/\beta_1]^{n/2-1}$. The region of existence of the modulated phase is modified accordingly: $0 < \Delta < 1$. For the free energy depending on the invariants defined by Eqs. (23) and (24), it is necessary to introduce, in addition to Δ, a parameter Δ' with β_n' replacing β_n. The case n = 3 requires a special discussion since, in the absence of the gradient invariant with the coefficient σ, the transition from the initial to the homogeneous phase is of first type because the free energy contains the third-order invariants. The transition to the modulated phase can be of second type provided the inequalities $\beta_3^2/4\beta_1\alpha_0 < 1$ and $\beta_3'^2 \cdot (4\beta_1\alpha_0)^{-1} < 1$ are satisfied.

Our model can be applied to ammonium fluoroberyllate,[6] which exhibits two phase transitions with the characteristic temperatures lying close to one another.[7] The symmetry of the high-temperature phase is D_{2h}^{18} and the symmetry of the low-temperature polarized phase (with the doubled translation period in the direction of the x axis) is C_{2v}^{17} (ref. 6). The symmetry of the intermediate phase lying between the two critical temperatures is not known. The only irreducible representation of the group D_{2h}^{18} which contains the group C_{2v}^{17} with the doubling of the lattice period in the direction of the x axis as its subgroup is two-dimensional and it admits the gradient invariant with the coefficient σ. The free energy corresponding to this representation is given by Eqs. (1) and (19). Our dependences of χ on T given by Eqs. (20) and (21) (Fig. 2b) reproduce well the measured dependences[8] in all phases. The intermediate phase should be modulated. The observation of such a modulated phase would confirm the validity of our model of the phase transitions in ammonium fluoroberyllate (the previously proposed models were discussed in ref. 9).

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Theory of nonequilibrium two-dimensional electrons in quantizing magnetic fields

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A study is made of the conductivity of nonequilibrium electrons (holes) whose energy spectrum consists of a set of discrete Landau levels. The range of frequencies near the cyclotron frequency is considered but the relation between the nonequidistant separation of the levels and their width is not restricted. The nonequidistant separation of the levels is caused by the nonparabolicity of the dispersion law and by the polaron effect. The relaxation is due to a strongly inelastic scattering by phonons. An analysis is presented of the steady-state conditions when the probability that an electron leaves the system (is transferred to a contact, captured by a trap, etc.) is independent of the electron state. In this case, most favorable conditions for the amplification of light are achieved when the difference between the gaps separating individual levels is much greater than the width of the levels (in particular, in the degenerate case). Under such conditions, it is necessary to impose restrictions on the average time spent by a carrier in the system for some types of the electron–phonon interaction in order to achieve the amplification. The amplification region grows rapidly when the magnetic field and the pumping energy are increased.

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When a strong transverse magnetic field H is applied to a size-quantized system (a thin crystal film or a narrow surface layer in a semiconductor), the energy spectrum of carriers (to be specific we shall consider electrons) transforms to a set of discrete Landau levels. It is possible to create an inverted population[1] in such a system with a discrete spectrum, which can then give rise to the amplification or even to the laser generation of light. This can occur only in very pure crystals; we shall not consider the scattering by impurities. For $\omega_c\tau_r \gg 1$ (where ω_c is the cyclotron frequency and τ_r the relaxation time), the electron relaxation is due to a strongly in-

elastic scattering by phonons.[2,3] The broadening of the Landau levels is, to some extent, analogous to the broadening of atomic levels, which is due to the inelastic scattering of electrons by photons (see, for example, ref. 4); however, in our case, the two-dimensional momentum of the system is conserved in the scattering.. Since the Landau levels are almost equidistant, the same phonon can be emitted due to transitions between different levels and, therefore, the electron–phonon interaction leads not only to damping of but also to an interference between electron states. The interference is governed by the ratio of the measure of the nonequidistant separation of the Landau levels $\Delta\omega_c$ to their width ($\sim\tau_r^{-1}$).

The spectrum is nonequidistant because of the electron dispersion and because of the polaron effect. It influences the profile of the cyclotron resonance peak at $T \sim \hbar\omega_c$ (ref. 2) when several excited levels are occupied by electrons. Roughly speaking, the spectrum is then the envelope of several lines corresponding to transitions from different levels, which are separated by a distance $\Delta\omega_c$. The effect of the nonequidistant separation of the levels under nonequilibrium conditions can best be studied at low temperatures, when there is no thermal smearing. In particular, in systems with $\Delta\omega_c \gg \tau_r^{-1}$, a strong amplification of light should occur (as shown below).

We shall consider mainly the case when nonequilibrium electrons occupy one two-dimensional subband and their density is so low that the electron–electron interaction can be neglected. For $|g\mu_B H - \hbar\omega_c| \gg \tau_r^{-1}$, the effects resulting in a spin flip have low probability and, therefore, we shall assume that all the electrons have equal spin projections. For simplicity, we shall restrict our discussion to a two-dimensional isotropic system of electrons and phonons (the results can then be easily generalized to the anisotropic case).

Assuming that the electron momentum p and its coordinate $\mathbf{r} = (x, y)$ as well as the vector potential $A = (0, Hx)$ are two-dimensional, we can write the Hamiltonian of the electron–phonon system in the following form (see ref. 2):

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_i; \quad \mathcal{H}_0 = \frac{1}{2m^*}P^2 + \frac{V}{8m^{*2}\omega_c^2}P^4 + \sum_{qj}\omega_{qj}a_{qj}^+a_{qj}; \tag{1}$$

$$P = p - \frac{e}{c}A; \quad \hbar = 1; \quad \omega_c = |e|H/(m^*c);$$

$$\mathcal{H}_i = \sum_{qj}\varepsilon_{qj}\exp(iqr)\,a_{qj}^+ + \text{H.c.}, \tag{2}$$

where q is the two-dimensional phonon wave vector and j labels the phonon branch, its polarization, and the wave number of the motion across a layer. It is assumed that the parameter governing the nonequidistant separation between the levels V and the interaction constants ε_{qj} are small so that the corrections $\sim |V|/\omega_c$, $1/\omega_c\tau_r$, and $|d\tau_r^{-1}/d\omega_c|$ can be neglected.

1. CONDUCTIVITY OF NONEQUILIBRIUM ELECTRONS

We shall assume that an electron at a time t_0 is described by a density matrix ρ_0 and the phonons are in thermal equilibrium. Many properties of a gas of nonequilibrium electrons can be described by the two-dimensional correlation function

$$\langle A(t-t_0)B(\tau-t_0)\rangle \equiv \mathrm{Tr}[\rho(t_0)A(t-t_0)B(\tau-t_0)], \tag{3}$$

where A and B are one-particle operators and ρ is the density matrix of the system consisting of an electron and the phonons, $\rho(t_0) = \rho_0\rho_{ph}$. In the most interesting time interval $t - t_0$, $\tau - t_0 \gg \omega_c^{-1}$, it is possible to carry out the averaging over phonons in Eq. (3) and express the correlation function $\langle A(t-t_0)B(\tau-t_0)\rangle$ in terms of the two-particle electron Green's function G(t), which satisfies a linear equation derived in ref. 3.[1)]

We shall study the linear response of a nonequilibrium electron to an electric field. The resonance case of field frequencies ω close to ω_c is of greatest interest. We shall introduce the nonequilibrium conductivity $\sigma(\omega, t-t_0) \equiv \sigma_{xx}(\omega, t-t_0)$ of a single electron

$$\langle j_x(t)\rangle_E = \sigma(\omega, t-t_0)E_x\exp(-i\omega t), \tag{4}$$

where $\langle j_x\rangle_E$ is the component of the current due to the field (the current due to nonequilibrium electrons can be nonzero even for E = 0). Using the standard methods, we can show that the following equations hold with the accuracy up to terms $\sim V/\omega_c$:

$$\left.\begin{aligned} \sigma(\omega, t-t_0) &= \frac{e^2\omega_c}{2m^*\omega}\int_{t_0}^{t} Q(t-\tau, \tau-t_0)\,e^{i\omega(t-\tau)}\,d\tau; \\ Q(t,\tau) &= Q_1(t,\tau) - Q_2(t,\tau); \\ \frac{2}{m^*\omega_c}\langle P_x(t-t_0)P_x(\tau-t_0)\rangle &= \begin{cases} Q_1(t-\tau, \tau-t_0), & t \geqslant \tau, \\ Q_2(\tau-t, t-t_0), & t \leqslant \tau. \end{cases} \end{aligned}\right\} \tag{5}$$

At a time t, the average (over a period) energy absorbed by an electron in a real classical field $E_x(\omega)$ is given by

$$W(\omega, t-t_0) = \frac{1}{2}E_x^2(\omega)\,\mathrm{Re}\,\sigma(\omega, t-t_0). \tag{6}$$

To calculate the luminescence of a nonequilibrium electron, we shall study its interaction with the quantized equilibrium radiation (at the phonon temperature) in the linear approximation following the method applied to an oscillator in ref. 5. The kinetics of the luminescence, the total luminescence spectrum during the relaxation time, and the conductivity (5) are governed by the correlation functions $Q_{1,2}$ (see ref. 5).

The dominant contribution to the luminescence and conductivity at frequencies $\omega \sim \omega_c$ is due to the terms in $Q_{1,2}$ similar to $\exp[-i\omega_c(t-\tau)]\bar{Q}_{1,2}(t-\tau, \tau-t_0)$, where $\bar{Q}_{1,2}$ are smooth functions which vary during a time $\sim\tau_r$ by $|V|^{-1}$. They can be expressed in the following form:

$$\bar{Q}_i(t-\tau, \tau-t_0) = \sum_{m,n=0}^{\infty}\varphi(m, n; t-\tau)\,\rho\left(n + \frac{1}{2} + \frac{1}{2}(-1)^i, \tau-t_0\right), \quad i = 1, 2. \tag{7}$$

The quantity $\rho(n, t-t_0)$ is equal to the sum of all the diagonal elements of the electron density matrix related to the n-th Landau level. It determines the probability density for an electron to be at the n-th level at a time t, provided it was described by the density matrix ρ_0 at a time t_0 [$\rho(n, 0) = \rho_0(n)$]. The matrix $\rho(n, t-t_0)$ satisfies

$$\frac{\partial \rho(n,\ t)}{\partial t} = -\Gamma_\rho(n)\,\rho(n,\ t)$$

$$+\sum_{k=-k_m}^{k_m}(1-\delta_{k,\,0})\,\Gamma_\rho\left(n+\frac{k}{2}-\frac{|k|}{2},\ k\right)\rho(n+k,\ t), \qquad (8)$$

which is a rate equation. The terms with $k \neq 0$ describe an electron transition from the (n + k)-th to the n-th level accompanied by the emission or absorption of a phonon. In the Born approximation, the transition probability (after averaging over the occupation numbers of phonons) is given by

$$\left.\begin{aligned}
&\Gamma_\rho(n,\ k) = 2\pi\sum_{q,j}|\varepsilon_{qj}|^2\,\delta(|k|\,\omega_c - \omega_{qj})\left(\frac{1}{2}lq\right)^{2|k|}\\
&\times\exp\left(-\frac{1}{2}l^2q^2\right)L_q^2(n,\ |k|)\left(\bar{n}_k+\frac{1}{2}+\frac{1}{2}\frac{k}{|k|}\right);\\
&\bar{n}_k = [\exp(|k|\,\omega_c/T)-1]^{-1};\quad l^2 = \hbar/m^*\omega_c;\\
&L_q(n,\ k) = L_n^k\left(\frac{1}{2}l^2q^2\right)\left[\frac{2^k n!}{(n+k)!}\right]^{1/2},\quad n\geqslant 0;\quad L_q(n,\ k)=0,\quad n<0,
\end{aligned}\right\} \qquad (9)$$

where L_n^k is a Laguerre polynomial. The probability that an electron leaves the n-th level is given by

$$\Gamma_\rho(n) = \sum_{k=-k_m}^{k_m}(1-\delta_{k,\,0})\,\Gamma_\rho\left(n+\frac{k}{2}-\frac{|k|}{2},\ k\right)\exp\left(-\frac{k\omega_c}{T}\right); \qquad (10)$$

k_m in Eqs. (9) and (10) determines the maximum gap between the Landau levels which are connected by one-phonon transitions.

The function φ in Eq. (7) is given by the following equation (see refs. 3 and 5):

$$\frac{\partial \varphi(m,\ n;\ t)}{\partial t} = -[\Gamma(m)+iP(m)]\,\varphi(m,\ n;\ t)$$

$$+2(m+1)\sum_{k=-k_m}^{k_m}(1-\delta_{k,\,0})\,\Gamma\left(m+\frac{k}{2}-\frac{|k|}{2},\ |k|\right)$$

$$\times\left(\bar{n}_k+\frac{1}{2}+\frac{1}{2}\frac{k}{|k|}\right)\varphi(m+k,\ n;\ t);\quad \varphi(m,\ n;\ 0) = (m+1)\,\delta_{m,\,n}. \qquad (11)$$

The parameters $\Gamma(m)$, $\Gamma(m, k)$ and $P(m)$ are given by Eqs. (4) and (5) of ref. 2. The system of linear equations (8)-(11) completely determines the kinetics of the conductivity and of the electron luminescence under arbitrary nonequilibrium conditions and for arbitrary ratio of the measure of nonequidistant separation to the width of the level.

2. STEADY-STATE CONDITIONS. AMPLIFICATION OF LIGHT

The conductivity of two-dimensional magnetized electrons under steady-state conditions is of considerable interest since: a) The field confining the carriers to a thin layer depends very often on the electron density, in which case the energy spectrum is well-defined only under steady-state conditions; b) it is desirable to develop steady-state sources of radiation with an easily tunable frequency. Equation (5) makes it possible to study the steady-state conductivity microscopically. We shall denote by $\zeta(t, t_0)$ the probability density that the electron whose conductivity is being considered at a time t had been created at a time t_0. Since, under steady-state conditions, the time of observation is the only distinguished moment of time, we can write $\zeta(t, t_0) = \zeta(t - t_0)$ and the steady-state conductivity is given by

$$\sigma_s(\omega) = \int_{-\infty}^{t} dt_0\,\zeta(t-t_0)\,\sigma(\omega,\ t-t_0)$$

$$\approx\frac{e^2}{2m^*}\int_0^\infty dt\int_0^\infty d\tau\exp[i\Omega t]\,[\bar{Q}_1(t,\ \tau)-\bar{Q}_2(t,\ \tau)]\,\zeta(t+\tau); \qquad (12)$$

$$\Omega = \omega-\omega_c,\quad |\Omega|\ll\omega_c.$$

We shall consider the simplest steady-state case, characterized by a recombination time τ_{rec}. In this context, the recombination means an arbitrary effect which results in the electron leaving the Landau levels (transition to a contact, capture at a trap, etc.). If τ_{rec} is independent of the electron state, the "recombination" and "creation" probabilities exhibit the same time dependences and

$$\zeta(t-t_0) = \lambda\exp[-\lambda(t-t_0)];\quad \lambda = \tau_{rec}^{-1},\quad \lambda\ll\omega_c. \qquad (13)$$

In the case (13), the absorption coefficient of light is given by

$$K(\lambda,\ \omega) = \mathrm{Re}\,\sigma_s(\omega) = \frac{\lambda e^2}{2m^*}\sum_{m,\,n}[\rho(n,\ \lambda)-\rho(n+1,\ \lambda)]\,\mathrm{Re}\,\varphi(m,\ n;\ \lambda-i\Omega), \qquad (14)$$

where $\rho(\lambda)$ and $\varphi(\lambda)$ are the Laplace transforms of the corresponding functions, which satisfy a system of linear algebraic equations [see Eqs. (8) and (11)].

Since the condition $\int_{-\infty}^{t}\zeta(t-t_0)dt_0 = 1$ is satisfied under arbitrary steady-state conditions, it follows from Eqs. (7) and (11) that $\int_0^\infty \mathrm{Re}\,\sigma_s(\omega)d\omega \approx (e^2/2m^*)\pi$, i.e., in the region of the cyclotron resonance peak [where the relation (12) is satisfied], the nonequilibrium electron absorbs the energy of the classical field.[2)] However, an amplification can occur in certain frequency intervals. Under the conditions (13), the maximum gain is obtained (see ref. 5) in the case when the transitions between neighboring levels exhibit a frequency spread:

$$|P(m\pm1)-P(m)|\gg\Gamma(m)+\Gamma(m\pm1)+\lambda,\quad |\Gamma(m,\ k)|. \qquad (15)$$

The quantity $K(\lambda, \omega)$ represents a set of almost overlapping fine-structure lines:

$$\mathrm{Re}\,\varphi(m,\ n;\ \lambda-i\Omega) = (m+1)\frac{\Gamma(m)+\lambda}{[\Gamma(m)+\lambda]^2+\Omega_m^2}\,\delta_{m,\,n};$$

$$\Omega_m = \omega-\omega_c-P(m). \qquad (16)$$

It should be noted that $2\Gamma(m) = \Gamma_\rho(m) + \Gamma_\rho(m+1)$ is the half-width of the line corresponding to the transition $|m\rangle \rightarrow |m+1\rangle$ calculated in the Born approximation for the equilibrium case, and $\omega_c + P(m)$ is the corresponding frequency (see refs. 2 and 3). It follows from Eq. (14) that the amplification of light can occur in the neighborhood of Ω_m only in the case of an inverted population $\rho(m, \lambda) - \rho(m+1, \lambda) < 0$. It follows from Eqs. (8), (12), and (14) that $\rho(m, \lambda) = \sum_{n=0}^{\infty} A_{mn}(\lambda)\rho_0(n)$ and, therefore, a necessary condition for the amplification of light in the case (15) is given by

$$\min[A_{mn}(\lambda)-A_{m+1\,n}(\lambda)]<0. \qquad (17)$$

The criterion (17) determines λ_{min} such that no amplification can occur for $\lambda < \lambda_{min}$ irrespective of the

pumping energy (which is governed by the density matrix ρ_0).

The matrix $A_{mn}(\lambda)$ can be evaluated in the important case of low temperatures, $\omega_c/T \gg 1$. If, for simplicity, we consider only the relaxation due to the transitions between neighboring Landau levels (i.e., we set $k_m = 1$, which often represents a good approximation for $\omega_c \gtrsim 10^{12}$ sec^{-1}, see refs. 2 and 3), we obtain from Eqs. (8)-(10) the following equation:

$$A_{mn}(\lambda) = [\Gamma_\rho(m)]^{-1} \prod_{k=m}^{n} \Gamma_\rho(k)\,[\lambda + \Gamma_\rho(k)]^{-1}, \quad m \leqslant n; \quad A_{mn}(\lambda) = 0, \quad m > n.$$

The criterion of amplification (17) then assumes the form

$$\max\,[\lambda + \Gamma_\rho(m) - \Gamma_\rho(m+1)] > 0. \tag{17a}$$

The conditions (17) and (17a) can be explained qualitatively as follows: Electrons should "recombine" rapidly enough (compared with the relaxation) to maintain a population inversion. It follows from the condition (17a) that, for $\lambda_{min} > 0$, the inequality $\omega_c/T \gg 1$ is satisfied provided the width of the Landau levels $\Gamma_\rho(m)$ increases monotonically as a function of the number of the level. When the inequalities (15) are weaker, the amplification can occur even for $\lambda < \lambda_{min}$ due to the interference of the electron states at different Landau levels. It should be also noted that the condition (17) is only necessary but not sufficient for a population inversion.

In the resonant scattering by acoustic phonons which are not localized in a two-dimensional layer, $\Gamma\rho(m)$ is a nondecreasing function in the deformation potential approximation.[3] For the scattering by phonons localized within the layer (which occurs, for example, in films with fixed or free surfaces), $\Gamma_\rho(m)$ is strongly nonmonotonic[3] and $\Gamma_\rho(m)/\Gamma_\rho(1)$ is very small for some m and a population inversion can be thus easily achieved. The widths of the levels $\Gamma_\rho(m)$ can be obtained from measurements of the equilibrium[2,3] and nonequilibrium cyclotron resonance spectra.

It can be shown that, if the conditions (15) and $\omega_c/T \gg 1$ are satisfied, our results can be applied to a degenerate gas provided the electron density at the excited Landau levels ($n > n_0$) is low and all the levels below and including n_0 are occupied. Since the scattering of an electron with $n > n_0$ by electrons with $n \leq n_0$ is forbidden by the law of conservation of the energy [because of the condition (15)], the electrons at the excited levels behave like nondegenerate carriers [the expressions for $\Gamma_\rho(n, k)$ and $\Gamma(n, k)$ are slightly modified since the scattering by phonons cannot result in electron transitions to an occupied state]. To achieve the amplification in the degenerate case, the inequality (15) has to be well satisfied since there is a very strong absorption peak (high density) at the frequency corresponding to the transition $|n_0\rangle \to |n_0 + 1\rangle$; the amplification region should be as far as possible from this peak.

Nonequilibrium magnetized electrons (holes) can be studied in experiments involving the flow of a current across a film (or a size-quantized semiconductor layer), optical excitation of the whole system, or abrupt switching of a magnetic field, etc. The effects discussed in the present paper can be detected only in pure crystals. The random field generated by imperfections should be very smooth and vary over distances much greater than the magnetic length l. For n-type InSb and $\omega_c = 5 \cdot 10^{12}$ sec^{-1}, we find that $l = \sqrt{\hbar/m^*\omega_c} \approx 3 \cdot 10^{-6}$ cm and $|V| = 2\omega_c^2/\varepsilon_g \approx 10^{11}$ sec^{-1}. The fluctuations of ω_c due to the impurity field E_i (inhomogeneous broadening) can be small compared with $|V|$ only if $\langle|\partial E_{ix}/\partial x|\rangle < |m^*\omega_c V/e| \approx 5 \cdot 10^6$ V/cm^2. Since this imposes very strong restrictions, a more favorable situation is created in impurity crystals with a weak degeneracy and a small screening radius. On the other hand, it is difficult to achieve amplification in the degenerate case (see our discussion). The optimum conditions for the amplification and generation of light by magnetized two-dimensional carriers are realized in systems where the impurities are screened by carriers of a different type and the spectrum of Landau levels is strongly nonequidistant. For such systems, a maximum gain can be obtained in a wide frequency range (proportional to $\Delta\omega_c$). The frequency range increases rapidly when the pumping energy is increased and the spectrum becomes more nonequidistant in stronger magnetic fields.

For $\omega_c > \omega_m$ (ω_m is the maximum phonon frequency), the relaxation occurs as a result of inelastic transitions with the participation of two phonons. Equations (8) and (11) remain valid and the corresponding expressions for the damping and the shift are obtained from ref. 3.

If the electrons occupy several two-dimensional subbands, the effects discussed in the present paper remain valid qualitatively but there are now several sets of Landau levels mutually coupled by phonons and, therefore, the expressions for the widths of the levels should be modified. The lines corresponding to transitions between different Landau levels also interfere in the case of the amplification of light polarized perpendicularly to the two-dimensional layer, which occurs as a result of the transitions between subbands. In the degenerate case with two subbands, amplification can occur only if the cyclotron frequencies or the shift of the levels due to nonparabolicity and due to the polaron effect are different in different subbands.

[1)] We shall not quote the corresponding general expression for $\langle A(t)B(\tau)\rangle$, since it is rather complex. It is similar to and can be obtained along the same lines as Eq. (4) of ref. 5, which describes the correlation function of the operators of a nonequilibrium nonlinear oscillator interacting with phonons. The energy spectrum of the oscillator is similar to the spectrum of a two-dimensional electron in a magnetic field. However, the electron wave function is governed not only by the number of the level but also (in the chosen gauge) by the projection of the center of the cyclotron orbit on the x axis (which does not affect the energy). Therefore, the two-particle Green's function f in ref. 5, which depends on four oscillator quantum numbers, should be replaced by the function G studied in ref. 3, which depends on eight electron quantum numbers.

[2)] This is due to the fact that the probability of a resonant transition from the n-th to the (n + 1)-th level accompanied by the absorption of light is greater than the probability of the transition $|n\rangle \to |n-1\rangle$ resulting in the emission of light ($n + 1 > n$). Such arguments led to the following conclusion in ref. 6: If the Landau levels are equidistant and, therefore, all the transitions have the same resonance frequency, isolated electrons ($\tau_r = \infty$) cannot give rise to the amplification of light. However, it can be shown that, if the relaxation is taken into account, the amplification can occur under steady-state conditions even for $\Delta\omega_c = 0$ (this was demonstrated in ref. 5 for a harmonic oscillator). It should be also noted that the steady-state luminescence due to the interaction with an equilibrium quantized field occurs for arbitrary $\Delta\omega_c\tau_r$.

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Self-localized states of conduction electron in ferromagnetic semiconductors at low temperatures

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A theoretical investigation is made of the fluctuon states of conduction electrons in the spin-wave range of conditions in a ferromagnetic semiconductor with a low Curie point. The analysis is made using the adiabatic and quasiclassical approximations allowing for the spinor nature of the electron wave function. Characteristics of the resultant self-localized states are investigated for various relationships between the conduction band width, spin–electron interaction constant, and Curie temperature. It is shown that in the case of sufficiently wide bands and moderately low temperatures allowance for the second component of the electron spinor has little influence on fluctuons outside the region of transition to fluctuon states, but it alters the temperature and nature of this transition, which is a broad phase transition of the first kind. In the case of moderately wide bands the influence of the second component of the electron spinor may be considerable throughout the low-temperature range.

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Interaction with a medium may result in self-localization of conduction electrons. Self-localized states were first investigated in ionic crystals, where – in the case of wide bands and sufficiently strong interaction with the crystal polarization – one should observe large-radius polarons.[1] Physically similar (and analyzed by similar methods) electron states may also form in other ordered systems. For example, in liquid helium[2] they are related to microcavities, in metal–ammonia solutions[3] to microcavities surrounded by a polarized medium, and in antiferromagnetic semiconductors[4–6] to a parallel spin region.

Self-localized large-radius states, which have much in common with polarons, may appear also in disordered or partly ordered systems with relatively easily variable parameters, for example, in semiconductor solutions or in systems which are close to a phase transition point.[7–9] They can be regarded as bound electron states and changes (fluctuations) of the parameter under consideration: This is why they are called fluctuons. A characteristic feature of fluctuons, which distinguishes them from other polaron-type electron excitations, is the property that the transition of most electrons to fluctuon states may occur in a narrow range of temperatures and appear as a broad phase transition in the electron system; moreover, fluctuons usually exist only in a limited temperature range outside absolute zero. Naturally, formation of fluctuons results from a gain in the thermodynamic potential (and not of energy, as in the case of self-localization in ordered systems) and fluctuons occur in highly excited states of a system.

Under suitable conditions fluctuons may appear also in ferromagnetic and paramagnetic semiconductors in the temperature range including the Curie point. In ferromagnets with low Curie points T_C this range covers the spin-wave region (but it does not reach T = 0). Fluctuons in such wide-band semiconductors are discussed in refs. 10-13 and 9, whereas those with narrow bands are considered in refs. 14 and 15 (the term "magnetic polaron" is used in refs. 11-13 and 15 for such self-localized states). Fluctuon states in these systems appear because of the s–d (or s–*f*) exchange interaction between electron spins and magnetic atoms.

The effective field due to this exchange interaction, is exerted by an s electron on the spins of magnetic atoms and it tends to orient these spins parallel to the magnetization; the effective field acting on the s electron orients its spin also parallel (or antiparallel) to this direction. It is assumed in refs. 10-13 that the spin is exactly parallel (antiparallel) to the magnetization and it is postulated that only one component of the spinor wave function of the electron differs from zero. As pointed out earlier,[9] we can ignore the second spinor component only if the characteristic size of a spin inhomogeneity (in the spin-wave region this size is of the order of the spin wavelength) is sufficiently small (then the rise of the kinetic energy of an electron, associated with the second component of the wave function varying rapidly in space, suppresses the inhomogeneity). However, at sufficiently low temperatures the characteristic size of a spin inhomogeneity is large and the s-electron spin becomes aligned with the local magnetization in various ways in different points in the localization region, i.e., the second component of the