MANY-ELECTRON TRANSPORT PHENOMENA IN NONDEGENERATE 2D ELECTRON SYSTEMS

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We consider static conductivity and cyclotron resonance in a normal two-dimensional electron fluid and Wigner crystal. The analysis is nonperturbative in the electron-electron interaction. It is based on the concept of a Coulomb force that drives an electron due to thermal fluctuations of electron density. This force controls electron dynamics in classical and semiclassical systems, where it is uniform over the electron wavelength, and strongly affects electron scattering by helium vapor atoms and ripplons, and thus electron transport. We derive and develop techniques for solving the many-electron quantum transport equation in the range from zero to quantizing magnetic fields B. We show that the static conductivity σ is nonmonotonous as a function of B, and that many-electron effects give rise to substantial narrowing of the cyclotron resonance absorption peak and to strong nonlinearity of the current-voltage characteristic even in the absence of electron heating. The results are in good qualitative and quantitative agreement with experiment and explain why different types of B-dependence of σ have been observed.

1. Introduction

Electrons above the surface of liquid helium provide an example of a nearly ideal two-dimensional (2D) electron system, where mobilities higher than in any solid state conductor have been obtained [1,2]. For characteristic electron densities $n_s \sim 10^7 - 10^8$ cm^{-2} and temperatures 0.1 K $< T <$ 2 K the interelectron distance $n_s^{-1/2}$ greatly exceeds the de Broglie wavelength $\lambda_T = \hbar/(2mT)^{1/2}$ (we set $k_B = 1$), and therefore the electron system is nondegenerate. At the same time, the ratio of the characteristic Coulomb

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energy of the electron-electron interaction to the kinetic energy, the plasma parameter

$$\Gamma = e^2(\pi n_s)^{1/2}/T \tag{1}$$

is usually large, $\Gamma \gtrsim 10$. Therefore the electron-electron interaction is by no means weak. The system is a strongly correlated normal fluid or, for $\Gamma \gtrsim 127$ (lower T), a Wigner crystal [3]-[5].

The normal electron fluid is a special type of many-electron system, which is very different from the much better understood Fermi liquid (and other quantum electron liquids) or low-density electron gas. Analysis of this fluid is complicated by the absence of "good" quasiparticles — the same problem encountered in the physics of liquids. In contrast to atomic or molecular liquids, in an electron fluid the interparticle forces are the long-range Coulomb forces. Another difference from 3D liquids is that relaxation of the total momentum is due to scattering (by ripplons and helium vapor atoms) of electrons which are inside the electron fluid, not on its boundary. Analysis of electron dynamics and transport phenomena in a normal electron fluid is necessary for understanding a large amount of experimental data on transport accumulated over the last few years [5]-[15].

To a large extent, transport is determined by the momentum exchange between electrons and scatterers. When the electron-electron interaction is strong, as in the fluid or Wigner crystal, it controls the collisions with the scatterers. The effect is expected to be particularly strong when a 2D electron system is placed into a transverse magnetic field B [16,17]. In the single-electron approximation the electron energy spectrum in the magnetic field is a set of discrete Landau levels, with separation $\hbar\omega_c$ (where $\omega_c = eB/m$ is the cyclotron frequency), and electrons do not have a finite group velocity. Therefore the standard picture of well separated in time elastic or quasielastic collisions of a moving electron with scatterers does not apply. As a consequence, the scattering is always strong, irrespective of the strength of the electron-scatterer coupling, with random potential of the scatterers being the only reason for the centers of cyclotron orbits to move.

The energy spectrum of the system of interacting electrons, on the contrary, is continuous even in the absence of scatterers. Therefore, although the electron-electron interaction does not change the total momentum of the electron system, it may mediate the momentum transfer to the scatterers and thus strongly affect the transport.

In this chapter we will provide an outline of the many-electron theory of static conductivity and cyclotron resonance of a normal electron fluid and a

Wigner crystal. In Sec. 2 we introduce and analyze the fluctuational electric field $\mathbf{E}_f$ that drives an electron because of the electron-electron interaction. In Sec. 3 we use the field $\mathbf{E}_f$ to provide qualitative picture of transport of a strongly correlated electron system, from weak to strong magnetic fields, and discuss nonlinear magnetoconductivity in a 2D electron fluid. In Sec. 4 we derive the many-electron quantum transport equation. The results are used in Sec. 5 for quantitative analysis of linear many-electron transport; at the end of Sec. 5 we discuss the Bragg-Cherenkov scattering by ripplons for a 2D Wigner crystal and the related nonlinearity of the conductivity. Sec. 6 contains concluding remarks. Detailed comparison of the theory of many-electron conductivity with experiment is given in the companion chapter by M.J. Lea.

2. Fluctuational Electric Field

2.1. CLASSICAL AND SEMICLASSICAL MANY-ELECTRON DYNAMICS

A theory of the dynamics and transport of a normal electron fluid can be formulated for not too low temperatures and/or for small enough electron densities or high magnetic fields, where the major effect of the electron-electron interaction on the electron dynamics may be described in terms of an electric field $\mathbf{E}_f$ [17] that drives each electron. Unlike the long-wavelength fluctuational field known in plasma physics [18], the field $\mathbf{E}_f$, although also of fluctuational origin, determines the Coulomb force on an individual electron. This force affects the electron motion during collisions with scatterers, and ultimately the momentum transfer from the many-electron system to the scatterers. A special significance of the field $\mathbf{E}_f$ for a 2D electron system in a magnetic field stems from the fact that a cyclotron orbit center drifts transverse to the fields $\mathbf{E}_f$, $\mathbf{B}$. A drifting electron occasionally collides with scatterers, as would a single electron in the absence of a magnetic field, and thus the field $\mathbf{E}_f$ may "restore" the simple Drude picture of electron relaxation that results from well separated in time collisions. Clearly, in this case $\mathbf{E}_f$ determines the collision probabilities and thus the transport coefficients.

The field $\mathbf{E}_f$ is particularly useful for characterizing the electron dynamics in a many-electron system provided this field is uniform over the electron wavelength $\lambda\!\!\!^{-}$ (otherwise the nonuniformity of the field would be as important as the field itself). A simple estimate of the field $\mathbf{E}_f$ and of the

parameter range where it is uniform can be obtained if one assumes that there is short-range order in the electron system in the interesting range $\Gamma \gtrsim 10$, as has been established by Monte Carlo calculations [19]-[23]. In this case the fields on the electrons are due to electron displacements from their (quasi)equilibrium positions (see Fig. 1). The characteristic thermal displacement δ and $\mathbf{E}_f$ can be estimated by linearizing electron equations of motion and by setting the potential energy of a displaced electron equal to T (cf. [17]):

$$eE_f\delta \sim e^2 \left| \frac{\partial^2}{\partial \mathbf{r}_n^2} \sum_m{}' |\mathbf{r}_n - \mathbf{r}_m|^{-1} \right|_{eq} \delta^2 \sim T, \tag{2}$$

(the derivative is evaluated for the equilibrium electron positions; clearly, the characteristic values of E_f, δ are independent of n). This gives

$$\langle E_f^2 \rangle \approx FTn_s^{3/2}, \quad \delta \sim T^{1/2} n_s^{-3/4} e^{-1} \tag{2}$$

(the coefficient F in (2) is discussed below).

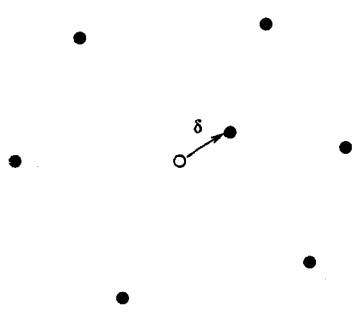

Figure 1. Fluctuational electron displacement in a strongly correlated system.

It is clear from Fig. 1 that the characteristic distance over which the field $\mathbf{E}_f$ varies is given by δ. The field is uniform over the wavelength $\lambda\!\!\!^{-}$ provided that $\lambda\!\!\!^{-} \ll \delta$. In the absence of the magnetic field the characteristic $\lambda\!\!\!^{-}$ is given by the thermal de Broglie wavelength $\lambda\!\!\!^{-}_T = \hbar(2mT)^{-1/2}$, whereas in a strong magnetic field it is given by the quantum magnetic length $l_B = (\hbar/m\omega_c)^{1/2}$. Therefore, with account taken of (2), the condition $\lambda\!\!\!^{-} \ll \delta$ can be written in the form

$$\lambda\!\!\!^{-} \ll \left(\frac{T}{m\omega_p^2}\right)^{1/2}, \; \lambda\!\!\!^{-} = l_B (2\bar{n}+1)^{-1/2}, \; \omega_p = \left(\frac{2\pi e^2 n_s^{3/2}}{m}\right)^{1/2}, \tag{3}$$

where

$$l_B = (\hbar/m\omega_c)^{1/2}, \; \omega_c = eB/m, \; \bar{n} = [\exp(\hbar\omega_c/T) - 1]^{-1}. \tag{4}$$

The condition (3) means that the electron motion is classical or, in a strong magnetic field, semiclassical. In the absence of the magnetic field ($\omega_c = 0$) (3) reduces to the inequality $T \gg \hbar\omega_p$. It is clear from Fig. 1

that ω_p is the frequency of electron vibrations about (quasi)equilibrium positions, and (3) is the condition for these vibrations to be classical. We note that for $T < \hbar\omega_p$, quantum effects come into play and the normal electron fluid becomes nonclassical. These quantum effects are not related to overlapping of the wave functions of different electrons; it is the motion of an electron in the field of other electrons that gets quantized.

In the presence of the magnetic field the fluid is classical provided

$$T \gg \hbar\omega_p, \hbar\omega_c. \tag{5}$$

For $\hbar\omega_c > T$ the fluid may be semiclassical. The motion of an electron in the field $\mathbf{E}_f$ is then a superposition of a quantum cyclotron motion with frequencies $\sim \omega_c$ and a semiclassical drift of the center of the cyclotron orbit. The frequency Ω that characterizes the drift can be estimated from Fig. 1 if one assumes that the field $\mathbf{E}_f$ is pointing towards the equilibrium position. Then the "displaced" electron drifts transverse to this field, with a velocity $eE_f/m\omega_c$, along a circle of radius δ. The frequency Ω is the reciprocal period of the drift. For

$$T \gg \hbar\Omega, \quad \Omega = \omega_p^2/\omega_c, \tag{6}$$

the drift is classical. The inequality (6) follows from (3) for $\hbar\omega_c \gtrsim T$. We note that (6) may be fulfilled in a sufficiently strong magnetic field, $\omega_c \gg \omega_p$, even if $T < \hbar\omega_p$, i.e., even if the fluid is non-classical for $B = 0$.

The conditions (5) and (6) show also where the dynamics of a Wigner crystal are classical and semiclassical, respectively. The spectrum of phonons of a crystal was analyzed in [24]; ω_p is a characteristic Debye frequency of the crystal for $B = 0$, whereas for $\omega_c \gg \omega_p$ the spectrum consists of the optical branch (that starts at ω_c) and a low frequency branch; the widths of the branches are $\sim \Omega$. We note that the melting temperature of the crystal T_m as given by the condition $\Gamma \approx 127$ [4,5] may be greater than or less than $\hbar\omega_p$ depending on the electron density ($T_m \propto n_s^{1/2}$, $\omega_p \propto n_s^{3/4}$; for electrons on helium $\hbar\omega_p/T_m \approx 1.3$ when $n_s = 10^8$ cm^{-2}). From this perspective it is particularly important that the magnetic field can be used to "switch" the 2D system, whether a fluid or a crystal, from the domain of quantum dynamics, $\hbar\omega_p \gg T$, to the semiclassical domain, $T \gg \hbar\Omega$.

2.2. DISTRIBUTION OF THE FLUCTUATIONAL FIELD

For classical and semiclassical electron systems the statistical averaging over the electron coordinates (or the positions of the centers of cyclotron orbits, in quantizing magnetic fields) may be performed independently from

the averaging over the electron momenta (the occupation numbers of the Landau levels, for $\hbar\omega_c > T$) [17,23]. In particular, the distribution of the Coulomb field on an electron, or of the components of the field, may be obtained by integrating the appropriate function over the electron coordinates with the weight $\exp(-H_{ee}/T)$, where

$$H_{ee} = \frac{1}{2}e^2 {\sum_{n,n'}}' |\mathbf{r}_n - \mathbf{r}_{n'}|^{-1} \tag{7}$$

It is straightforward to show, by changing to dimensionless coordinates $e\mathbf{r}_n n_s^{3/4} T^{-1/2}$, that the distribution of the dimensionless field $\mathbf{E}_f/n^{3/4}T^{1/2}$ is determined by the single parameter Γ, and in particular the coefficient F in Eq. (2) is a function of Γ only. We note that $\langle E_f^2 \rangle$ can be expressed in terms of the two-particle distribution function of the electron system $\mathcal{P}(\mathbf{r}_1, \mathbf{r}_2)$:

$$e^2\langle E_f^2 \rangle \equiv \langle (\nabla_n H_{ee})^2 \rangle = -eT\langle \nabla_n \mathbf{E}_n \rangle \equiv \frac{e^2 T}{n_s S} \int \frac{\mathcal{P}(\mathbf{r}_1, \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3} d\mathbf{r}_1 d\mathbf{r}_2 \tag{8}$$

(S is the area of the system).

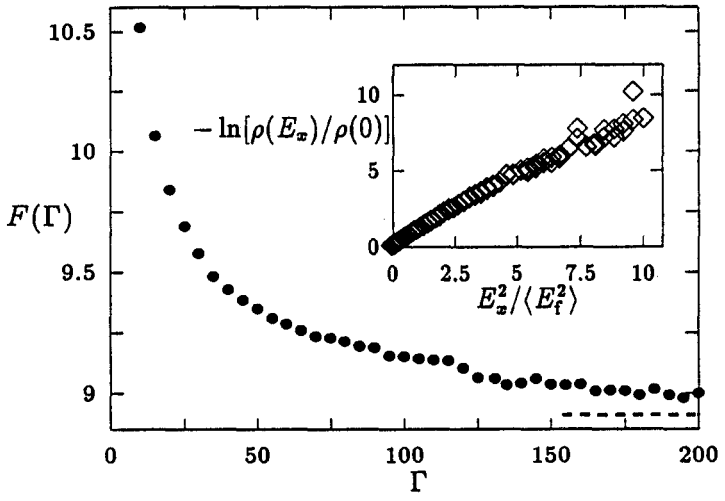

Figure 2. The scaled mean square fluctuational field $F(\Gamma) = \langle E_f^2 \rangle / T n_s^{3/2}$ from Monte Carlo calculations [23]. The asymptotic value of F for a harmonic Wigner crystal is shown dashed. Inset: the logarithm of the distribution of a component of the field for $\Gamma = 60$.

The function F in (2) and the distribution of the field $\mathbf{E}_f$ can be easily found for large Γ (low T) where electrons form a Wigner crystal and electron vibrations about equilibrium positions can be described in the harmonic approximation [25](b). Both transverse and longitudinal modes of the crystal contribute to the field, and the numerical value of F is ≈ 8.91. The distribution of the field is Gaussian, which is a standard result for the distribution of the force per particle in a classical solid.

In the opposite limit of small Γ the major contribution to the field $\mathbf{E}_f$ comes from pair collisions, and

$$F(\Gamma) \approx 2\pi^{3/2}\Gamma^{-1}, \quad \Gamma \ll 1.$$

In the most interesting range of the electron fluid and the melting transition, the function F and the distribution of the field were obtained from Monte Carlo simulations [23]. We used the technique similar to that in [19], with periodic boundary conditions and with the number of particles equal to 324. The field on an electron was evaluated as the gradient of the potential in which the electron was moving.

The results for the scaled mean square fluctuational field $F(\Gamma)$ and the logarithm of the probability density of a component of the field are shown in Fig. 2. For $\Gamma \gtrsim 10$, the function F decreases monotonically with increasing Γ. Quite remarkably (but in qualitative agreement with the above small-Γ estimate which, when extrapolated to $\Gamma \sim 1$, gives $F(1) \approx 11$), the variation of F is small in this range, although the structure of the system changes dramatically, from a liquid where correlations in electron positions decay within twice the mean electron separation, to a crystal. The function $F(\Gamma)$ has a smeared singularity at the melting point $\Gamma \approx 127$. We observed a small hysteresis of F as well as hysteresis of the mean electron potential energy and the effective diffusion constant for crystal and random initial configurations in the range of the transition (the hysteresis of the mean energy and the diffusion constant were observed earlier in molecular dynamic simulations [20,22]). The behavior of F is a consequence of $\langle E_f^2 \rangle$ being determined primarily by the short-range order in the system, according to Eq. (8).

We note that, with account taken of Eqs. (2), (8) and the above data for $\langle E_f^2 \rangle$, the criterion for the fluctuational field to be uniform over the electron wavelength

$$\lambda\!\!\!^{-} \, |\langle \nabla_n \mathbf{E}_n \rangle| = e\langle E_f^2 \rangle \lambda\!\!\!^{-} \, T^{-1} \ll \langle E_f^2 \rangle^{1/2} \qquad (3a)$$

takes on a form that coincides with the inequality (3).

The shape of the distribution of the field in its central part is close to Gaussian for $\Gamma > 10$, cf. Fig. 2. The tail of the distribution could be investigated analytically by evaluating the minimal energy $H_{ee}^{(min)}[\mathbf{E}_f]$ of the many-electron configuration in which one of the electrons is driven by a given field $E_f \gg \langle E_f^2 \rangle^{1/2}$ [23]. The logarithm of the distribution on the tail is given by $-\left(H_{ee}^{(min)}[\mathbf{E}_f] - H_{ee}^{(min)}[0]\right)/T$. The results are in a very good agreement with the results of Monte Carlo simulations.

As Γ decreases, the deviation of the field distribution from the Gaussian shape becomes more substantial. However, the difference between the mean reciprocal field $\langle E_f^{-1} \rangle$, which is of interest for transport (see Secs. 4 and 5), and its value for a Gaussian distribution $\pi^{1/2} \langle E_f^2 \rangle^{-1/2}$ is less than 10% for $\Gamma \gtrsim 20$.

3. Qualitative Picture of Many-Electron Transport

3.1. WEAK TO MODERATELY STRONG MAGNETIC FIELDS

For several types of 2D electron systems, and for electrons on helium in particular, electron scattering is due to collisions with short-range scatterers, and the scattering is elastic or quasielastic. Clearly, in a strongly correlated electron fluid at most one electron at a time collides with a given short-range scatterer. If the characteristic duration of a collision t_{coll} is small compared to the characteristic time over which the field $\mathbf{E}_f$ varies in order of magnitude (the correlation time in the electron system), the effect of the electron-electron interaction on the collisions may be fully described in terms of $\mathbf{E}_f$. Indeed, in this case the field $\mathbf{E}_f$ is all that an electron "knows" about other electrons during a collision.

We will first analyze the effect of the field $\mathbf{E}_f$ on the collisions with short-range scatterers for not too strong magnetic fields where

$$T \gg e\langle E_f^2 \rangle^{1/2} \lambda\!\!\!^{-}_T \sim \hbar\omega_p \gg \hbar\omega_c, \quad \lambda\!\!\!^{-}_T = \hbar(2mT)^{-1/2}. \tag{9}$$

We note that (9) does not mean that the magnetic field is weak. The field may well be classically strong, i.e., there may hold the inequality $\omega_c \tau \gg 1$, where τ^{-1} is the scattering rate. In what follows we use the term "moderately strong fields" for classically strong magnetic fields that satisfy condition (9).

In the range (9) the electron motion is classical (cf. (5)), and an electron has a well-defined kinetic energy $p^2/2m \sim T$ and a well-defined potential energy in the field of other electrons. Uncertainty of each of these energies is

determined by smearing $\lambda\!\!\!^{-}_T$ of the electron wave packet. For an electron in an electric field $\mathbf{E}_f$ this uncertainty is given by $eE_f\lambda\!\!\!^{-}_T$ and is small compared to T. This means that, in spite of the electron system being strongly correlated, the electron-electron interaction has little effect on collisions with short-range scatterers in the absence of a magnetic field. One can also see this from the following arguments. The duration of a collision is determined by the time it takes an electron to fly past the scatterer. For short-range scatterers and for electrons with thermal velocities $v_T = (2T/m)^{1/2}$ this time is $t_{\rm coll} \sim \lambda\!\!\!^{-}_T/v_T \sim \hbar/T$. The acceleration of the electron in the field E_f over this time is $\sim eE_f\lambda\!\!\!^{-}_T v_T/T \ll v_T$. We note that the condition $T \gg \hbar\omega_p$ guarantees that $t_{\rm coll} = \hbar/T$ is small compared to the velocity correlation time ω_p^{-1} [21] (cf. Fig. 1).

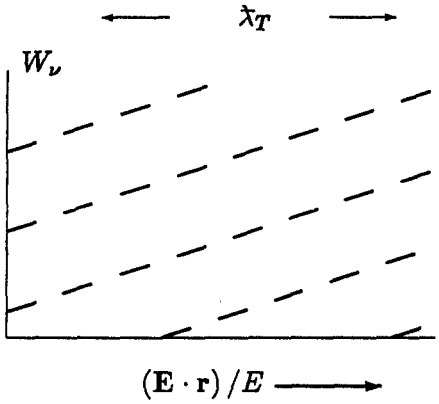

Figure 3. Single-electron energy levels W_ν in the electric field $\mathbf{E}$ and transverse magnetic field (tilted Landau levels). Uncertainty of the electron kinetic energy exceeds $\hbar\omega_c$ for the shown size of the electron wave packet $\lambda\!\!\!^{-}_T$.

The role of the field $\mathbf{E}_f$ becomes very different in the presence of the magnetic field, since the field $\mathbf{E}_f$ tilts Landau levels and makes the electron energy spectrum continuous. It is clear from Fig. 3 that for an electron wave packet of size $\lambda\!\!\!^{-}_T$, the discreteness of the one-electron energy spectrum due to Landau quantization is washed out by many-electron effects if $eE_f\lambda\!\!\!^{-}_T \gg \hbar\omega_c$. One would therefore expect that even in classically strong magnetic fields, $\omega_c\tau \gg 1$, collisions with scatterers will occur nearly as if there were no magnetic field at all [12]. Then the many-electron system should not display magnetoresistance, and in the whole range (9) the static conductivity σ is given by a simple expression

$$\sigma \equiv \sigma_{xx}(\omega = 0) = \frac{e^2 n_s}{m}\frac{\tau_{B=0}}{1+\omega_c^2\tau_{B=0}^2}, \quad e\langle E_f^2\rangle^{1/2}\lambda\!\!\!^{-}_T \gg \hbar\omega_c, \hbar\tau_{B=0}^{-1} \qquad (10)$$

where $\tau_{B=0}^{-1}$ is the scattering rate in the absence of the magnetic field calculated in the approximation where the effect of the electron-electron interaction on collisions with scatterers is ignored.

We emphasize that the absence of magnetoresistance in the range (9) for

classically strong magnetic fields, known experimentally since [6], is a purely many-electron effect. In the single-electron approximation, the character of electron scattering for $\omega_c\tau \gg 1$ is qualitatively different from that in the absence of the magnetic field even in the range of high temperatures $T \gg \hbar\omega_c$, where there applies the notion of a classical electron orbit. For an electron colliding with a short-range scatterer the orbit has the shape of a rosette [26]. It is a nearly closed circle, with the characteristic cyclotron radius $R_B \sim (T/m)^{1/2}\omega_c^{-1}$ and with the center slowly rotating around the scatterer. The electron is coming back to the scatterer, over and over again. Therefore it experiences *multiple* collisions with the same scatterer, in contrast to a single collision in the absence of the magnetic field. In fact, in the single-electron approximation the number of collisions is determined by the probability to find another scatterer while spinning around the given scatterer.

Single-electron magnetotransport is usually analyzed using the self-consistent Born approximation (SCBA) [2,10,11]. In this approximation the relaxation rate $\tau_{SCBA}^{-1}(B)$ is given by the relaxation rate $\tau_{B=0}^{-1}$ multiplied by the factor of the increase of the density of states that results from "squeezing" of the energy spectrum into (broadened) Landau levels. This factor, in turn, is given by the ratio between the interlevel distance and the level broadening which is itself due to scattering, $\omega_c/\tau_{SCBA}^{-1}(B)$. The result for the relaxation rate is of the form

$$\tau_{SCBA}^{-1}(B) = \vartheta_{SCBA}\omega_c^{1/2}\tau_{B=0}^{-1/2}, \quad \vartheta_{SCBA} \sim 1, \quad \omega_c \gg \tau_{SCBA}^{-1}(B). \qquad (11)$$

Detailed analysis of the SCBA for a nondegenerate 2D system is given in [10]; an alternative approach to the single-electron theory is based on the method of moments for the frequency-dependent conductivity $\sigma_{xx}(\omega)$, and for quantizing magnetic fields it gives results similar to the SCBA [27]. If $\tau_{SCBA}(B)$ is used instead of $\tau_{B=0}$ in Eq. (10), the magnetoconductivity in classically strong fields is $\propto B^{-3/2}$ (the resistivity is $\propto B^{1/2}$); it differs from the result (10) by a large factor $(\omega_c\tau_{B=0})^{1/2} \gg 1$.

3.2. "STRONG" STRONG MAGNETIC FIELDS

Onset of magnetoresistance in classically strong magnetic fields, $T > \hbar\omega_c > e\langle E_f^2\rangle^{1/2}\lambda_T$, can be qualitatively understood in the following way [12]. If there were no fluctuational electric field, an electron in the magnetic field would be moving along a rosette described above, coming back to the scatterer with period $2\pi/\omega_c$. In the presence of the field $\mathbf{E}_f$ the center of the

electron cyclotron orbit drifts with a velocity $v_d = E_f/B$. Therefore the number of times the scatterer is encountered is finite. It is clear from Fig. 4 that in order of magnitude, this number is $\zeta = \lambda\!\!\!^{-}_T(2\pi E_f/B\omega_c)^{-1}$ (here, $\lambda\!\!\!^{-}_T$ stands for the characteristic "size" of the scatterer; if scatterers are not point-like and their size exceeds $\lambda\!\!\!^{-}_T$, the above expression should be appropriately modified). One would expect classical magnetoresistance to arise in the many-electron system for $\zeta > 1$.

The magnetoconductivity σ can be estimated using the Einstein relation between the conductivity and the diffusion coefficient D, $\sigma = e^2 n_s D/T$. It is seen from Fig. 4 that scattering results in a shift of the electron orbit by the cyclotron radius R_B. Therefore $R_B^2/2$ may be associated with the squared diffusion length, and then $D = R_B^2/2\tau$. The scattering rate τ^{-1} is proportional to the encountering factor ζ [28], and the expression for σ takes on the form

$$\sigma = \frac{e^2 n_s}{2T} R_B^2 \tau^{-1}, \quad \tau^{-1} \sim \zeta \tau_{B=0}^{-1}, \quad (12)$$

$$\zeta = \lambda\!\!\!^{-}\, \omega_c B/2\pi \langle E_f^2 \rangle^{1/2}.$$

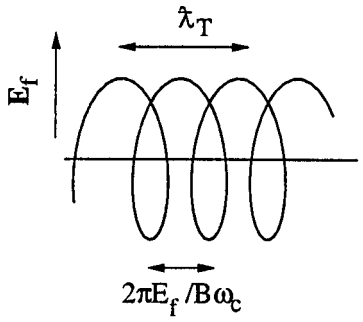

Figure 4. Classical electron trajectory in the fluctuational electric field $\mathbf{E}_f$ and transverse magnetic field B. The radius of the spiral $R_B \sim (T/\hbar\omega_c)\lambda\!\!\!^{-}_T$.

A distinctive feature of the many-electron magnetoconductivity (12) is its *independence* of the field B for classically strong fields where $R_B \sim (T/\hbar\omega_c)\lambda\!\!\!^{-}_T \propto B^{-1}$ and $\zeta \propto B^2$ ($\lambda\!\!\!^{-} = \lambda\!\!\!^{-}_T$ for $\hbar\omega_c \ll T$).

The arguments used to obtain an estimate of σ apply also if the electron system is in a quantizing magnetic field. For strongly quantizing fields, $\hbar\omega_c \gg T$, an electron is a "hard disk" with characteristic size $l_B = (\hbar/m\omega_c)^{1/2}$. It drifts transverse to the magnetic field with a velocity E_f/B, and the characteristic duration of a collision is [17]

$$t_e = l_B B \langle E_f^{-1} \rangle.$$

The scattering rate is increased compared to $\tau_{B=0}^{-1}$ by the encountering factor $\zeta \sim \omega_c t_e \propto B^{3/2}$ (the same estimate can be obtained using density-of-states arguments: the kinetic energy uncertainty of an electron wave packet of a size l_B in the field E_f is $\sim eE_f l_B$, and therefore the density of states into which the electron may be scattered is increased by a factor $\zeta \sim \hbar\omega_c/eE_f l_B$).

In the whole domain $\hbar\omega_c \gtrsim T$, the value of R_B in Eq. (12) is given by the characteristic radius of the electron wave function, whereas $t_{\rm coll}$ is given by the time-of-flight over the wavelength λ (3),

$$R_B = l_B(2\bar{n}+1)^{1/2}, \qquad t_{\rm coll} = l_B(2\bar{n}+1)^{-1/2}B\langle E_f^{-1}\rangle. \qquad (13)$$

It follows from (10), (12), (13) that the magnetoconductivity σ is non-monotonic as a function of B. It decreases as B^{-2} in the range (10), reaches a minimum for "strong" classically strong fields where $\zeta \gg 1$, and then increases as $B^{1/2}$ for $\hbar\omega_c \gg T$ (see Fig. 6).

Eq. (12) for τ^{-1} gives also the characteristic value of the halfwidth γ of the peak of cyclotron resonance of a many-electron system in a strong magnetic field. We note that in the classical range $T \gg \hbar\omega_c$ the expressions for γ and for the relaxation rate in Eqs. (10), (12) for the static conductivity coincide with each other. This is no longer true in the quantum range, although still $\gamma \sim \tau^{-1}$ [29].

3.3. INTERELECTRON MOMENTUM EXCHANGE

The exchange of momentum between electrons does not affect the long-wavelength conductivity directly [30], since it does not change the total momentum of the electron system. However, its role in the transport may be substantial. This is well-known in the theory of low-density electron plasma in semiconductors [31] from the analysis of the case where the single-electron rate of collisions with scatterers $\tau_s^{-1}(\epsilon)$ depends on the electron energy ϵ. In the single-electron approximation the static conductivity σ (for $B = 0$) is a sum of the conductivities of electrons with different energies and therefore different scattering rates; it is given by the averaged (over ϵ) *reciprocal* scattering rate, $\sigma = e^2 n_s \overline{\tau_s(\epsilon)}/m$. The interelectron momentum exchange occurs via pair electron-electron collisions. If the frequency of these collisions greatly exceeds $\tau_s^{-1}(T)$, then $\sigma = e^2 n_s/m\overline{\tau_s^{-1}(\epsilon)}$. These results were applied to 2D electrons on helium in [9].

Based on the discussion in Sec. 3.1 one would expect that similar arguments apply to the static conductivity of a *strongly correlated* classical electron fluid for weak magnetic fields. Here, an electron exchanges its momentum with other electrons not via pair collisions but by being accelerated by the Coulomb force from these electrons. The rate of interelectron momentum exchange $\tau_{\rm ex}^{-1}$ is given by the frequency of the electron vibrations ω_p, as it is clear from Fig. 1. If $\omega_p\tau \gg 1$, as it was assumed in Eq. (10), the conductivity is determined by the relaxation rate of the total momentum

of the many-electron system, i.e., by the average rate $\overline{\tau_s^{-1}(\epsilon)}$.

The role of interelectron momentum exchange in strong fields B, where collisions with scatterers are mediated by the electron-electron interaction, is clear from the analysis of cyclotron resonance. Resonant absorption at frequency ω_c is due to transitions between neighboring tilted Landau levels in Fig. 3, $|\nu\rangle \to |\nu+1\rangle$. "Partial spectra" which correspond to different transitions are broadened because of collisions with scatterers. Prior to averaging over the many-electron ensemble the broadening of a spectrum $\gamma_\nu(\mathbf{E}_f)$ depends both on the level number ν and $\mathbf{E}_f$. Even if all partial spectra are Lorentzian, but with different widths, the total spectrum may be non-Lorentzian (see [32] for a review of the theory of systems with equidistant or nearly equidistant energy levels).

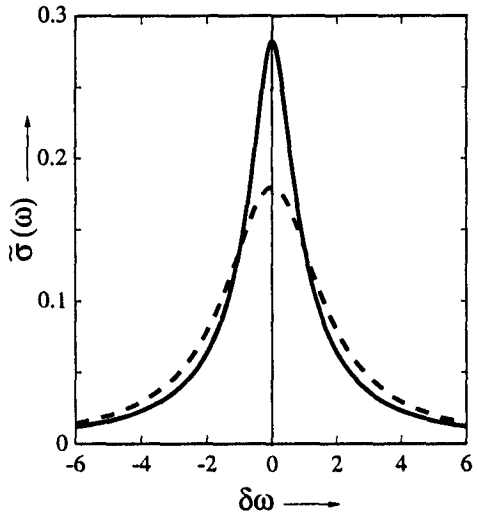

Figure 5. **Reduced high-frequency conductivity (14) near the cyclotron resonance peak as a function of the reduced frequency $\delta\omega = (\omega - \omega_c)/\gamma_0$ for Gaussian distribution of $\mathbf{E}_f$ (solid line); $\tilde{\sigma}(\omega) = 2m\gamma_0\sigma_{xx}(\omega)/\pi e^2 n_s$ ($\gamma_0 \equiv \gamma_0(\langle E_f^2\rangle^{1/2})$). Lorentzian distribution with the same area and with the halfwidth $\pi^{1/2}\gamma_0$ is shown with a dashed line (from [29]).**

Electron-electron interaction gives rise to transitions between the Landau levels of individual electrons and to drift of the cyclotron orbit centers. Electron motion results also in averaging of the widths $\gamma_\nu(\mathbf{E}_f)$. The characteristic frequency of the corresponding interelectron momentum exchange is seen from Fig. 1 to be $\tau_{ex}^{-1} = \Omega = \omega_p^2/\omega_c$ for $\omega_p \ll \omega_c$. For fast momentum exchange, $\tau_{ex}^{-1} \gg \tau^{-1}$, this is relaxation of the total momentum of the electron system that determines the shape of the cyclotron resonance spectrum, and the spectrum is Lorentzian with a width given by the appropriately averaged $\gamma_\nu(\mathbf{E}_f) \propto E_f^{-1}$ [29]. In the opposite case $\tau_{ex}^{-1} \ll \tau^{-1}$ the spectrum is non-Lorentzian. For $T \ll \hbar\omega_c$ the conductivity is given by the expression

$$\sigma_{xx}(\omega) \approx \frac{e^2 n_s}{2m}\left\langle \frac{\gamma_0(\mathbf{E}_f)}{\gamma_0^2(\mathbf{E}_f) + (\omega-\omega_c)^2}\right\rangle. \qquad (14)$$

The shape of the peak of $\sigma_{xx}(\omega)$ depends on the shape of the distribution of the fluctuational field. For the case of Gaussian distribution it is shown in Fig. 5.

3.4. NONLINEAR EFFECTS IN A COLD 2D ELECTRON FLUID

An interesting feature of nondegenerate 2D electron systems is the possibility to observe strong nonlinear effects without heating the system. One of such effects is the occurrence of *negative differential conductivity* in quantizing magnetic fields [16,17(a),33] in a 2D electron fluid. It arises because an external electric field $\mathbf{E}_{\rm ext}$ affects the electron drift in the fluctuational field $\mathbf{E}_{\rm f}$ and thus changes the rate of collisions with scatterers and the electron motion during a collision. The nonlinearity may be expected to be substantial if the field $E_{\rm ext}$ becomes of the same order of magnitude as the fluctuational field $\langle E_{\rm f}^2\rangle^{1/2}$. We will use the arguments of Sec. 3.2 and Eq. (12) to estimate the nonlinear longitudinal conductivity σ which determines the current density along the field $\mathbf{j}_{\|} = \sigma \mathbf{E}_{\rm ext}$.

One of the factors in Eq. (12) which are affected by the field $E_{\rm ext}$ is the encountering factor ζ. Indeed, the mean square drift velocity of an electron in the fluctuational and external fields, $\langle v_d^2(E_{\rm ext})\rangle = \langle(\mathbf{E}_{\rm ext}+\mathbf{E}_{\rm f})^2\rangle/B^2$, increases with increasing $E_{\rm ext}$. This results in the decrease of ζ, since ζ is proportional to the time of flight past the scatterer and thus inversely proportional to the drift velocity, $\zeta(E_{\rm ext}) \sim \zeta(0)\langle v_d^{-1}(E_{\rm ext})\rangle/\langle v_d^{-1}(0)\rangle$. Clearly, $\zeta(E_{\rm ext}) \propto E_{\rm ext}^{-1}$ for strong fields.

In the analysis of elastic or quasi-elastic collisions with short-range scatterers in the field $\mathbf{E}_{\rm ext}$ one should also take into account that, because of the energy conservation law, an electron can "jump" away from the scatterer only transverse to the total field $\mathbf{E}_{\rm ext}+\mathbf{E}_{\rm f}$. Therefore the jumps in the direction of the field $\mathbf{E}_{\rm ext}$ are suppressed, and the characteristic squared diffusion length $R_B^2/2$ for the jumps along the field in Eq. (12) should be multiplied by a factor of the type $\langle E_{\rm f}^2\rangle/\langle(\mathbf{E}_{\rm ext}+\mathbf{E}_{\rm f})^2\rangle$. In strong fields $\mathbf{E}_{\rm ext}$ this gives an extra factor $\propto E_{\rm ext}^{-2}$ in the conductivity as given by Eq. (12).

On the whole, the nonlinear conductivity decreases with the increasing external field in a nondegenerate 2D electron fluid, and $\sigma \propto E_{\rm ext}^{-3}$ for $E_{\rm ext} \gg \langle E_{\rm f}^2\rangle^{1/2}$ [17]. Negative differential conductivity in the magnetic field was first found in [16] phenomenologically by assuming that the effect of the electron-electron interaction may be described by a finite lifetime of an individual electron. In this model $\sigma \propto E_{\rm ext}^{-2}$ for strong field, in contradiction with the above result. Recently negative differential conductivity was obtained in the single-electron approximation using an appropriately mod-

ified self-consistent Born approximation [33]. We note that, for the fields $E_{\rm ext}$ which are so strong that $|eE_{\rm ext}|R_B \gtrsim \hbar\omega_c$, the current should sharply increase [34], since elastic scattering may be accompanied by transitions between the Landau levels.

4. Many-Electron Quantum Transport Equation

The analysis of the long-wavelength conductivity $\sigma_{xx}(\omega)$ can be done using the Kubo formula which relates $\sigma_{xx}(\omega)$ to the Fourier transform of the correlation function of the total many-electron momentum $\hat{\mathbf{P}}$

$$\left\langle e^{i\hat{H}t/\hbar}\hat{P}_x e^{-i\hat{H}t/\hbar}\hat{P}_x\right\rangle = {\rm Tr}_e\left[e^{i\hat{H}_0 t/\hbar}\hat{P}_x e^{-i\hat{H}_0 t/\hbar}\hat{\mathcal{G}}_x(t)\right],\ \hat{\mathbf{P}} = \sum_n \hat{\mathbf{p}}_n, \quad (15)$$

$$\hat{H}_0 = \frac{1}{2m}\sum_n \hat{\mathbf{p}}_n^2 + \hat{H}_{\rm ee}, \qquad \hat{\mathbf{p}}_n = -i\hbar\boldsymbol{\nabla}_n - e\mathbf{A}(\hat{\mathbf{r}}_n).$$

Here, $\hat{H}$ is the Hamiltonian of the whole system of electrons and scatterers, ${\rm Tr}_e$ is taken over the wave functions of the isolated many-electron system with the Hamiltonian $\hat{H}_0$, and $\mathbf{A}(\mathbf{r})$ is the vector-potential of the magnetic field. The Hamiltonian of the electron-electron interaction $H_{\rm ee}$ is given in Eq. (7).

Interaction of the electrons with scatterers (ripplons, helium vapor atoms) has been moved in (15) into the operator $\hat{\mathcal{G}}_x(t)$,

$$\hat{\mathcal{G}}_x(t) = Z^{-1}{\rm Tr}_b\left[\hat{S}(t)\hat{P}_x e^{-\beta\hat{H}}\hat{S}^+(t)\right],\ \hat{S}(t) = e^{i\hat{H}_0 t/\hbar}e^{-i\hat{H}t/\hbar}, \quad (16)$$

$$\hat{H} = \hat{H}_0 + \hat{H}_i + \hat{H}_b, \qquad \hat{H}_i = \sum_{\mathbf{q}}\hat{V}_{\mathbf{q}}\hat{\rho}_{\mathbf{q}}, \quad \hat{\rho}_{\mathbf{q}} = \sum_n e^{i\mathbf{q}\hat{\mathbf{r}}_n}$$

where the trace ${\rm Tr}_b$ is taken over the wave functions of ripplons and/or the positions of helium vapor atoms, $\hat{H}_b$ is the Hamiltonian of ripplons (vapor atoms), H_i is the interaction Hamiltonian ($\hat{V}_{\mathbf{q}}$ depends on the dynamical variables of ripplons or vapor atoms), and $Z = {\rm Tr}_e{\rm Tr}_b\exp(-\beta\hat{H})$. In the case of scattering by a static random field other than that of ripplons or vapor atoms the expression for $\hat{\mathcal{G}}_x(t)$ (16) should include averaging over realizations of the field. The results below apply to this case provided the random field is Gaussian and has a small correlation length.

We assume that the interaction $\hat{H}_i$ is weak enough so that the duration of an electron collision with a scatterer $t_{\rm coll}$ is much smaller than the intercollision interval which is given by the relaxation time τ,

$$t_{\rm coll} \ll \tau \tag{17}$$

(both $t_{\rm coll}$ and τ have to be found). In the range (17) the collisions occur independently from each other, and therefore the transport equation for the operator $\hat{\mathcal{G}}_x(t)$ should be Markovian.

The many-electron quantum transport equation (QTE) to second order in $\hat{H}_i$ can be written in a standard form which is known for systems with a small number of degrees of freedom, like one-electron systems or an atom coupled to radiation, and which contains the double commutator $\left[\hat{H}_i(t), \left[\hat{H}_i(t'), \hat{\mathcal{G}}_x(t)\exp(-\beta\hat{H}_b)\right]\right]$ (cf. [32,35]). We will assume that the collisions are quasielastic, and in particular the frequencies of ripplons with the characteristic q are small, $\omega_q \ll t_{\rm coll}^{-1}, T/\hbar$. In this case ripplons create a quasistatic random Gaussian field, as do vapor atoms. Then the QTE is of the form [29]

$$\frac{\partial\hat{\mathcal{G}}_x}{\partial t} = -\hbar^{-2}\sum_{\mathbf{q}}\overline{|V_{\mathbf{q}}|^2}\sum_{n,n'}\int_0^t dt'\left[e^{i\mathbf{q}\hat{\mathbf{r}}_n(t)}, \left[e^{-i\mathbf{q}\hat{\mathbf{r}}_{n'}(t')}, \hat{\mathcal{G}}_x(t)\right]\right], \tag{18}$$

$$\hat{\mathbf{r}}_n(t) = e^{i\hat{H}_0 t/\hbar}\hat{\mathbf{r}}_n e^{-i\hat{H}_0 t/\hbar}; \quad \overline{|V_{\mathbf{q}}|^2} \equiv \langle\hat{V}_{\mathbf{q}}\hat{V}_{-\mathbf{q}}\rangle, \quad \hat{\mathcal{G}}_x(0) = Z_e^{-1}\hat{P}_x e^{-\beta\hat{H}_0}.$$

Here, $\overline{|V_{\mathbf{q}}|^2}$ is the mean square Fourier component of the random field of the scatterers; in the case of scattering by ripplons $\overline{|V_{\mathbf{q}}|^2} \propto T$; $Z_e = \mathrm{Tr}_e\exp(-\beta\hat{H}_0)$. We note that the time t in (18) is large, $t \sim \tau \gg t_{\rm coll}$.

In what follows we assume that the characteristic momentum transfer in a collision $q \gg n_s^{1/2}$ (short-wavelength scattering), which means that one electron at a time collides with a given scatterer. In this case only the diagonal terms with $n' = n$ should be retained in the sum over n, n' in (18). Still the QTE remains substantially many-electron, since time evolution of the operators $\hat{\mathbf{r}}_n(t)$ may be strongly affected by the electron-electron interaction.

4.1. SOLUTION OF THE QTE FOR FAST INTERELECTRON MOMENTUM EXCHANGE

The standard way of solving quantum transport equations for systems with a small number of degrees of freedom (e.g., one-electron systems) is to change from the QTE in the operator form of the type (18) to the set of equations for the matrix elements of the density operator on the wave functions of the system isolated from scatterers; these equations may then

be solved analytically or numerically [32,35]. This method does not apply in the present case, since the wave functions of the many-electron system (the eigenfunctions of the operator $\hat{H}_0$) are not known. If we use approximate functions, an error in the matrix elements of the operators $\exp[i\mathbf{q}\hat{\mathbf{r}}_n(t)], \exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t')]$ will be large for large $t, t' \sim \tau$.

One way to avoid the problem is to change from $\hat{\mathcal{G}}_x(t)$ to the operator

$$\hat{G}_x(t) \equiv e^{-i\hat{H}_0 t/\hbar}\hat{\mathcal{G}}_x(t)e^{i\hat{H}_0 t/\hbar}, \ \langle \hat{P}_x(t)\hat{P}_x(0)\rangle = \mathrm{Tr}_e\left[\hat{P}_x\hat{G}_x(t)\right]. \qquad (19)$$

The equation for $\hat{G}_x(t)$ is of the form

$$\frac{\partial \hat{G}_x}{\partial t} = \frac{i}{\hbar}\left[\hat{G}_x(t), \hat{H}_0\right] + \left[\frac{\partial \hat{G}_x}{\partial t}\right]_{\mathrm{coll}}, \qquad (20)$$

$$\left[\frac{\partial \hat{G}_x}{\partial t}\right]_{\mathrm{coll}} == -\hbar^{-2}\sum_{\mathbf{q}}\overline{|V_{\mathbf{q}}|^2}\sum_n \int_0^t dt' \left[e^{i\mathbf{q}\hat{\mathbf{r}}_n(0)}, \left[e^{-i\mathbf{q}\hat{\mathbf{r}}_n(t'-t)}, \hat{G}_x(t)\right]\right],$$

The collision term in (20) contains the operators $\exp[\pm i\mathbf{q}\hat{\mathbf{r}}_n(t_1)]$ for $t_1 = 0$ and $t_1 = t' - t$. Although the time t is large, $t \sim \tau \gg t_{\mathrm{coll}}$, the interval $t - t'$ that contributes to the integral over t' in (20) is small, as it is given by the duration of a collision t_{coll} (in fact, it was assumed in deriving (18) that $\hat{\mathcal{G}}_x(t)$ remains nearly constant for the time $\sim t - t'$). Smallness of $t - t'$ makes it possible to express $\hat{\mathbf{r}}_n(t' - t)$ in terms of the electron operators for $t = 0$ and then to find the matrix elements of $\exp[-i\mathbf{q}\hat{\mathbf{r}}_n(t' - t)]$ on appropriately chosen wave functions.

It is straightforward to estimate the two terms in the rhs of Eq. (20). The operator $\hat{G}_x(t)$ depends on the coordinates and momenta of all electrons. The electron-electron interaction results in the change of the momenta of individual electrons, and therefore $\hbar^{-1}[\hat{G}_x, \hat{H}_{\mathrm{ee}}] \sim \tau_{\mathrm{ex}}^{-1}\hat{G}_x$ [29], where τ_{ex}^{-1} is the interelectron momentum exchange rate discussed in Sec. 3.3. The collision term gives rise, additionally, to a change of the total momentum of the electron system. This term is $\sim \tau^{-1}\hat{G}_x$, and for fast interelectron momentum exchange rate it is smaller than the first term in the rhs of (20).

If we drop the collision term, the solution of (20), which has the required symmetry of the x-component of a vector and satisfies the initial condition (18), will be of the form

$$\hat{G}_x(t) = \sum_{\alpha=\pm} g_\alpha(t)\hat{P}_\alpha e^{-i\alpha\omega_c t}e^{-\beta\hat{H}_0}, \ \tau_{\mathrm{ex}}^{-1} \gg \tau^{-1}, \qquad (21)$$

$$\hat{P}_\alpha = 2^{-1/2}\left(\hat{P}_x - i\alpha\kappa\hat{P}_y\right), \ \left[\hat{P}_\alpha, \hat{H}_0\right] = -\alpha\hbar\omega_c\hat{P}_\alpha, \ \kappa = eB_z/m\omega_c \ (|\kappa| = 1)$$

with time-independent (and equal, see below) $g_\pm$. Time evolution of $g_\pm(t)$ is due to the collision term in (20). This evolution may depend on $\hat{H}_0$ and $\hat{P}_+\hat{P}_-$. However, in the case of elastic scattering the form of $\hat{G}_x$ as a function of the total energy of the electron system $\hat{H}_0$ should not change in time; one can also show that, in the statistical limit of a large number of electrons $n_s S$, the terms $\propto \left(\hat{P}_+\hat{P}_-\right)^m$ with $m > 1$ that could be present in $g_\pm(t)$ are decoupled from the term allowed for in (21) and thus do not arise.

The equations for $g_\pm(t)$ follow from (20), (21). They are derived below for classical and quantizing magnetic fields, and in both cases, to the leading order of perturbation theory, for $t \gg t_{\rm coll}$ they have a simple form

$$\frac{dg_\alpha}{dt} = -\gamma g_\alpha(t),\ g_+(0) = g_-(0) = 2^{-1/2} Z_e^{-1},\ Z_e = \mathrm{Tr}_e \exp(-\beta \hat{H}_0),$$

with the appropriate relaxation rate γ (see Eqs. (25), (45)). We note that Eqs. (20), (21) do *not* describe static conductivity in quantizing magnetic fields, which is evaluated using a different approach in Sec. 4.3.1. However, they describe the cyclotron resonance, and in the analysis of the cyclotron resonance for $\omega \approx \omega_c$ the nonresonant term $\propto g_- \exp(i\omega_c t)$ in (21) should be dropped.

4.2. THE COLLISION TERM FOR A CLASSICAL SYSTEM

In the range $T \gg \hbar\omega_p, \hbar\omega_c$ (5) the electron motion on the time scale $\sim t_{\rm coll}$ is a semiclassical motion in crossed uniform electric and magnetic fields. Therefore it is straightforward to express the electron coordinates $\hat{\mathbf{r}}_n(t_1)$ in the collision term in (20), for small $|t_1| \equiv |t' - t| \sim t_{\rm coll}$, in terms of $\hat{\mathbf{r}}_n \equiv \hat{\mathbf{r}}_n(0)$ and $\hat{\mathbf{p}}_n \equiv \hat{\mathbf{p}}_n(0)$. As a result we obtain

$$e^{-i\mathbf{q}\hat{\mathbf{r}}_n(t_1)} \approx e^{-i\mathbf{q}\hat{\mathbf{r}}_n(0)} \exp\left[-i\mathbf{q}\mathbf{F}\left(t_1, \hat{\mathbf{p}}_n(0)\right) + i\frac{\hbar q^2}{2m\omega_c}\sin\omega_c t_1\right]$$

$$\mathbf{F}\left(t, \hat{\mathbf{p}}_n\right) = \mathbf{f}\left(t, \hat{\mathbf{p}}_n\right) - \mathbf{f}\left(t, m\mathbf{v}_n^{(d)}\right) + \mathbf{v}_n^{(d)} t, \tag{22}$$

$$\mathbf{f}\left(t, \hat{\mathbf{p}}_n\right) = \frac{\hat{\mathbf{p}}_n}{m\omega_c}\sin\omega_c t + e\frac{\hat{\mathbf{p}}_n \times \mathbf{B}}{m^2\omega_c^2}(1 - \cos\omega_c t), \quad \mathbf{v}_n^{(d)} = \left(\mathbf{E}_n \times \mathbf{B}\right)/B^2.$$

It is convenient to evaluate matrix elements of the electron operators in the Wigner representation,

$$L\left(\{\mathbf{p}_n\}, \{\mathbf{r}_n\}\right) = \int \left[\prod_n d\zeta_n \exp\left(i\zeta_n \mathbf{r}_n\right)\right] \left\langle \{\mathbf{k}_n + \frac{1}{2}\zeta_n\} \middle| \hat{L} \middle| \{\mathbf{k}_n - \frac{1}{2}\zeta_n\} \right\rangle,$$

$$|\{\mathbf{k}_n\}\rangle \equiv \prod_n (2\pi)^{-1} \exp(i\mathbf{k}_n\mathbf{r}_n), \quad \mathbf{p}_n \equiv \mathbf{k}_n - e\mathbf{A}(\mathbf{r}_n). \tag{23}$$

In the WKB approximation, which applies in the classical range (5), the matrix elements (23) of the operator $\hat{G}_x(t)$ (21) and of the collision term in (20) can be obtained, with account taken of (22), by replacing the operators $\hat{\mathbf{r}}_n \equiv \hat{\mathbf{r}}_n(0)$ and $\hat{\mathbf{p}}_n \equiv \hat{\mathbf{p}}_n(0)$ by c-numbers $\mathbf{r}_n$ and $\mathbf{p}_n$, respectively. In particular

$$G_x(t; \{\mathbf{p}_n\}, \{\mathbf{r}_n\}) = \sum_{\alpha=\pm} g_\alpha(t) e^{-i\alpha\omega_c t} \exp[-\beta H_0(\{\mathbf{p}_n\}, \{\mathbf{r}_n\})] P_\alpha. \tag{24}$$

where $P_\alpha = 2^{-1/2} \sum_n (p_{nx} - i\alpha p_{ny})$.

The collision term is given by the (appropriately weighted) sum over $\mathbf{q}$ of the difference between the matrix elements (24) and the matrix elements of $\hat{G}_x(t)$ for the same energy (the collisions are elastic), but for the total momentum of the electron system changed by $\mathbf{q}$. In evaluating this difference one should replace $\mathbf{q} \Rightarrow \mathbf{P}[(\mathbf{qP})/P^2]$, $P^2 \approx \langle P^2 \rangle \approx 2n_s SmT$ (to some extent, this is similar to what is done in deriving the collision term in the single-electron Boltzmann equation for elastic scattering, see [31]; for $\tau_{ex}^{-1} \gg \tau^{-1}$ the only singled out momentum is the total momentum of the many-electron system $\mathbf{P}$).

Further calculations are quite straightforward. The functions $g_\pm(t)$ decay in time as $\exp(-t/\tau) g_\pm(0)$. The expression for the relaxation rate τ^{-1} is of the form:

$$\tau^{-1} = \frac{1}{2} \varkappa_T^2 \hbar^{-2} \sum_{\mathbf{q}} q^2 \overline{|V_\mathbf{q}|^2} \xi(\mathbf{q}), \quad \xi(\mathbf{q}) \equiv \int_{-\infty}^{\infty} dt \, \langle e^{i\mathbf{q}\hat{\mathbf{r}}_n(t)} e^{-i\mathbf{q}\hat{\mathbf{r}}_n(0)} \rangle \tag{25}$$

(in deriving (25) we took into account that the major contribution to the integral over t' in the collision term (20) comes from the range of small $t - t' \sim t_{coll}$ and extended the limits of the integration over t' to infinity). The expression for the correlator $\xi(\mathbf{q})$ immediately follows from (22),

$$\xi(\mathbf{q}) = \int_{-\infty}^{\infty} dt \, \phi(t) \Phi_E(t), \tag{26}$$

$$\phi(t) = \exp\left[-\frac{q^2 T}{m\omega_c^2} (1 - \cos\omega_c t) - i \frac{\hbar q^2}{2m\omega_c} \sin\omega_c t \right]$$

$$\Phi_E(t) = \left\langle \exp\left[i \frac{\mathbf{q}\mathbf{v}_n^{(d)}}{\omega_c} (\omega_c t - \sin\omega_c t) - i \frac{e\mathbf{B}}{m\omega_c^2} \mathbf{q} \times \mathbf{v}_n^{(d)} (1 - \cos\omega_c t) \right] \right\rangle.$$

Here, we have taken into account that statistical averagings over the electron momenta $\mathbf{p}_n$ and coordinates $\mathbf{r}_n$ are performed independently, for a classical system.

The factor $\Phi_E(t)$ describes the effect of electron-electron interaction on collisions with scatterers. Averaging in $\Phi_E(t)$ is performed over electron configurations, with the weight $Z_e^{-1}\exp\left[-\beta H_{ee}\left(\{\mathbf{r}_n\}\right)\right]$ (the drift velocity of an nth electron $\mathbf{v}_n^{(d)}$ (22) is determined by the fluctuational field on the electron and depends on the positions of all electrons). Clearly, this averaging can be reduced to averaging over the distribution of the field $\mathbf{E}_f$ discussed in Sec. 2. We note that, if not for the factor $\Phi_E(t)$, the integral over time in $\xi(\mathbf{q})$ would diverge for $B \neq 0$, which is the mathematical manifestation of the inapplicability of the single-electron Boltzmann equation in the magnetic field. The explicit form of $\xi(\mathbf{q})$ in various limiting cases will be analyzed in Sec. 5.

4.3. QUANTUM THEORY

In quantizing magnetic fields, $\hbar\omega_c \gtrsim T$, the evaluation of the collision term in (20) for fast interelectron momentum exchange, $\tau^{-1} \ll \Omega = \omega_p^2/\omega_c$, should be done in a way different from that used above. Qualitatively, this is a consequence of the electrons no longer having well-defined trajectories shown in Fig. 4 and implied in Eq. (22). Formally, the non-commutativity of the operators $\hat{\mathbf{p}}_n$ and $\hat{\mathbf{p}}_n \times \mathbf{B}$ in $\mathbf{f}$ in (22) becomes substantial for t and $\mathbf{q}$ that contribute to the collision term, and quantization of the kinetic energy should be taken into account when averaging is performed.

In the semiclassical range (6) electron motion is a superposition of "vibrations" around quasiequilibrium positions with frequencies $\Omega \ll \omega_c$ (see Fig. 1) and fast cyclotron motion. It is convenient to introduce the operators $\hat{\tilde{\mathbf{r}}}_n$ of the positions of the centers of electron wave packets,

$$\hat{\tilde{\mathbf{r}}}_n = \hat{\mathbf{r}}_n + e\frac{\hat{\mathbf{p}}_n \times \mathbf{B}}{m^2\omega_c^2},\ \left[\hat{\tilde{r}}_{nj}, \hat{p}_{nj'}\right] = 0\ (j, j' = x, y),\ \left[\hat{\tilde{x}}_n, \hat{\tilde{y}}_n\right] = -i\kappa l_B^2 \quad (27)$$

($\kappa = \pm 1$), and the raising and lowering operators $\hat{p}_{n\alpha}$ that move an electron to an upper (for $\alpha = +$) or lower (for $\alpha = -$) Landau level

$$\hat{p}_{n\alpha} = (2\hbar m\omega_c)^{-1/2}\left(\hat{p}_{nx} - i\alpha\kappa p_{ny}\right),\ \alpha = \pm,\ [\hat{p}_{n-}, \hat{p}_{n+}] = 1. \quad (28)$$

The operators $\hat{p}_{n\alpha}$ commute with the operators $\hat{\tilde{\mathbf{r}}}_n$.

The centers of electron orbits $\tilde{\mathbf{r}}_n$ move because of electron-electron interaction. In the range (6) the characteristic values of the momenta p_n are

$\sim \hbar/\lambda$, and the p_n-dependent terms in $\hat{\tilde{\mathbf{r}}}_n$, $l_B^2 p_n/\hbar \sim l_B$, are much smaller than the characteristic radius δ (2) of the vibrations of the orbit centers in Fig. 1. To lowest order in $\lambda/\delta \lesssim eE_f l_B/T$ the motion of $\tilde{\mathbf{r}}_n$ is described by the Hamiltonian $H_{ee} \equiv H_{ee}(\{\mathbf{r}_n\})$ (7) in which the electron coordinates $\mathbf{r}_n$ are replaced by $\hat{\tilde{\mathbf{r}}}_n$. The increment of $\hat{\tilde{\mathbf{r}}}_n$ over a small time interval $|t_1| \sim t_{coll} \ll \Omega^{-1}$ (the duration of a collision t_{coll} is given in (13)) may be expressed in terms of the field on an electron as

$$\hat{\tilde{\mathbf{r}}}_n(t_1) \approx \hat{\tilde{\mathbf{r}}}_n(0) + \tilde{\mathbf{v}}_n^{(d)}(0)t_1, \ \tilde{\mathbf{v}}_n^{(d)} = \frac{\tilde{\mathbf{E}}_n \times \mathbf{B}}{B^2}, \ \tilde{\mathbf{E}}_n = -\frac{1}{e}\frac{\partial H_{ee}(\{\tilde{\mathbf{r}}_n\})}{\partial \tilde{\mathbf{r}}_n}. \quad (29)$$

The many-electron field $\tilde{\mathbf{E}}_n \equiv \tilde{\mathbf{E}}_n(\{\tilde{\mathbf{r}}_n\})$ may be assumed to be a c-number, to lowest order in $eE_f l_B/T$. Clearly, this is just the fluctuational field.

The effect of electron-electron interaction on time evolution of the operators $\hat{p}_{n\alpha}$ is small over the times $|t_1| \lesssim t_{coll} \ll \tau_{ex} \sim \Omega^{-1}$. Therefore for such t_1 the operators in the collision term in (20) are of the form

$$e^{-i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(t_1)} \approx e^{-i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(0)} \exp\left[-\sum_\alpha \alpha l_B q_{-\alpha} \hat{p}_{n\alpha}(0) e^{i\alpha\omega_c t_1} - i\mathbf{q}\tilde{\mathbf{v}}_n^{(d)}(0)t_1\right] \quad (30)$$

where $q_\alpha = (q_x - i\alpha\kappa q_y)/\sqrt{2}$.

Although the collision time t_{coll}, i.e., the range of t' that contributes to the integral over t' in (20), is small compared to the relaxation time τ and Ω^{-1}, it substantially exceeds ω_c^{-1}. The major contribution to the integral over t' comes from the terms in (30) that do not contain fast oscillating factors $\exp[\pm i\omega_c(t-t')]$ (in (30), as well as in (22), we have set $t_1 \equiv t' - t$). These terms are obtained if we replace in (30)

$$\exp\left[-\sum_\alpha \alpha l_B q_{-\alpha} \hat{p}_{n\alpha} e^{i\alpha\omega_c t_1}\right] \Rightarrow \hat{M}_n\left(\frac{1}{2}l_B^2 q^2\right) \exp\left[-\frac{1}{4}l_B^2 q^2\right], \quad (31)$$

$$\hat{M}_n(\mu) = \sum_{s=0}^{\infty} \left[(-\mu)^s/(s!)^2\right] \hat{p}_{n+}^s \hat{p}_{n-}^s$$

(the operator $\exp[i\mathbf{q}\hat{\tilde{\mathbf{r}}}_n(0)]$ in (20) should be expressed in terms of $\hat{\tilde{\mathbf{r}}}_n$, $\hat{p}_{n\pm}$ in a way similar to (30), (31)).

The equation for the function $g_+(t)$ in (21), which describes the shape of the cyclotron resonance spectrum, can be obtained by substituting (21) into Eq. (20), multiplying the resulting equation by $\hat{P}_-$ (from the left) and taking the trace over the states of the many-electron system (the contribution

to the conductivity $\sigma_{xx}(\omega)$ from the other terms in $\hat{P}_x$, $\hat{G}_x$ is small for $\omega \approx \omega_c \gg \tau^{-1}$). Since the operators $\hat{p}_{n\pm}$ and $\hat{\tilde{\mathbf{r}}}_n$ commute and the motion of the centers of cyclotron orbits $\tilde{\mathbf{r}}_n$ is semiclassical, the trace Tr_e (15) is reduced to the sum over the wave functions of the Landau levels in the occupation number representation $|\nu_n\rangle$ and the integral over $\tilde{\mathbf{r}}_n$. It can be shown [17,29] that the Boltzmann factor is factorized in the semiclassical range,

$$\exp[-\beta\hat{H}_0] \approx \exp\left[-\beta\hbar\omega_c \sum_n \left(\hat{p}_{n+}\hat{p}_{n-} + \frac{1}{2}\right)\right] \exp\left[-\beta H_{ee}\left(\{\tilde{\mathbf{r}}_n\}\right)\right]. \quad (32)$$

The matrix elements of the operators $\hat{p}_{n\alpha}$ on the wave functions $|\nu_n\rangle$ are of the standard form $\langle \nu_n + 1|\, \hat{p}_{n+}\, |\nu_n\rangle = (\nu_n + 1)^{1/2}$.

With account taken of (30)–(32) the calculations become straightforward, although cumbersome. The result is that the function $g_+(t)$ decays in time exponentially,

$$g_+(t) = g_+(0)\exp(-\gamma t), \quad t \gg t_{\mathrm{coll}}. \quad (33)$$

The shape of the cyclotron resonance peak that corresponds to (33) is Lorentzian, and γ is the halfwidth of the peak. The explicit form of γ is discussed in Sec. 5 below. We note that in the approximation (30), (31) the only time-dependent term that remains in the collision integral over t' in (20) and that enters the expression for γ is seen from (30) to be of the form of $\exp\left[-i\mathbf{q}\tilde{\mathbf{v}}_n^{(d)}(0)(t-t')\right]$. Integration and subsequent averaging of this term gives

$$\mathrm{Re}\int_0^\infty dt \left\langle \exp\left[-i\mathbf{q}\tilde{\mathbf{v}}_n^{(d)}t\right]\right\rangle = \pi\left\langle \delta\left(\mathbf{q}\tilde{\mathbf{v}}_n^{(d)}\right)\right\rangle = \frac{t_e}{ql_B},\ t_e = l_B B\langle E_f^{-1}\rangle. \quad (34)$$

For short-range scatterers the characteristic value of the transferred momentum $\hbar q$ is given by the reciprocal electron wavelength $\hbar\lambda\!\!\!^{-1}$, and therefore from Eq. (34) we obtain the expression (13) for the duration of a collision t_{coll}.

4.3.1. *Static Conductivity*

The many-electron static conductivity σ is not described by Eq. (33) because of the frequency dispersion of the polarization operator in quantizing magnetic fields. This dispersion is substantial in the frequency range

$\Delta\omega \gtrsim t_{\rm coll}{}^{-1}$, and since $\omega_c t_{\rm coll} \gg 1$ for $\hbar\omega_c \gg T$, the characteristic scattering rate in the expression for the static conductivity should differ from the broadening of the cyclotron resonance peak.

For weak enough coupling, where $\omega_c\tau \gg 1$ and, in addition, $t_{\rm coll} \ll \tau$ so that electron-scatterer collisions are well separated in time, σ can be obtained from simple perturbation theory in the coupling to scatterers (see [29]). To lowest order in $(\omega_c\tau)^{-1}$ and for the case of short-range scatterers the result is

$$\sigma \equiv \sigma_{xx}(0) = \frac{e^2 n_s}{4m^2\omega_c^2 T} \sum_{\mathbf{q}} q^2 \overline{|V_{\mathbf{q}}|^2} \xi(\mathbf{q}) \tag{35}$$

where the correlator $\xi(\mathbf{q})$ is defined in Eq. (25).

Eq. (35) applies for both classically strong and quantizing magnetic fields. For classically strong fields it gives the same result as that which follows from Eqs. (25), (26). The evaluation of $\xi(\mathbf{q})$ in the range $\hbar\omega_c \gtrsim T$ can be done using Eqs. (30)–(32). We note that the derivation of Eq. (35) requires that collisions be short compared to the intercollision intervals ($\sim \tau$); however, the relation between the rate of interelectron momentum exchange and τ^{-1} may be arbitrary.

5. Results and Discussion

5.1. STATIC CONDUCTIVITY.

It follows from the solution of the transport equation and from Eq. (35) that the static conductivity of the many-electron system can be written in the form

$$\sigma \equiv \sigma_{xx}(0) = \frac{e^2 n_s}{m} \frac{\tau}{1+\omega_c^2\tau^2}, \quad \omega_c\tau \lesssim 1, \quad \omega_p\tau \gg 1; \tag{36}$$

$$\sigma = \frac{\hbar e^2 n_s}{2mT\omega_c}(2\bar{n}+1)\tau^{-1}, \ \omega_c\tau \gg 1; \quad \tau^{-1} = \frac{1}{2}\lambda\!\!\!^{-}{}^2\hbar^{-2} \sum_{\mathbf{q}} q^2 \overline{|V_{\mathbf{q}}|^2} \xi(\mathbf{q}).$$

Here $\overline{|V_{\mathbf{q}}|^2}$ is the mean square Fourier component of the field of the scatterers, $\bar{n} = [\exp(\hbar\omega_c/T) - 1]^{-1}$, and $\lambda\!\!\!^{-} = l_B(2\bar{n}+1)^{-1/2}$. Eq. (36) has the same form as Eqs. (10), (12), (13) obtained above from qualitative arguments, with τ^{-1} having the meaning of the electron relaxation rate.

The effect of electron-electron interaction on the conductivity is contained in the *electron density correlator* $\xi(\mathbf{q})$ which is defined in (25). For

$T \gg eE_f\lambda_T \gg \hbar\omega_c$ the correlator $\xi(\mathbf{q})$ is given by Eq. (26). The integral over t in (26) can be evaluated by the steepest descent method, and it is sufficient to allow for one saddle point in the integrand (at $t = -i\hbar/2T$). As expected (see Sec. 3.1), because of the many-electron factor $\Phi_E(t)$ in the integrand in (26) the value of τ^{-1} turns out to be nearly B-independent (the corrections are $\sim (1/48)(\hbar\omega_c/T)^2$),

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} \exp\left[-\frac{\hbar^2 q^2}{8mT}\right], \quad T \gg e\langle E_f^2\rangle^{1/2}\lambda_T \gg \hbar\omega_c. \tag{37}$$

For stronger magnetic fields the scattering rate becomes B-dependent and also density-dependent. The integrand in (26) has saddle points at $t = 2\pi s\omega_c^{-1} - i\hbar(2T)^{-1}$, $s = 0, \pm 1, \ldots$, and

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} \exp\left[-\frac{\hbar^2 q^2}{8mT}\right] \sum_{s=-\infty}^{\infty} \left\langle \exp\left[i\mathbf{q}\mathbf{E}_f \frac{2\pi s}{\omega_c B}\right]\right\rangle, \; B \gtrsim B_0, \tag{38}$$

where

$$B_0 = \left(2\pi m^3 T/\hbar^2 e^2 \langle E_f^{-1}\rangle^2\right)^{1/4}. \tag{39}$$

The encountering factor ζ in Eq. (12) is given by the number of terms in the sum over s that contribute substantially to $\xi(\mathbf{q})$ in Eq. (38), $\zeta - 1 \sim B^2/B_0^2$ in classical magnetic fields. It is seen from (38) that, in agreement with the arguments in Sec. 3.2, it is for $B^2 \gtrsim B_0^2$ that the dependence of $\xi(\mathbf{q})$, and thus of τ^{-1}, on B becomes substantial.

Eq. (38) is further simplified if $B/B_0 \gg 1$,

$$\xi(\mathbf{q}) = \left(\frac{2\pi m}{Tq^2}\right)^{1/2} \exp\left[-\frac{\hbar^2 q^2}{8mT}\right] \frac{\omega_c B}{\pi q}\langle E_f^{-1}\rangle, \; B \gg B_0, T \gg \hbar\omega_c. \tag{40}$$

As expected, the scattering rate as given by (36), (40) is proportional to B^2 and inversely proportional to the fluctuational field. Therefore decrease of σ with the increasing B displays characteristic *saturation*. The limiting B-independent value of the conductivity, $\sigma_{\text{sat}} \propto \langle E_f^{-1}\rangle$, is determined by the fluctuational field.

The expression for σ is simplified in the important case of a δ-correlated random potential,

$$\overline{|V_\mathbf{q}|^2} = \frac{\hbar^3}{m} S^{-1} \tau_{B=0}^{-1} \tag{41}$$

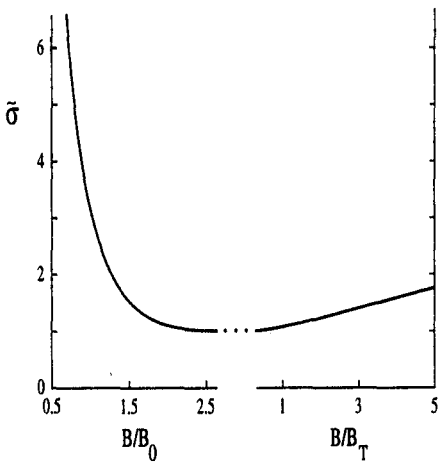

Figure 6. Reduced conductivity $\tilde{\sigma} = \sigma/\sigma_{\rm sat}$ as a function of B/B_0 for classical fields and of $B/B_T \equiv \hbar\omega_c/T$ for $B \gg B_0$, for a δ-correlated random potential and for a Gaussian distribution of the fluctuational field (from [29]).

($\tau^{-1}_{B=0}$ introduced in (41) is equal to the scattering rate τ^{-1} as given by (36), (37) for $B = 0$ and q-independent $\overline{|V_{\mathbf{q}}|^2}$ of the form (41)). In particular

$$\sigma_{\rm sat} = \left(mn_s/\pi B_0^2\right)\tau^{-1}_{B=0}. \qquad (42)$$

If one further assumes that the distribution of the fluctuational field is Gaussian, the averaging over $\mathbf{E}_f$ in (38) and integration over $\mathbf{q}$ in (36) can be done explicitly. The result is shown by the left curve in Fig. 6.

In the range of quantizing magnetic fields we have

$$\xi(\mathbf{q}) = \frac{2B}{q}\langle E_f^{-1}\rangle e^{-\frac{1}{2}l_B^2 q^2(2\bar{n}+1)} \times \sum_{m=0}^{\infty}\left(\frac{1}{2}l_B^2 q^2\right)^{2m}\frac{[\bar{n}(\bar{n}+1)]^m}{(m!)^2} \qquad (43)$$

Eq.(43) goes over into (40) in the limit $T \gg \hbar\omega_c$. In the opposite limit of low T (strongly quantizing B), where $\bar{n} \ll 1$, only the term with $m = 0$ should be kept in the sum in (43). For a δ-correlated random potential the integral over $\mathbf{q}$ in (36) can be evaluated explicitly. The result is shown by the right curve in Fig. 6. In this case $\sigma \approx (\pi\sigma_{\rm sat}/4)\,(\hbar\omega_c/T)^{1/2} \propto B^{1/2}$ for $\bar{n} \ll 1$. The overall dependence of σ on B displays a characteristic minimum, as seen from Fig. 6. In quantizing fields, as well as in strong classical fields, the value of σ depends explicitly, and in a simple way, on the fluctuational field, $\sigma \propto \langle E_f^{-1}\rangle$, which makes it possible to measure $\langle E_f^{-1}\rangle$ experimentally.

5.2. CYCLOTRON RESONANCE

For fast interelectron momentum exchange compared to the momentum relaxation due to collisions with the scatterers (i.e., for $\tau^{-1} \ll \omega_p$ in the range $\omega_p \gtrsim \omega_c$, and for $\tau^{-1} \ll \omega_p^2/\omega_c$ in the range $\omega_c \gg \omega_p$) the peak of cyclotron resonance has a Lorentzian shape in strong magnetic fields,

$$\sigma_{xx}(\omega) \approx \frac{e^2 n_s}{2m} \frac{\gamma}{(\omega - \omega_c)^2 + \gamma^2} \quad \text{for} \quad |\omega - \omega_c|, \tau^{-1} \ll \omega_c. \qquad (44)$$

In the classical range $T \gg \hbar\omega_c$ the halfwidth of the peak $\gamma = \tau^{-1}$, where τ^{-1} is given by Eq. (36). The dependence of γ on the magnetic field, temperature, and electron density is described in this range by Eqs. (36)—(40): $\gamma = \tau_{B=0}^{-1}$ is independent of B for $eE_f\lambda_T \gg \hbar\omega_c$, whereas, for a δ-correlated potential, $\gamma = (B^2/\pi B_0^2)\,\tau_{B=0}^{-1}$ for $B \gg B_0$.

In a quantizing magnetic field the expression for γ is of the form [29]

$$\gamma = \frac{l_B B}{4\hbar^2} \langle E_f^{-1} \rangle \sum_{\mathbf{q}} (l_B q)^3 \overline{|V_{\mathbf{q}}|^2} \exp\left[-\frac{1}{2} l_B^2 q^2 (2\bar{n}+1)\right] \times \sum_{m=0}^{\infty} \left(\frac{1}{2} l_B^2 q^2\right)^{2m} \frac{[\bar{n}(\bar{n}+1)]^m}{m!(m+1)!} \qquad (45)$$

The dependence of γ on the fluctuational field, and thus on the electron density, is given by $\gamma \propto \langle E_f^{-1} \rangle$. In the classical limit $\bar{n} \gg 1$ Eq. (45) goes over into the expression for τ^{-1} given by (36), (40).

In the limit of $\bar{n} \ll 1$ the only term to be retained in the sum over m in (45) is that with $m = 0$. The results for γ and σ in this case were obtained earlier [17]. The corresponding expression for γ and its specific dependence on electron density have been shown [25](b) to be in good agreement with the experimental data on cyclotron resonance [7](a) published simultaneously with [17](a). The value of γ is a factor $t_{\text{coll}}\tau_{SCBA}^{-1} \ll 1$ smaller than the width of the cyclotron resonance peak in the self-consistent Born approximation. For a δ-correlated random potential $\gamma \propto B^{3/2}$.

We note that the above expressions for static conductivity and cyclotron resonance can be obtained in a different way if electrons form a Wigner crystal. In this case electron relaxation is described in terms of the decay of phonons of the Wigner crystal due to electron coupling to scatterers (ripplons) [3(b),25,36–38]. Phonon-ripplon coupling is strongly nonlinear in the phonon operators due to divergence of the mean square electron displacement from a lattice site in a 2D crystal for $T > 0$. The Green function techniques which allow for this nonlinearity in a nonperturbative way were considered in [25,36,39]. The technique [25] can be extended to the parameter range (3) investigated in the present paper (however, the above analysis does not require that electrons form a Wigner crystal). In this range the decay processes are substantially multi-phonon. For example, broadening of the cyclotron resonance peak is due to the ripplon-induced decay of the long-wavelength phonon with a cyclotron frequency into a

short-wavelength phonon, which is accompanied by birth and death of many other short-wavelength phonons [25].

A corollary from the above arguments is that, in the case where Wigner crystallization occurs in a strong magnetic field, so that $\hbar\omega_c, T \gg \hbar\omega_p^2/\omega_c$ and the conditions of applicability of the semiclassical theory are fulfilled, the static conductivity and the cyclotron resonance lineshape should remain nearly constant through the crystallization transition. This is in contrast with a comparatively sharp change in the transport properties observed in [5] for crystallization at comparatively high electron densities, $\hbar\omega_p/T \sim 2.5$, and $B = 0$, where the semiclassical theory does not apply.

5.3. APPLICATION OF RESULTS TO ELECTRONS ON HELIUM

For electrons on helium with a density $n_s = 10^8$ cm^{-2} and $T = 1$ K the fluctuational field is $\langle E_f^2\rangle^{1/2} = F^{1/2}(\Gamma) n_s^{3/4} T^{1/2} \approx 11$ V/cm, and the characteristic magnetic field for the onset of magnetoresistance is $B_0 \approx (2m^3F/\hbar^2e^2)^{1/4} n_s^{3/8} T^{1/2} \approx 0.54$ T. Therefore the specific features of many-electron transport are accessible to experimental investigation. The squared matrix elements of the random potential of helium vapor atoms and ripplons are, respectively, [40]

$$\overline{|V_{\mathbf{q}}|_v^2} = \frac{3\pi\hbar^4}{8m^2}\gamma_\perp b_{\mathrm{He}}^2 N_v S^{-1} = \frac{\hbar^3}{m} S^{-1} \left[\tau_{\mathrm{B}=0}^{-1}\right]_v , \tag{46}$$

$$\overline{|V_{\mathbf{q}}|_r^2} = S^{-1}\frac{Te^2}{\alpha q^2}\left[E_\perp^2 + 2E_\perp E_{\mathrm{pol}} + E_{\mathrm{pol}}^2\right], \tag{47}$$

$$E_{\mathrm{pol}} \equiv E_{\mathrm{pol}}(q) = \frac{\hbar^2\gamma_\perp^{(0)}}{2me} q^2 \varphi\left(\frac{q}{2\gamma_\perp}\right).$$

Here, $\gamma_\perp$ and $\gamma_\perp^{(0)}$ are the variational parameters of the electron wave function transverse to the layer $\psi(z) \propto z\exp(-\gamma_\perp z)$, in the presence and absence of the electric field $E_\perp$ that presses the electrons against the helium surface, $\gamma_\perp^{(0)} = (me^2/4\hbar^2)(\varepsilon - 1)/(\varepsilon + 1)$ (ε is the dielectric constant of helium). In Eq. (46), b_{He}^2 is the helium atom cross-section, $b_{\mathrm{He}}^2 \approx 5\text{Å}^2$, N_v is the (3D) vapor density. In Eq. (47), α is surface tension; the function $\varphi(x)$ is given in [40], $\varphi(x) \approx \ln(2/x)$ for $x \ll 1$.

In the temperature range $T \gtrsim 1$ K electrons are primarily scattered by helium vapor atoms. These atoms create a nearly ideal δ-correlated random potential, and the results for this potential discussed in Secs. 5.1, 5.2 directly apply, with the effective strength of the potential given by

(46). Experimental investigation of magnetotransport [6,12,13,15,41], using different techniques, has shown that there is a range of strong magnetic fields, $\omega_c\tau > 1$, where there is no magnetoresistance. Recent experiments [42] have demonstrated that, for electron scattering by vapor atoms, the range of the magnetic fields where $\omega_c\tau \gg 1$ and yet the conductivity $\sigma \propto B^{-2}$ may be in fact so broad that in this range σ varies by more than two orders of magnitude. This Drude-like behavior in a 2D system is in full agreement with Eqs. (36), (37) of the many-electron theory.

Both the relaxation rate τ^{-1} and duration of a collision $t_{\rm coll} = \lambda\!\!\!^{-}B\langle E_{\rm f}^{-1}\rangle$ increase with increasing B. Therefore, the criterion for applicability of the above theory $t_{\rm coll} \ll \tau$ does not hold for very strong B. The role of many-electron effects then becomes less substantial, and transport becomes essentially single-electron [17](b). The crossover from many-electron to single-electron transport in strong magnetic fields was observed in [12,13]. It was described in a modified self-consistent Born approximation which allowed for many-electron effects. The theory and experiment were shown to be in a good quantitative agreement with each other.

We note that at high temperatures where, because of high vapor density, the electron relaxation rate is high, $\tau_{\rm B=0}^{-1} > \omega_p^{-1}$, the condition of the absence of magnetoresistance $e\langle E_{\rm f}^2\rangle^{1/2}\lambda\!\!\!^{-}_T \sim \hbar\omega_p \gg \hbar\omega_c$ does not apply for strong magnetic fields $\omega_c\tau \gg 1$. Therefore, in contrast to the case discussed above, magnetoresistance should arise as soon as the magnetic field becomes strong. This is in agreement with the observations [10,11].

Experimental investigation of electron transport for lower temperatures, $T < 1$ K, where ripplon scattering is dominating, has made it possible to analyze the region of small scattering rates $\tau_{\rm B=0}^{-1}$ and to perform detailed quantitative comparison with the many-electron theory of magnetoconductivity in Sec. 5.1 [42,43].

For $B \ll B_0$ the conductivity as given by Eqs. (36), (37), (47) is described by the universal Drude law, $\sigma \propto \tau_{\rm B=0}/[1+\omega_c^2\tau_{\rm B=0}^2]$, where the scattering rate $\tau_{\rm B=0}^{-1}$ is given by the sum of the contributions from all three terms in the mean square ripplon potential (47). On the other hand, in the range of strong magnetic fields, $B \gtrsim B_0$, the B-dependences of the contributions to the conductivity from each term in (47) are different. They were considered in [29]. For the ripplon scattering described by the term $\propto E_\perp^2$ in Eq. (47), the sum over $\mathbf{q}$ in Eqs. (36), (40) for the classical relaxation rate in the range $B \gg B_0$ diverges logarithmically at small q. The effect of this term should be analyzed using quantum theory (43). The corresponding contribution to the scattering rate $\tau_{E_\perp}^{-1}$ is of the form

$$\tau_{E_\perp}^{-1} = \frac{e^2 E_\perp^2}{\pi\hbar^2\alpha} \frac{T l_B B \langle E_f^{-1}\rangle}{[2(2\bar{n}+1)]^{3/2}} \sum_{m=0}^{\infty} \left[\frac{\bar{n}(\bar{n}+1)}{(2\bar{n}+1)^2}\right]^m \frac{\left(2m-\frac{1}{2}\right)!}{(m!)^2} \qquad (48)$$

For large $\bar{n}$ the sum over m goes over into $(2/\pi)^{1/2}\ln[T/\hbar\omega_c]$, and the corresponding term in the conductivity $\sigma_{E_\perp}$ depends on B only logarithmically, $\sigma_{E_\perp} \propto \ln[T/\hbar\omega_c]$. In quantizing fields ($\bar{n} \ll 1$) we have $\sigma_{E_\perp} \propto B^{-1/2}$. On the whole, $\sigma_{E_\perp}$ slowly decreases with increasing B.

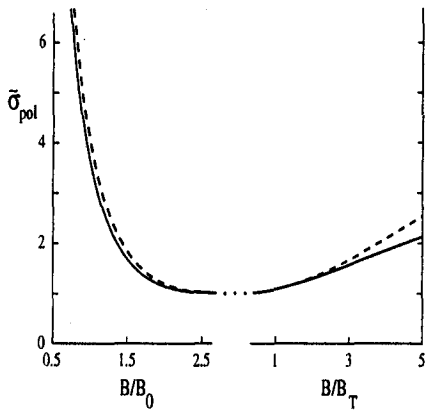

Figure 7. Reduced conductivity $\tilde{\sigma}_{pol} = \sigma_{pol}/\sigma_{pol,sat}$, which is determined by the third (polarization) term in the ripplon field intensity (47), as a function of B/B_0 for classical fields and of $B/B_T \equiv \hbar\omega_c/T$ for $B^3 \gg (B_0^4/B_T)(\bar{n}+\frac{1}{2})$. The solid and dashed lines refer to $2mT/\hbar^2\gamma_\perp^2 = 0.1$ and 0.01, respectively (from [29]).

The contributions to σ from the terms $\propto E_{pol}$ and $\propto E_{pol}^2$ in (47) display a nonmonotonic dependence on B. These contributions can be found numerically both in classical and quantizing magnetic fields. In the classical fields they saturate for $B \gtrsim B_0$. For $B \gg B_0$ and $\bar{n} \ll 1$ they increase with B, as in the case of a δ-correlated random potential. The B-dependence of the contribution σ_{pol} from the third term in (47) is shown in Fig. 7; the value of σ_{pol} is scaled by

$$\sigma_{pol,sat} = \frac{2mn_s\left(\gamma_\perp^{(0)}T\right)^2}{\pi\hbar\alpha B_0^2}\tilde{\varphi}\left(\frac{2mT}{\hbar^2\gamma_\perp^2}\right),$$

$$\tilde{\varphi}(z) = \int_0^\infty dx\, xe^{-x}\varphi^2\left[(zx)^{1/2}\right].$$

The overall ripplon conductivity as a function of B displays a characteristic minimum similar to that shown in Figs. 6, 7. The position of the minimum depends on temperature and the field $E_\perp$.

The nonmonotonic behavior of the static conductivity as a function of B has been observed in the experiments [14,42]. The experimental results, which have been obtained for several values of the electron density and temperature, are in good quantitative agreement with the theory. The data has also made it possible to extract the value of the mean square fluctuational field $\langle E_f^2\rangle$ and to compare it with Monte Carlo data in Fig. 2 [43]. Detailed

discussion of the experiment is given in the chapter by M.J. Lea in this book.

5.4. BRAGG-CHERENKOV SCATTERING FOR A WIGNER CRYSTAL

Recently several experimental groups observed [44–46] that, when electrons above helium surface form a two-dimensional Wigner crystal, they display strongly nonlinear magnetoconductivity for comparatively weak driving fields. In [45] the effect was interpreted as due to the electron crystal sliding out of the periodic array of polaron-type "dimples" on the helium surface at a threshold value of the driving field $\mathbf{E}_{\mathbf{ext}}$. This nonlinearity mechanism was first suggested in [3(b)]. In this subsection we show that there is a different mechanism which is due to strong nonlinearity of electron *losses*. This mechanism, which we call the *Bragg-Cherenkov scattering*, describes experimental observations reasonably well [47], including the observation that the Hall velocity of a Wigner crystal displays saturation with the increasing $\mathbf{E}_{\mathbf{ext}}$ [46].

The Bragg-Cherenkov scattering is a *coherent many-electron* scattering mechanism specific for electron solids. The conditions for the "conventional" single-electron Cherenkov emission are that the electron velocity $\mathbf{v}$ exceed the phase velocity of irradiated waves (ripplons), and the transferred momentum $\hbar\mathbf{q}$ be small compared to the electron momentum, in which case the energy conservation law is of the form $\mathbf{qv} = \omega(q)$ ($\omega(q)$ is the wave frequency). If electrons form a solid and the above conditions hold for the wave vector of the irradiated waves equal to a reciprocal lattice vector of the solid $\mathbf{G}$, then the Cherenkov waves emitted by different electrons are coherent and interfere with each other. This is similar to the interference in Bragg scattering. The Bragg-Cherenkov scattering should result in a strong increase in the emission rate when the velocity of the electron solid is such that $\mathbf{vG}$ becomes close to $\omega(G)$. Respectively, the reaction (friction) force $\mathbf{F}(\mathbf{v})$ should also dramatically increase for such velocities.

For finite temperatures the density correlation function in 2D crystals decays, and the Bragg peaks have power-law tails [48]. One may expect that the Bragg-Cherenkov peaks in the friction force $\mathbf{F}(\mathbf{v})$ (at $\mathbf{Gv} = \omega(G)$) also have tails as a function of the crystal velocity $\mathbf{v}$. These tails are important for the analysis of nonlinear conductivity of a Wigner crystal.

The force $\mathbf{F}$ from ripplons on an electron system is given by the time derivative of the total electron momentum $\hat{\mathbf{P}}$ defined in Eq. (15), $\mathbf{F} = -i\hbar^{-1}\langle\left[\hat{\mathbf{P}}, \hat{H}_i\right]\rangle$, where $\hat{H}_i$ is the Hamiltonian of electron-ripplon interac-

tion. In evaluating $\mathbf{F}$ we will assume that the electron system as a whole is moving with a velocity $\mathbf{v}$ with respect to the helium surface, but other than that, the two systems are close to thermal equilibrium. Then, to the lowest order in $\hat{H}_i$, the force $\mathbf{F}$ can be expressed in terms of the electron density correlator $\langle\hat{\rho}_{\mathbf{q}}(t)\hat{\rho}_{-\mathbf{q}}(0)\rangle_0$ for the isolated electron system [49]:

$$\mathbf{F} \approx -\hbar^{-1}\sum_{\mathbf{q}}\mathbf{q}\frac{\bar{n}(\omega(q))+1}{\bar{n}(\mathbf{qv})+1}\left|\tilde{V}_{\mathbf{q}}\right|^2\int_{-\infty}^{\infty}dt\,e^{i[\mathbf{qv}-\omega(q)]t}\langle\hat{\rho}_{\mathbf{q}}(t)\hat{\rho}_{-\mathbf{q}}(0)\rangle_0. \quad (49)$$

Here, $\bar{n}(\omega)=[\exp(\hbar\omega/T)-1]^{-1}$ is the Planck number, and $\tilde{V}_{\mathbf{q}}$ is the matrix element of the electron-ripplon interaction. In a slightly different form (for the electron losses due to $\mathbf{F}$), Eq. (49) was obtained in [17(a)] for the electron system in crossed electric and magnetic fields, in which case $\mathbf{v}$ is the transverse drift velocity (in [17(a)] it was assumed that $\hbar qv \ll T$).

In contrast to Sec. 4 where we were interested in the Fourier-transformed electron density correlator $\xi(\mathbf{q})$ (25) for large $q \gg n_s^{1/2}$, in the analysis of the peaks of $\mathbf{F}\equiv\mathbf{F}(\mathbf{v})$ near Bragg-Cherenkov resonances we are interested in evaluating the electron density correlator for $q\sim n_s^{1/2}$, and the major contribution to the integral over time in (49) comes from large times $|t|\sim|\mathbf{Gv}-\omega(G)|^{-1}$. The problem of the resonant friction force has much in common with the problem of the tails of resonant absorption due to coupled phonon-ripplon modes [3(b)], which was considered in [39]. The resulting expression for the resonant term $\Delta\mathbf{F}_{\mathbf{G}}$ in the force $\mathbf{F}(\mathbf{v})$ for $\mathbf{Gv}\approx\omega(G)$ is of the form [47]

$$\Delta\mathbf{F}_{\mathbf{G}} = -\frac{n_s^2S^2}{\hbar\omega_{\mathrm{m}}}\varsigma(G)\mathbf{G}\left|\tilde{V}_{\mathbf{G}}\right|^2\left|\frac{\omega_{\mathrm{m}}}{\mathbf{Gv}-\omega(G)}\right|^{1-\mu(G)},\ \mu(G)=\frac{TG^2}{4\pi mc_t^2n}, \quad (50)$$

$$\varsigma(G)=2\sin[\pi\mu(G)/2]\,\Gamma(1-\mu(G)),\quad \omega_{\mathrm{m}}=\min(\omega_p,\omega_p^2/\omega_c,T/\hbar)$$

(c_t is the transverse sound velocity of the Wigner crystal).

It follows from Eq. (50) that the force on the Wigner crystal increases as a power of the reciprocal detuning $|\mathbf{Gv}-\omega(G)|^{-1}$, with a fractional temperature-dependent exponent $1-\mu(G)$. Clearly, the friction force becomes large *before* the crystal reaches the velocity $\omega(G)/G$ (for $\mathbf{v}$ parallel to $\mathbf{G}$), and it further increases as v approaches the resonant value.

Since the ripplon dispersion law is superlinear for relevant q ($\omega(q)\propto q^{3/2}$), the resonant condition $\mathbf{vG}=\omega(G)$ is first met for the minimal reciprocal lattice vector $G=G_{\min}=(8\pi^2n_s/\sqrt{3})^{1/2}$. This means that, if

the crystal is driven along one of the vectors $\mathbf{G}_{\min}$, its velocity as a function of the driving field E_{ext} will display saturation at the critical value $v_c = \omega(G_{\min})/G_{\min}$. Saturation of the Hall velocity v_H of a Wigner crystal at $v_H \approx v_c$ was observed in [46]. Detailed comparison with the experiment [46] is given in [47].

We note that the Bragg-Cherenkov scattering does not arise if electrons form a fluid. The nonlinearity in a fluid, which was discussed in Sec. 3.4, arises for higher driving fields E_{ext}.

6. Conclusions

We have presented an intuitive picture and detailed microscopic theory of many-electron dynamics and transport in a normal electron fluid and a Wigner crystal. In our approach, strong correlations in the electron system are taken into account explicitly, in a nonperturbative way. We show that electron-electron interaction can dramatically affect transport. The effect is of fluctuational origin, for classical and semiclassical systems. It can be described in terms of a characteristic parameter, the electric field that drives an electron because of electron density fluctuations. The fluctuational field is accessible to experimental measurement. The variation of its distribution with electron density and temperature has been analyzed using Monte Carlo simulations. The dependence of transport coefficients on electron density, temperature, and magnetic field has been investigated in a broad range of parameters over which the system displays different types of behavior. Theoretical results are in good agreement with experiments.

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