### Many-Electron System on Helium and Color Center Spectroscopy

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Electrons on the helium surface display sharp resonant absorption lines related to the transitions between the subbands of quantized motion transverse to the surface. A magnetic field parallel to the surface strongly affects the absorption spectrum. We show that the effect results from admixing the intersubband transitions to the in-plane quantum dynamics of the strongly correlated electron liquid or a Wigner crystal. This is similar to the admixing of electron transitions in color centers to phonons. The spectrum permits a direct characterization of the many-electron dynamics and also enables testing the theory of color centers in a system with controllable coupling.

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Electrons above the surface of liquid helium are localized in a one-dimensional potential well, which is formed by the high repulsive barrier at the surface and the image potential. The energy levels in the well are quantized. The electrons occupy the lowest level forming a twodimensional system [1,2]. The spectroscopic observation of transitions between the quantized energy levels [3] was a direct proof of the picture of the electron confinement and the overall nature of the potential. Since then much work has been done on the exact positions and the widths of the spectral lines and their dependence on the temperature and the electron density [4–12].

The electron system on helium is free from static disorder. It is also weakly coupled to the vibrational excitations in helium, ripplons and phonons. The observed spectral lines are narrow, with width as small as  $\sim 2$  MHz for T = 0.3 K [8]. In the nomenclature of the solid-state spectroscopy they correspond to zero-phonon lines. Such lines in the spectra of point defects result from transitions between the defect energy levels with no energy transfer to or from phonons [13]. The physics of point defects and the defect spectroscopy have been the focus of attention recently in the context of quantum computing and quantum sensing [14]. On their side, electrons on helium themselves have been also considered as a viable candidate system for a scalable quantum computer [15–19].

One of the major attractive features of electrons on helium is the possibility to study many-electron effects. The electron-electron interaction is strong, the ratio of its energy to the electron kinetic energy is  $\Gamma = e^2 (\pi n_s)^{1/2} / k_B T > 30$ for the electron density  $n_s \ge 10^7$  cm<sup>-2</sup> and  $T \le 0.3$  K. The electrons form a Wigner crystal [20,21] or a classical or quantum nondegenerate liquid with unusual transport properties, cf. Refs. [22–29] and references therein. Spectroscopy would be expected to provide a most detailed insight into the correlated many-electron dynamics. However, the only spectral effect of the electron-electron interaction studied so far is a small density-dependent line shift [5,10].

In this Letter we show that, by applying a magnetic field along the helium surface, one can use spectroscopy to study quantum dynamics of a nondegenerate electron liquid and a Wigner solid. Importantly, in the cases where this dynamics has been already understood, the system can serve as a quantum simulator of color center spectroscopy, with the unique opportunity of controlling the strength of the coupling of the electron transition and many-body excitations in the system. The importance of such simulations follows from the broad applications of color centers, including the color centers in diamond such as NV centers, cf. Refs. [14,30,31].

The change of the interband absorption spectrum by an in-plane magnetic field has been studied for degenerate quasi-two-dimensional electron systems in semiconductors, see Ref. [32] and references therein. The results were interpreted in the mean-field approximation. The field-induced high-temperature spectral broadening was also reported for electrons on helium [33,34]. Here we show that, for electrons on helium in the quantum regime, the spectrum is qualitatively different from what the mean-field theory predicts. It has to be analyzed using an approach that explicitly takes into account the interplay of the strong correlations and fluctuations in the quantum electron system.

The effect of the parallel magnetic field on the electron spectrum and the similarity with the physics of color centers can be understood from Fig. 1. We choose the z axis as the direction of quantized motion normal to the surface. In different quantized states of the out-of-plane motion  $|\mu\rangle$  the electron is at a different average distance



FIG. 1. Left: The many-electron system on helium in a magnetic field with components parallel  $(B_{\parallel} \equiv B_x)$  and perpendicular  $(B_{\perp} \equiv B_z)$  to the helium surface. Right: The energy spectrum of an electron in the two lowest bands of motion normal to the surface. The energy difference between the bands  $\varepsilon_{21}$  is the distance between the levels of the quantized motion along the *z* axis. The single-electron kinetic energy of motion along the surface is quadratic in the in-plane momentum **p** for  $B_{\perp} = 0$ . The field  $B_{\perp}$  transforms the spectrum into the Landau levels, which are broadened by the electron-electron interaction. The field  $B_{\parallel}$  shifts the bands of the in-plane motion by  $\Delta p_B$ , see Eq. (1).

from the surface. If a magnetic field  $B_{\parallel}$  is applied parallel to the surface, an interstate transition leads to the electron shift transverse to  $B_{\parallel}$ . Therefore the electron in-plane momentum is changed by the Lorentz force in the  $\hat{\mathbf{z}} \times \mathbf{B}_{\parallel}$  direction. For the transition  $|1\rangle \rightarrow |2\rangle$  from the ground to the first excited state the change  $\Delta p_B$  is

$$\begin{split} \Delta p_B &= m \omega_{\parallel} \Delta_z, \qquad \Delta_z = \bar{z}_{22} - \bar{z}_{11}, \\ \omega_{\parallel} &= e B_{\parallel} / m c, \qquad \bar{z}_{\mu\mu} = \langle \mu | z | \mu \rangle \quad (\mu = 1, 2). \end{split}$$
(1)

Thus the minima of the energy bands  $\varepsilon_1(\mathbf{p})$  and  $\varepsilon_2(\mathbf{p})$  of the in-plane motion ( $\mathbf{p}$  is the in-plane momentum) are shifted with respect to each other. We assume  $m\omega_{\parallel}^2 \Delta_z^2 \ll \varepsilon_{21} \equiv \min[\varepsilon_2(\mathbf{p}) - \varepsilon_1(\mathbf{p})]$ .

The right panel of Fig. 1 has the familiar form of the sketch of the energy of a point defect coupled to a vibrational mode in a crystal [13]. In the case of a defect, the horizontal axis is the coordinate of the vibrational mode, and the parabolas show the potential energy of the mode in the two electron states with the energy difference  $\varepsilon_{21}$ . The zero-phonon spectral line corresponds to a transition at frequency  $\varepsilon_{21}/\hbar$  between the minimum of the parabolas. The vertical transition from the minimum of the lower parabola (the Franck-Condon transition) occurs at a higher energy. Usually the electron is coupled to many modes (phonons), which significantly complicates the analysis, as has been known since the work of Pekar [35] and Huang and Rhys [36].

In distinction from a defect, the parabolas in Fig. 1 show the single-electron energy as a function of the in-plane momentum. In a strongly correlated electron system the momentum can be transferred to other electrons. Such recoil reminds us of the recoil from the absorption of a gamma-quantum by an impurity in a crystal, which underlies the Mössbauer effect. By analogy with the Mössbauer effect and the spectra of color centers, the absorption spectrum of electrons on helium should strongly depend on the in-plane many-electron dynamics.

To analyze the spectrum in the presence of strong electron correlations, one should start with the full Hamiltonian of the system. It is a sum of the terms  $H_{\parallel}$ ,  $H_{\perp}$ , and  $H_i$  that describe, respectively, the in-plane motion, the motion normal to the helium surface in the image potential [1,2], and the coupling of these two motions by the in-plane field  $B_{\parallel} \equiv B_x$ . In the presence of a magnetic field  $B_{\perp} \equiv B_z$  normal to the surface

$$\begin{split} H &= H_{\parallel} + H_{\perp} + H_{i}, \\ H_{\parallel} &= \sum_{n} \frac{\pi_{n}^{2}}{2m} + \frac{1}{2} \sum_{n,m'} \frac{e^{2}}{|\mathbf{r}_{n} - \mathbf{r}_{m}|}, \\ H_{\perp} &= \sum_{n} \left[ \frac{p_{nz}^{2}}{2m} + U(z_{n}) \right], \qquad H_{i} = \sum_{n} \omega_{\parallel} \pi_{ny} (z_{n} - \bar{z}_{11}). \end{split}$$

Here *n* enumerates electrons,  $\mathbf{r}_n \equiv (x_n, y_n)$  and  $\pi_n = -i\hbar \nabla_n + (e/c) \mathbf{A}_{\perp}(\mathbf{r}_n)$  are the in-plane electron coordinate and kinematic momentum  $[\mathbf{A}_{\perp}(\mathbf{r})$  is the vector-potential of the field  $B_{\perp} \equiv B_z]$ , whereas U(z) is the confining potential. The leading-order part of  $H_i$  is diagonal with respect to the states  $|\mu\rangle_n$  of the out-of-plane motion,  $H_i = \omega_{\parallel} \Delta_z \sum_n \pi_{ny} |2\rangle_{nn} \langle 2|$ , see Supplemental Material [37].

The frequency  $\varepsilon_{21}/\hbar$  of the interstate transition largely exceeds all characteristic frequencies of the in-plane electron motion. One therefore can think of the adiabatic approximation in which the transition  $|1\rangle \rightarrow |2\rangle$  occurs "instantaneously" for a given in-plane many-electron state. The transition frequency depends on this state. It is this dependence that determines the shape of the spectrum.

Formally, the absorption of microwaves polarized in the *z* direction is determined by the real part of the conductivity  $\sigma_{zz}(\omega)$ . For a nondegenerate electron system it is given by the sum of the contributions from individual electrons, i.e., by the conductivity of an *n*th electron multiplied by the inplane electron density  $n_s$ . From the Kubo formula

$$\operatorname{Re}_{\sigma_{zz}}(\omega) = C_{\sigma}\operatorname{Re}\int_{0}^{\infty} dt e^{i\omega t} \langle [z_{n}(t), z_{n}(0)] \rangle.$$
(2)

Here,  $C_{\sigma} = e^2 n_s \omega / \hbar \approx e^2 n_s \varepsilon_{21} / \hbar^2$  in the considered range of resonant absorption.

The evaluation of the conductivity depends on whether the electron system is a liquid or a crystal. For a Wigner crystal the operators  $\pi_n$  are linear combinations of the creation and annihilation operators of the Wigner crystal phonons, making the form of the coupling  $H_i$  and the problem as a whole largely the same as that of the spectra of color centers [37]. However, in our experiment the electron system is a strongly correlated liquid in a strong transverse magnetic field  $B_{\perp}$ . In such a field the in-plane electron motion is a superposition of a fast quantized cyclotron motion at frequencies  $\sim \omega_c = eB_{\perp}/mc$  and a slow semiclassical drift of the guiding centers of the cyclotron orbits. The drift comes from the fluctuational electric field caused by the electron density fluctuations. The field on an *n*th electron is  $\mathbf{E}_n = -e \sum'_m (\mathbf{r}_n - \mathbf{r}_m)/|\mathbf{r}_n - \mathbf{r}_m|^3$ . It varies on the timescale  $\omega_c/\omega_p^2 \gg \omega_c^{-1}$ , where  $\omega_p = (2\pi e^2 n_s^{3/2}/m)^{1/2}$  [22].

The timescale separation allows describing the peak of the absorption spectrum of the electron liquid in an explicit form [37]. It is convenient to single out in the integrand in Eq. (2) the factor that oscillates at the resonant frequency,  $\langle [z_n(t), z_n(0)] \rangle = |\langle 1|z|2 \rangle|^2 \exp(-i\varepsilon_{21}t/\hbar)Q(t)$ . The function Q(t) describes the effect of the in-plane many-electron dynamics,

$$Q(t) = e^{i\delta_{\parallel}t} \exp[-(\gamma^2/2)w(t)],$$
  

$$\delta_{\parallel} = m\omega_{\parallel}^2 \Delta_z^2/2\hbar, \qquad \gamma^2 = \delta_{\parallel}\omega_p^2 k_B T/2\pi\hbar\omega_c^2,$$
  

$$w(t) = (n_s^{3/2}k_B T)^{-1} \iint_0^t dt_1 dt_2 \langle \mathbf{E}_n(t_1)\mathbf{E}_n(t_2) \rangle.$$
(3)

We assumed Gaussian distribution of the fluctuational field  $\mathbf{E}_n$ . In a broad parameter range relevant for the experiments on electrons on helium  $\langle \mathbf{E}_n^2 \rangle \approx F(\Gamma) n_s^{3/2} k_B T$ , where  $F(\Gamma) \simeq 9$ .

If the coupling to the in-plane fluctuations is strong,  $\gamma \gg \omega_p^2/\omega_c$ , from Eq. (3) the main part of the absorption spectrum (2) is a Gaussian peak, reminiscent of the spectrum of color center. The typical width of the peak in the frequency units is  $\gamma F(\Gamma)^{1/2}$ .

The absorption spectrum also has an analog of the zerophonon line. It is described by the long-time behavior of w(t) and dominates the spectrum for small  $B_{\parallel}$ . In the electron liquid the line is Lorentzian with a half-width which, unexpectedly, is determined by the self-diffusion and is equal to  $m\delta_{\parallel}D/2$ , where D is the self-diffusion coefficient [37]. One can switch from a Lorentzian to a Gaussian spectrum by increasing the field  $B_{\parallel}$ .

In the experiment, the absorption spectrum is measured by varying the electric field  $E_z$  applied perpendicular to the helium surface, using that the level spacing  $\varepsilon_{21}$  linearly depends on  $E_z$  within the linewidth. In the units of  $E_z$ , the typical width of the Gaussian peak is

$$\delta E_z = \frac{B_{\parallel}}{B_{\perp}\sqrt{2}} [k_B T n_s^{3/2} F(\Gamma)]^{1/2}.$$
 (4)

All parameters in Eq. (4) can be controlled in the experiment. This enables testing the theoretical prediction with high accuracy.

We measured the change of the low-frequency helium cell admittance *Y* due to absorption of microwave

radiation, as explained in the Supplemental Material [37]. Such photoassisted transport spectroscopy provides a sensitive way to measuring resonant microwave absorption [10]. The method has been used to study the rich out-ofequilibrium physics and a variety of nontrivial nonlinear effects associated with moderately strong resonant microwave excitation of the electron system [10, 12, 27, 28]. Here we focus on the linear response. The microwave power was attenuated down to  $\mu$ W levels. The experimental technique used here is very close to [12], however, improvements were made to work at very low microwave power and to ensure that the helium filling level in the sample cell was close to 50% to provide a good compensation between the electric field created by the top and bottom image charges. These steps are described in detail in the Supplemental Material [37].

The spectra of the resonant  $|1\rangle \rightarrow |2\rangle$  photoexcitation are shown in Fig. 2. For  $B_{\parallel} \gtrsim 0.4$  T, where the strong-coupling condition holds, the observed shape of the spectra is very well described by a Gaussian fit (dashed lines) with the variance  $\delta E_z$  given by Eq. (4), with no fitting parameters. The overall area of the spectral peaks is determined by the photoassisted transport response of electrons on helium, which depends on  $B_{\parallel}$ ; the discussion of this dependence is beyond the scope of this Letter.

In Fig. 3 we show the linewidth  $\delta E_z$  as a function of  $B_{\parallel}$  for several refrigerator temperatures. The observed linear dependence quantitatively agrees with Eq. (4) in the strong-coupling regime, which corresponds to  $B_{\parallel}T^{1/2} \gtrsim 0.15 \text{ T} \times \text{K}^{1/2}$ , for the used  $n_s$  and  $B_{\perp}$ . The linewidth at  $B_{\parallel} = 0$  is attributed to residual inhomogeneous broadening in our system. The linear fits to the data at different temperatures all intersect near  $B_{\parallel} = 0$ , supporting this interpretation. The inset shows the ratio  $\delta E_z/B_{\parallel}$  as a function of the square root of the temperature. The black line depicts this ratio as given by Eq. (4)



FIG. 2. The spectra of the relative microwave-induced change of the low-frequency admittance Y for different  $B_{\parallel} \equiv B_x$ . The data refer to the microwave frequency f = 150 GHz, T = 0.2 K,  $B_{\perp} \equiv B_z = 0.5$  T, and  $n_s = 21.5 \times 10^6$  cm<sup>-2</sup>. The ac bias is 30 mV. The dashed lines show Gaussian fit to the data with the variance  $\delta E_z^2$  given by Eq. (4).



FIG. 3. The dependence of the typical width of the spectral peaks  $\delta E_z$  on  $B_{\parallel} \equiv B_x$  for different temperatures. The other parameters are the same as in Fig. 2. The dashed lines are the linear fit. In the strong-coupling range they are described by Eq. (4). The inset shows the slope of  $\delta E_z/B_{\parallel}$  as a function of  $T^{1/2}$ . The dashed black line is given by Eq. (4) with no adjustable parameters.

with no adjustable parameters [Eq. (4) holds for  $T^{1/2} < (\hbar \omega_c / k_B)^{1/2} \approx 0.6 \text{ K}^{1/2}$ ].

To further check Eq. (4) we investigated the density dependence of the linewidth for different magnetic fields  $B_{\parallel} \equiv B_x$  and  $B_{\perp} \equiv B_z$ . In order to reduce the averaging time and increase the sensitivity for small  $n_s$  we used a stronger microwave power, in the 100  $\mu$ W range. This resulted in an additional spectral broadening, which we attribute to an effective electron temperature  $T_e = 0.6$  K (the refrigerator temperature is 0.3 K; the dependence of the linewidth on the microwave power is shown in the inset in Fig. 4). With this assumption the data are in full agreement with Eq. (4). As shown in Fig. 4,  $(\delta E_z)^2 \propto n_s^{3/2}$ .



FIG. 4. The dependence of the squared linewidth scaled by the ratio of the magnetic fields  $B_{\parallel} \equiv B_x$  and  $B_{\perp} \equiv B_z$  on the electron density  $n_s$ . The excitation frequency is 174 GHz except for the data at  $B_{\perp} \equiv B_z = 0.3$  T, which refers to f = 144 GHz. The black line shows the prediction of Eq. (4) for the effective electron temperature  $T_e = 0.6$  K. Inset: the linewidth as a function of the microwave power for  $B_z = 0.5$  T,  $B_x = 0.75$  T, f = 150 GHz, and  $n_s = 21.5 \times 10^6$  cm<sup>-2</sup>.

By rescaling the linewidth, we see that the results for different  $B_{\parallel}$  and  $B_{\perp}$  collapse onto the same curve.

The many-electron theory of the interband absorption spectra developed in this Letter and the experimental observations are in full quantitative agreement, with no adjustable parameters. In contrast to the previous work on the electron absorption spectra, the theory explicitly takes into account strong electron correlations. The experimental data were obtained by extending the measurements to low microwave power, which made it possible to investigate the spectra in the linear-response regime.

The experimental data provide the first direct measurement of the fluctuational electric field which an electron is experiencing in a nondegenerate electron liquid and which, as we show, determines the shape of the spectrum. The results refer to a broad range of the electron densities, temperature, and the coupling strength of the in-plane and out-of-plane motions, where the in-plane motion is quantized by the magnetic field. Such quantization is advantageous for revealing nontrivial aspects of the many-electron dynamics in a strongly correlated two-dimensional system.

Our results demonstrate that, by applying an in-plane magnetic field, one can directly study intimate features of the quantum physics of an electron liquid and a Wigner crystal. The regimes other than the one explored here experimentally can be also investigated with the developed technique. Those include the regime of Wigner crystallization, in which case the closed-form expression for the spectrum is obtained. Self-diffusion in the electron liquid, which is hard to characterize otherwise, can be also explored. Importantly, the results demonstrate that electrons on helium can be used as a test bed for the quantum theory of the effect of the electron-phonon coupling on the optical spectra of color centers. The system provides a unique setting where both the effective coupling strength and the spectrum of elementary excitations coupled to the electron transition can be varied in situ by varying the inplane and out-of-plane magnetic fields.

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# Supplemental Material for the paper Many-electron system on helium and color center spectroscopy

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### I. MANY-ELECTRON THEORY OF THE ABSORPTION SPECTRUM

#### A. The Hamiltonian

We recall for completeness that we consider the absorption spectrum due to transitions between the states  $|1\rangle$  and  $|2\rangle$  of the out-of-plane electron motion and the effect on this motion of the magnetic field  $B_{\parallel}$  parallel to the helium surface. In the presence of the in-plane field  $B_{\parallel} \equiv B_x$  and a magnetic field  $B_{\perp} \equiv B_z$  normal to the surface, the Hamiltonian of the many-electron system is

$$H = H_{\parallel} + H_{\perp} + H_{i}, \quad H_{\parallel} = \sum_{n} \frac{\pi_{n}^{2}}{2m} + \frac{1}{2} \sum_{n,m}' \frac{e^{2}}{|\mathbf{r}_{n} - \mathbf{r}_{m}|},$$
$$H_{\perp} = \sum_{n} \left[ \frac{p_{nz}^{2}}{2m} + U(z_{n}) \right], \quad H_{i} = \sum_{n} \omega_{\parallel} \pi_{ny} (z_{n} - \bar{z}_{11}).$$
(1)

Here *n* enumerates electrons,  $\mathbf{r}_n \equiv (x_n, y_n)$  and  $\pi_n = -i\hbar \nabla_n + (e/c) \mathbf{A}_{\perp}(\mathbf{r}_n)$  are the in-plane electron coordinate and kinematic momentum  $[\mathbf{A}_{\perp}(\mathbf{r})$  is the vectorpotential of the field  $B_{\perp} \equiv B_z$ ] [1]. The coordinate *z* is normal to the helium surface and U(z) is the confining potential. The states  $|\mu\rangle$  (with  $\mu = 1, 2$ ) are the quantized states of motion in this potential, and in Eq. (1) we use notations

$$\omega_{\parallel} = eB_{\parallel}/mc, \quad \bar{z}_{\mu\mu} = \langle \mu | \, z \, | \mu \rangle \,.$$

The matrix elements of the normal coordinate  $z_n$  are the same for all electrons; therefore we skip the subscript n in the definition of  $\bar{z}_{\mu\mu}$  as well as in the off-diagonal matrix elements  $\langle \mu | z | \mu' \rangle$  below.

The electron confinement in the z-direction is determined by a (practically) infinite barrier at the helium surface at z = 0 and the attractive potential for z > 0[2, 3]. The potential U(z) for z > 0 is formed by the image force and the electric field  $E_z$  normal to the surface, which is applied to further confine the electrons. In the presence of the magnetic field  $B_{\parallel}$  the potential acquires the term  $m\omega_{\parallel}^2(z - \bar{z}_{11})^2/2$ . This term leads to a comparatively small change of the distance  $\varepsilon_{21}$  between the energy levels of the out-of-plane motion,

$$\varepsilon_{21} \to \varepsilon_{21} + (m\omega_{\parallel}^2/2)[\langle 2| z^2 | 2 \rangle - \langle 1| z^2 | 1 \rangle - 2\bar{z}_{11}(\bar{z}_{22} - \bar{z}_{11})].$$

In Eq. (1) we have disregarded the part of  $H_i$  that is non-diagonal with respect to the states  $|\mu\rangle_n$  of individual electrons. This part can be treated by the perturbation theory for the considered low temperatures,  $k_BT \ll \varepsilon_{21}$ . To the leading order, it causes a shift of the Landau levels of the in-plane motion in the field  $B_{\perp}$  normal to the surface. In the interesting case where  $\varepsilon_{21} \gg \hbar \omega_c$  (here  $\omega_c = eB_{\perp}/mc$ ) this shift has an opposite sign for the Landau levels in the states  $|1\rangle$  and  $|2\rangle$ . For the kth Landau level (k = 0, 1, ...) in the out-of-plane state  $|1\rangle$  it is  $\approx -(\hbar \omega_c/2\varepsilon_{21})m\omega_{\parallel}^2(2k+1)|\langle 1|z|2\rangle|^2$ . The analysis can be extended to the case where the ratio  $\hbar \omega_c/\varepsilon_{21}$  is not small.

We have also omitted in REq. (1) the out-of-plane component of the electron-electron interaction. It makes  $\varepsilon_{21}$ weakly density-dependent [4, 5]. The corresponding shift of  $\varepsilon_{21}$  is  $\sim m\omega_p^2(\bar{z}_{22} - \bar{z}_{11})^2$ . The full expression is given in [5].

#### B. The adiabatic approximation

The first step toward calculating the absorption spectrum at frequency  $\omega \sim \varepsilon_{21}/\hbar$  is to note that this frequency largely exceeds all characteristic frequencies of the in-plane electron motion. Therefore one can think of the adiabatic approximation in which a transition between the states  $|1\rangle$  and  $|2\rangle$  occurs "instantaneously" for a given in-plane many-electron state. The transition frequency depends on this state, and it is this dependence that determines the shape of the spectrum.

Formally, the shape of the spectrum of absorption of radiation polarized in the z-direction is given by the Fourier transform of the commutator  $\langle [z_n(t), z_n(0)] \rangle$  at frequency  $\omega \approx \varepsilon_{21}/\hbar$ ,

$${\rm Re}\,\sigma_{zz}(\omega) \propto {\rm Re}\,\int_0^\infty dt e^{i\omega t} \langle [z_n(t),z_n(0)]\rangle$$

As indicated in the main text, this immediately follows from the standard Kubo formula. To describe the absorption peak, we need to single out in the above commutator the resonant term  $\propto \exp(-i\varepsilon_{21}t/\hbar)$ . This can be conveniently done by switching to the interaction representation with the operator  $U_{\rm int}(t) = \exp[-i(H_{\parallel} + H_{\perp})t/\hbar]$ . In this representation the corresponding term in the commutator takes the form

$$\langle [z_n(t), z_n(0)] \rangle = \left| \langle 1 | z | 2 \rangle \right|^2 \exp(-i\varepsilon_{21}t/\hbar)Q(t),$$
$$Q(t) = \left\langle T_\tau \exp\left[ -i(\omega_{\parallel}\Delta_z/\hbar) \int_0^t d\tau \pi_{ny}(\tau) \right] \right\rangle.$$
(2)

We assumed here that the electrons are in the ground state of the out-of-plane motion,  $\exp(-\varepsilon_{21}/k_BT) \ll 1$ .

Therefore the averaging  $\langle \cdot \rangle$  is the thermal averaging over the in-plane many-electron states;  $T_{\tau}$  is the time-ordering operator.

The Fourier transform of function Q(t) at frequency  $\omega - \varepsilon_{21}/\hbar$  gives the shape of the absorption spectrum. As seen from Eq. (2), the effect of the in-plane motion on the spectrum is controlled by the magnetic field  $B_{\parallel}$  and the parameter

$$\Delta_z = \bar{z}_{22} - \bar{z}_{11},\tag{3}$$

which gives the change of the mean distance of the electron from the helium surface in the excited and ground states of the out-of-plane motion. We note that Q(t) is independent of the electron number n, it is the same for all electrons.

We note also that we have not taken into account the fact that the lifetime of the electrons in the state  $|2\rangle$  is finite. Therefore for  $B_{\parallel} = 0$ , where Q(t) = 1, the real part of the Fourier transform of the commutator (2) becomes  $\propto \delta[\omega - \varepsilon_{21}/\hbar]$ , i.e., the absorption peak is a  $\delta$ -function at the transition frequency.

## C. Slow variables in the electron liquid in a strong magnetic field

The calculation of Q(t) is done differently for an electron liquid and a Wigner solid, with the calculation for a Wigner solid being simpler, see Sec. I.E. In the both cases the result depends on the interrelation between the frequency  $\omega_{\parallel}$ , the frequencies that determine the electron dynamics

$$\omega_p = (2\pi e^2 n_s^{3/2}/m)^{1/2}, \qquad \omega_c = eB_\perp/mc, \qquad (4)$$

and the temperature. Here  $n_s$  is the electron density and  $\omega_c$  is the cyclotron frequency. The frequency  $\omega_p$  is the characteristic short-wavelength plasma frequency of the 2D electron system in the absence of a magnetic field; it is obtained from the standard expression for the long-wavelength plasma frequency as a function of the in-plane wave number (cf. [2]) by setting this wave number equal to  $n_s^{1/2}$ . This is the analog of the Debye frequency of the Wigner crystal in the absence of a magnetic field.

The experimental results in the paper refer to the case where the electrons form a liquid and this liquid is placed into a strong a strong magnetic field  $B_{\perp}$ , where the cyclotron frequency  $\omega_c$  exceeds  $\omega_p$ . An important frequency in this case is  $\omega_p^2/\omega_c$ . This is the limiting frequency of the lower branch of phonons in the Wigner crystal in a strong field  $B_{\perp}$  [6]. In the case of the electron liquid, this frequency gives the reciprocal time scale for the motion of the guiding centers of the cyclotron orbits, as shown below. We will concentrate on calculating Q(t) for the case that was studied in the experiment in most detail,

$$\omega_c \gtrsim k_B T / \hbar \gg \omega_p^2 / \omega_c. \tag{5}$$

where the electrons are mostly in the ground Landau level, which is smeared by the electron-electron interaction, and the motion of the guiding centers of the cyclotron orbits is semiclassical.

#### 1. Dynamics of the guiding center

To calculate the function Q(t) it is necessary to find the time evolution of the operators of the kinematic momentum of an electron  $\pi_n(t)$  in the interaction representation. In the parameter range Eq. (5), it is convenient to change to new variables from the in-plane coordinate  $\mathbf{r_n}$  and kinematic momentum  $\pi_n$ . These variables are the fast oscillating momentum components  $\pi_{n\pm}$  and the slowly varying guiding center coordinates  $R_{n\pm}$ , which commute with  $\pi_{n\pm}$ ,

$$\pi_{n\pm} = (\ell/\hbar\sqrt{2})(\pi_{nx} \mp i\pi_{ny}), \qquad [\pi_{n-}, \pi_{n'+}] = \delta_{nn'}.$$
(6)

Here  $\ell = (\hbar c/eB_{\perp})^{1/2}$  is the quantum magnetic length (we chose the sign of the magnetic field  $B_{\perp} \equiv B_z$  so that  $eB_{\perp} > 0$ ). The guiding center operators are

$$R_{n\pm} = x_n \mp i y_n \pm i \ell \sqrt{2} \, \pi_{n\pm}, \qquad [R_{n\pm}, \pi_{n'\pm}] = 0,$$
  
$$[R_{n+}, R_{n'-}] = 2\ell^2 \delta n n'. \tag{7}$$

The underlying physical picture is that the guiding centers of the electron cyclotron orbits fluctuate about their quasiequilibrium position in the electron liquid or equilibrium positions in the Wigner crystal. The fluctuations are thermal, with the typical mean square displacement  $\sim k_B T/m\omega_p^2$ . This estimate is obtained by considering a displacement of an electron from its (quasi)equilibrium position in the field of other electrons, given that this displacement is small compared to the inter-electron distance  $\sim n_s^{-1/2}$ . Still the displacement largely exceeds the quantum magnetic length  $\ell$  in the parameter range (5). Therefore the dynamics of the guiding centers in this range is semiclassical [7].

On the formal side, the electron kinetic energy in terms of the operators  $\pi_{n\pm}$  is  $\sum_n (\pi_n^2/2m) \rightarrow \hbar\omega_c \sum_n \pi_{n+}\pi_{n-} + \text{const.}$  Operators  $\pi_{n\pm}$  play the role of the ladder operators with respect to the Landau energy levels. The equation of motion for  $R_{n\pm}$  in the interaction representation is  $\dot{R}_{n\pm} = -(i/\hbar)[R_{n\pm}, H_{\parallel}]$  with  $H_{\parallel}$  given by Eq. (1). As seen from Eq. (7), to the leading order in  $\ell^2 n_s$  this equation can be written as

$$\dot{R}_{n\pm} = \mp i c \tilde{E}_{n\pm} / B_{\perp}, \quad \tilde{E}_{n\pm} = -e \sum_{m}' R_{nm\pm} / |R_{nm+}|^3,$$
$$R_{nm\pm} = R_{n\pm} - R_{m\pm}, \quad \tilde{E}_{n\pm} \equiv (\tilde{E}_{nx} \mp i \tilde{E}_{ny}). \tag{8}$$

Here we have used that the magnetic length  $\ell$  is small compared to the interelectron distance  $|\mathbf{r}_n - \mathbf{r}_m| \approx |R_{nm\pm}|$ . If we disregard corrections  $\sim \ell^2 n_s$ , the field  $\tilde{\mathbf{E}}_n$  coincides with the field  $\mathbf{E}_n$  used in the main text. In the approximation  $\ell^2 n_s \ll 1$  we have also disregarded the non-commutativity of the position operators of the guiding centers  $R_{n+}$  and  $R_{n-}$ . This semiclassical approximation breaks down for  $k_B T < \hbar \omega_p^2 / \omega_c$ . The analysis of the low-temperature case can be done assuming that the electrons form a Wigner crystal, see Sec. I.E.

With the account taken of the relation  $|R_{nm\pm}| \gtrsim n_s^{-1/2}$ , one can see from Eq. (8) that the time scale on which the guiding orbit centers  $R_{n\pm}$  are changing is given by  $\omega_c/\omega_p^2$ . The field  $\tilde{E}_{n\pm}$  varies on the same time scale.

To the first order in  $\ell n_s^{1/2}$ , the equation of motion for the operators  $\pi_{n\pm}$  in the interaction representation is

$$\dot{\pi}_{n\pm} \equiv -\frac{i}{\hbar} [\pi_{n\pm}, H_0] = \pm i\omega_c \pi_{n\pm} - \frac{\ell}{\hbar\sqrt{2}} e E_{n\pm}, \qquad (9)$$

$$E_{n\pm} \approx \tilde{E}_{n\pm} \mp i \frac{e\ell}{\sqrt{2}} \sum_{m}' \left[ \frac{\pi_{nm\pm}}{|R_{nm+}|^3} - 3 \frac{R_{nm\pm}^2 \pi_{nm\mp}}{|R_{nm+}|^5} \right],$$

where  $\pi_{nm\pm} = \pi_{n\pm} - \pi_{m\pm}$ .

It is seen from Eq. (9) that the operators  $\pi_{n\pm}$  oscillate in time as  $\exp(\pm i\omega_c t)$ . The field  $\tilde{E}_{n\pm}$ , on the other hand, varies on a much slower time scale  $\sim (\omega_p^2/\omega_c)^{-1}$ . For  $\omega_c \gg \omega_p^2/\omega_c$  we can write the solution of Eq. (9) in the adiabatic approximation as

$$\pi_{n\pm}(t) \approx \tilde{\pi}_{n\pm}(t) e^{\pm i\omega_c t} \mp i \frac{e\ell}{\hbar\omega_c \sqrt{2}} \tilde{E}_{n\pm}(t) \qquad (10)$$

The time dependence of the operators  $\tilde{\pi}_{n\pm}$  is determined, to the lowest order, by the first term in the square bracket in the expression (9) for  $E_{n\pm}$ . To the zeroth order in the electron-electron interaction, this term is oscillating as  $\exp(\pm i\omega_c t)$ , i.e., in the same way as  $\pi_{n\pm}$ , whereas the last term in the expression for  $E_{n\pm}$ is counter-rotating, it oscillates as  $\exp(\mp i\omega_c t)$ . As seen from Eq. (9),  $\tilde{\pi}_{n\pm}$  varies on the time scale  $(\omega_n^2/\omega_c)^{-1}$  [7].

### D. Averaging over fluctuations in the quantum nondegenerate electron liquid

An important consequence of the results of the previous section is that in the considered parameter range (5), the averaging over the quantum cyclotron motion and over the positions of the guiding centers in Eq. (2) can be done separately. The first averaging is essentially the summation over the Landau levels, and for  $\exp(\hbar\omega_c/k_BT) \gg 1$  the major contribution to the average comes from the lowest Landau level. The second averaging is the integration over the electron positions in the electron liquid,

$$Q(t) \approx I^{(1)}(t)I^{(2)}(t); \qquad I^{(1)}(t) = \left\langle T_{\tau} \exp\left[\frac{\omega_{\parallel}\Delta_z}{\ell\sqrt{2}}\right] \times \int_0^t d\tau \left(\tilde{\pi}_{n+}(\tau)e^{i\omega_c\tau} - \tilde{\pi}_{n-}(\tau)e^{-i\omega_c\tau}\right)\right] \right\rangle,$$
$$I^{(2)}(t) = \left\langle T_{\tau} \exp\left[i\frac{e\omega_{\parallel}\Delta_z}{\hbar\omega_c}\int_0^t d\tau \tilde{E}_{nx}(\tau)\right] \right\rangle. \tag{11}$$

Function Q(t) should be found for t of the order of the reciprocal width of the absorption spectrum. We will assume that this width is much smaller than  $\omega_c$ . Respectively, we are interested in the values of  $I^{(1,2)}$  for  $\omega_c t \gg 1$ . To find  $I^{(1)}$  in this range one can use the approximation of non-intersecting diagrams. Odd-order terms in the series expansion of  $I^{(1)}(t)$  in  $\tilde{\pi}_{n\pm}$  vanish: they contain an odd number of the operators  $\tilde{\pi}_{n\pm}$ , and their diagonal matrix element on the wave functions of the Landau levels is zero. We can write the 2kth term in the expansion of  $I^{(1)}(t)$  as

$$I_{k}^{(1)}(t) = \alpha^{2k} \int_{t_{2}}^{t} dt_{1}\hat{f}(t_{1}) \int_{t_{3}}^{t} dt_{2}\hat{f}(t_{2}) \int_{0}^{t} dt_{3}\hat{f}(t_{3})...$$
$$\times \int_{0}^{t_{2k-1}} dt_{2k}\hat{f}(t_{2k}), \quad \alpha = \omega_{\parallel}\Delta_{z}/\ell\sqrt{2},$$
$$\hat{f}(t) = \tilde{\pi}_{n+}e^{i\omega_{c}t} - \tilde{\pi}_{n-}e^{-i\omega_{c}t}$$
(12)

Integration over  $dt_1$  gives four terms. The first two of them oscillate as  $\exp(\pm i\omega_c t)$  whereas the second two of them oscillate as  $\exp(\pm i\omega_c t_2)$ . When multiplied by  $\hat{f}(t_2)$ , the first two terms will be fast oscillating as  $\exp[\pm i\omega_c(t\pm t_2)]$ . Their integral over  $t_2$  will be  $\propto \omega_c^{-1}$ . In contrast, the second two terms, when multiplied by  $\hat{f}(t_2)$ , will lead to the onset of smooth terms. Integrating them over  $t_2$ will give a factor  $\propto t - t_3 \gg \omega_c^{-1}$ ,

$$\int_{t_2}^t dt_1 \hat{f}(t_1) \int_{t_3}^t dt_2 \hat{f}(t_2)$$
  

$$\approx \frac{i}{\omega_c} \int_{t_3}^t dt_2 [\tilde{\pi}_{n-}(t_2), \tilde{\pi}_{n+}(t_2)] = \frac{i}{\omega_c} (t - t_3).$$

Substituting this expression into Eq. (12), differentiating over t, and integrating the resulting chain of equations for  $I_k^{(1)}(t)$ , we obtain:

$$\frac{dI_k^{(1)}}{dt} = (i\alpha^2/\omega_c)I_{k-1}^{(1)}, \qquad I_k^{(1)} = (i\alpha^2 t/\omega_c)^k/k! \quad (13)$$

This gives, to the leading order in  $\alpha^2$ ,

$$I^{(1)}(t) = \exp(i\delta_{\parallel}t),$$
  

$$\delta_{\parallel} = \omega_{\parallel}^2 \Delta_z^2 / 2\ell^2 \omega_c \equiv m \omega_{\parallel}^2 \Delta_z^2 / 2\hbar.$$
(14)

The term  $I^{(2)}(t)$  can be calculated assuming that the distribution of the fluctuational field in the electron liquid is Gaussian. This assumption was used in the main text. Numerical simulations [8] have shown that the singletime distribution of the fluctuational field is indeed very close to Gaussian except for very far tails. The Gaussian distribution is to be expected, since the electron dynamics is primarily random harmonic vibrations about a (quasi)equilibrium position in the electron liquid or a Wigner crystal. This argument also suggests that the probability density functional of the fluctuational field  $\mathcal{P}(\mathbf{E}_n)$  is Gaussian,

$$\mathcal{P} = \exp\left[-\frac{1}{2}\iint dt dt' \mathbf{E}_n(t)\hat{\Lambda}(t-t')\mathbf{E}_n(t')\right]$$
(15)

where  $\hat{\Lambda}(t)$  is the reciprocal correlation function of the fluctuational field. By symmetry,

 $Lambda_{xx} = \Lambda_{yy}$ , whereas  $\Lambda_{xy} = \Lambda_{yx} = 0$ . It is seen from Eqs. (11) and (15) that

$$I^{(2)}(t) = \exp[-(\gamma^2/2)w(t)], \quad \gamma^2 = \delta_{\parallel}\omega_p^2 k_B T/2\pi\hbar\omega_c^2,$$
  
$$w(t) = (n_s^{3/2}k_B T)^{-1} \iint_0^t dt_1 \, dt_2 \, \langle \mathbf{E}_n(t_1)\mathbf{E}_n(t_2) \rangle. \quad (16)$$

Equations (14) and (16) give the correlation function Q(t) in the explicit form, which coincides with Eq. (3) of the main text.

The correlation function of the fluctuational field in Eq. (16) can be written as

$$\iint_{0}^{t} dt_{1} dt_{2} \langle \mathbf{E}_{n}(t_{1}) \mathbf{E}_{n}(t_{2}) \rangle \approx \iint_{0}^{t} dt_{1} dt_{2} \langle \tilde{\mathbf{E}}_{n}(t_{1}) \tilde{\mathbf{E}}_{n}(t_{2}) \rangle$$
$$= (B_{\perp}/c)^{2} \iint_{0}^{t} dt_{1} dt_{2} \langle \dot{\mathbf{R}}_{n}(t_{1}) \dot{\mathbf{R}}_{n}(t_{2}) \rangle$$
$$= (B_{\perp}/c)^{2} \langle [\mathbf{R}_{n}(t) - \mathbf{R}_{n}(0)]^{2} \rangle.$$
(17)

This expression relates  $I^{(2)}(t)$ , and thus Q(t) to the mean square displacement of a guiding center of the cyclotron orbit, providing an intuitive insight into the nature of the broadening of the absorption spectrum by the electronelectron interaction.

In the strong-coupling limit,  $\gamma^2 \gg 1$ , the function  $I^{(2)}(t)$  is fast decaying with time. For comparatively short times we have  $\langle [\mathbf{R}_n(t) - \mathbf{R}_n(0)]^2 \rangle \approx \langle \dot{\mathbf{R}}_n^2 \rangle t^2$ , that is, the decay of  $I^{(2)}(t)$  is Gaussian. Respectively, the peak in the absorption spectrum is also Gaussian. When calculating the dependence of the position of the peak on the in-plane magnetic field one has to take into account the factor  $\exp(i\delta_{\parallel}t)$  in the expression for  $I^{(1)}(t)$  and also the renormalization of  $\varepsilon_{21}$  due to the term  $m\omega_{\parallel}^2(z-\bar{z}_{11})^2/2$  in the potential U(z). This gives for the resonant frequency

$$\omega_{\rm res} \approx \hbar^{-1} \varepsilon_{21} + \hbar^{-1} (m \omega_{\parallel}^2/2) (\langle 2 | z^2 | 2 \rangle - \langle 1 | z^2 | 1 \rangle - \langle 2 | z | 2 \rangle^2 + \langle 1 | z | 1 \rangle^2)$$
(18)

On the time scale much longer than  $\omega_c/\omega_p^2$ , if electrons form a liquid, they are diffusing. Self-diffusion involves correlated many-electron motion, as seen in the simulations [9]. Therefore it is reasonable to assume that the distribution of the diffusion trajectories is Gaussian. We can then again do the averaging over the fluctuational field in  $I^{(2)}(t)$  assuming the field distribution to be Gaussian. From Eq. (17), on a long time scale, the leadingorder term in the integral of the correlation function of the fluctuational field is

$$\iint_{0}^{t} dt_1 dt_2 \langle \mathbf{E}_n(t_1) \mathbf{E}_n(t_2) \rangle \approx 2(B_{\perp}/c)^2 Dt \qquad (19)$$

where D is the self-diffusion coefficient. Equation(19) was used in the main text to describe the long-time limit of the correlator Q(t) and thus the many-electron analog of the zero-phonon line in the electron liquid. As in the case of color centers, this line has an exponentially small intensity for strong coupling.

#### E. Wigner crystal

In the case of a Wigner crystal, we write the Hamiltonian  $H_{\parallel}$  as

$$H_{\parallel} = \hbar \sum_{\mathbf{k}\nu} \omega_{\mathbf{k}\nu} a_{\mathbf{k}\nu}^{\dagger} a_{\mathbf{k}\nu}.$$
 (20)

Here, **k** is the wave vector of a phonon of the crystal and  $\nu = 1, 2$  is the phonon branch. Operators  $a_{\mathbf{k}\nu}^{\dagger}$  and  $a_{\mathbf{k}\nu}$  are the phonon creation and annihilation operators and  $\omega_{\mathbf{k}\nu}$  is the phonon frequency. The operator of the kinematic momentum of an *n*th electron is

$$\boldsymbol{\pi}_n = -im \sum_{\mathbf{k}\nu} e^{i\mathbf{k}\mathbf{X}_n} \omega_{\mathbf{k}\nu} \mathbf{A}_{\mathbf{k}\nu} a_{\mathbf{k}\nu} + \text{H.c.} \qquad (21)$$

where  $\mathbf{X}_n$  is the lattice site position. The coefficients  $\mathbf{A}_{\mathbf{k}\nu} \propto (n_s S)^{-1/2}$  (S is the area of the system) give the electron displacement in terms of  $a_{\mathbf{k}\nu}, a_{\mathbf{k}\nu}^{\dagger}$ . For the Wigner crystal in a strong magnetic field  $B_{\perp}$  they were obtained in Ref. 6.

Substituting Eq. (21) into Eq. (2) for the correlator Q(t) and calculating the trace over the phonons in a standard way, we obtain

$$Q(t) = \exp\left[-(m\omega_{\parallel}^{2}\Delta_{z}^{2}/\hbar\omega_{c})g(t)\right],$$
  

$$g(t) = \frac{1}{2\ell^{2}}\sum_{\mathbf{k}\nu}|\mathbf{A}_{\mathbf{k}\nu}|^{2}\left[i\left(\sin\omega_{\mathbf{k}\nu}t - \omega_{\mathbf{k}\nu}t\right) + (2\bar{n}_{\mathbf{k}\nu} + 1)(1 - \cos\omega_{\mathbf{k}\nu}t)\right],$$
(22)

where  $\bar{n}_{\mathbf{k}\nu} \equiv \bar{n}(\omega_{\mathbf{k}\nu})$  is the phonon Planck number,  $\bar{n}(\omega) = [\exp(\hbar\omega/k_BT) - 1]^{-1}$ . Equation (22) is not limited to the case of a strong magnetic field  $B_{\perp}$ . It also applies for an arbitrary temperature as long as the electrons form a Wigner crystal.

Before discussing other limiting cases we show that Eq. (22) coincides with Eqs. (11), (14) and (16) in a strong magnetic field  $B_{\perp}$ , when  $\omega_c \gg \omega_p$ , and when  $k_BT \gg \hbar \omega_p^2/\omega_c$  [except that Eq. (22) does not describe electron diffusion in the liquid phase]. We note first that, for a strong field  $B_{\perp}$ , the phonon spectrum of the Wigner crystal consists of a high-frequency magnetoplasmon branch  $\nu = 1$  and a low-frequency branch  $\nu = 2$ . The widths of the both branches are  $\sim \omega_p^2/\omega_c$ . The branch  $\nu = 1$  is an analog of the optical phonon branch,  $\omega_{\mathbf{k}\,\nu=1} \rightarrow \omega_c$  for  $k \rightarrow 0$ , whereas  $\omega_{\mathbf{k}\,\nu=2} \propto k^{3/2}$  for  $k \rightarrow 0$ .

The contribution of the branch  $\nu = 1$  to g(t) consists of fast-oscillating terms  $\propto \exp(\pm i\omega_c t)$ , which make a small contribution to the smooth part of Q(t), and also

of a non-oscillating term  $\propto t$ . Using the results [6], one can show that  $|A_{\mathbf{k}1}|^2 \approx \ell^2/n_s S$ , to the leading order in  $\omega_p/\omega_c$ . Therefore the term  $\propto t$  in g(t) is  $\approx -i\omega_c t/2$  and its effect on Q(t) is described by the factor  $\exp(i\delta_{\parallel}t)$ ; the correction to  $\delta_{\parallel}$  from the branch  $\nu = 2$  can be shown to be  $\propto \omega_p^2/\omega_c^2 \ll 1$ .

The branch  $\nu = 2$  corresponds to vibrations of the electron guiding centers. For  $k_B T \gg \hbar \omega_p^2 / \omega_c$  these vibrations are semiclassical and are described by Eq. (8) linearized in  $\mathbf{R}_n - \mathbf{X}_n$ . The mean-square electron displacement is  $\langle (\mathbf{R}_n - \mathbf{X}_n)^2 \rangle \gg \ell^2$ , it is determined by the branch  $\nu = 2$  if we neglect corrections  $\sim \ell^2$ . Writing

$$\mathbf{R}_n - \mathbf{X}_n = \sum_{\mathbf{k}} e^{i\mathbf{k}\mathbf{X}_n} \mathbf{A}_{\mathbf{k}\,\nu=2} a_{\mathbf{k}\,\nu=2} + \text{H.c.},$$

we obtain from Eq. (17)

$$\iint^{t} dt_{1} dt_{2} \langle \mathbf{E}_{n}(t_{1}) \mathbf{E}_{n}(t_{2}) \rangle \approx (B_{\perp}/c)^{2} \\ \times \sum_{\mathbf{k}} |\mathbf{A}_{\mathbf{k}\,\nu=2}|^{2} (2\bar{n}_{\mathbf{k}\,\nu=2}+1)(1-\cos\omega_{\mathbf{k}\,\nu=2}t)$$

(strictly speaking, we should have replaced  $2\bar{n}_{\mathbf{k}\nu=2}+1 \rightarrow 2k_B T/\hbar\omega_{\mathbf{k}\nu=2}$ ). This expression has the same form as the corresponding term in Eq. (22). It shows that, indeed, Eq. (3) of the main text coincides with Eq. (22) where electrons form a Wigner crystal.

A somewhat unexpected result follows from Eq. (22) in the classical limit,  $k_B T \gg \omega_{\mathbf{k}\nu}$ . Using the sum rule  $\sum_{\mathbf{k}\nu} |\mathbf{A}_{\mathbf{k}\,\nu}|^2 \omega_{\mathbf{k}\nu} = \hbar/m$  we find that

$$g(t) \approx (\omega_c k_B T/2\hbar) t^2, \qquad \omega_{\mathbf{k}\nu} t \ll 1.$$
 (23)

Then from Eq. (22),  $Q(t) \approx \exp[-(m\omega_{\parallel}^2 \Delta_z^2 k_B T t^2/2\hbar^2]$ and the spectrum  $\sigma_{zz}(\omega)$  is Gaussian with typical width

$$\gamma_G^{\text{classic}} = (m\omega_{\parallel}^2 \Delta_z^2 k_B T/\hbar^2)^{1/2}.$$

The expansion where we keep only the term  $\propto t^2$  in g(t) applies provided  $\gamma_G^{\text{classic}} \gg \max \omega_{\mathbf{k}\nu}$ .

Interestingly, the same result for the absorption spectrum follows from the general expression (3) of the main text if one assumes that there is no electron-electron interaction and no transverse magnetic field  $B_{\perp}$ , so that  $\pi_{ny}$  is independent of time and the momentum distribution is of the Maxwell-Boltzmann form. This corresponds to the Doppler broadening of the absorption spectrum due to the thermal distribution of the electron momentum.

Even for a strong in-plane magnetic field  $B_{\parallel}$  and high temperatures, where Eq. (23) applies, the many-electron interaction leads to the onset of an analog of the zerophonon line. For a Wigner crystal this line has zero width. This means that, for low temperatures, the width of this line in the system of electrons on helium is determined by the electron scattering by ripplons and phonons; for  $T \gtrsim 0.7$  K it is determined by scattering by the helium vapor atoms [2, 3].

### F. Estimation of the factor $F(\Gamma)$ for a Wigner crystal

As indicated in the main text, the mean square fluctuational field that drives an electron due to the density fluctuations has the form  $\langle \mathbf{E}_n^2 \rangle = F(\Gamma) n_s^{3/2} k_B T$ . The factor  $F(\Gamma)$  in this expression can be explicitly calculated if the electrons form a Wigner crystal. It is given by the lattice sum [7, 8]

$$F(\Gamma) = n_s^{-3/2} \sum_m' |\mathbf{X}_m - \mathbf{X}_0|^{-3}.$$
 (24)

For a hexagonal lattice the lattice sites are  $\mathbf{X}_m = (2n_s)^{-1/2} 3^{-1/4} [(2m_1 + m_2)\hat{\mathbf{x}} + m_2\sqrt{3}\hat{\mathbf{y}}]$  with integer  $m_{1,2}$ and with  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  being orthogonal unit vectors. Then

$$F(\Gamma) = 2^{3/2} 3^{3/4} \sum_{m_1, m_2} [(2m_1 + m_2)^2 + 3m_2^2]^{-3/2}, \quad (25)$$

where the sum over  $m_1, m_2$  runs from  $-\infty$  to  $\infty$  and the prime indicates that  $m_1^2 + m_2^2 > 0$ .

In Refs. [7, 8] the lattice sum (25) was calculated by summing over a few terms with  $(2m_1 + m_2)^2 + 3m_2^2 \leq K$ , integrating over  $dm_1 dm_2$  for larger  $|m_{1,2}|$ , and checking that the result weakly depended on K. A different way of calculating the sum is to write it as

$$F(\Gamma) = 2^{1/2} 3^{3/4} \sum_{m,n}' \frac{1 + (-1)^{m+n}}{(n^2 + 3m^2)^{3/2}} = 2 \frac{12^{3/4}}{\sqrt{\pi}} \int_0^\infty \sum_{m,n}' [1 + (-1)^{m+n}] e^{-(n^2 + 3m^2)x^2} x^2 dx$$
  
$$= 2 \frac{12^{3/4}}{\sqrt{\pi}} \int_0^\infty \left[ \theta_3 \left( 0, e^{-x^2} \right) \theta_3 \left( 0, e^{-3x^2} \right) + \theta_4 \left( 0, e^{-x^2} \right) \theta_4 \left( 0, e^{-3x^2} \right) - 2 \right] x^2 dx.$$
(26)

Here we are using the theta functions,

$$\theta_3(0,q) = \sum_{n=-\infty}^{\infty} q^{n^2}, \quad \theta_4(0,q) = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2}.$$

The advantagious feature of Eq. (26) is that it expresses the lattice sum in terms of the standard functions. It gives  $F(\Gamma) \approx 8.893$ .

#### II. EXPERIMENTAL TECHNIQUES

The experimental method is similar to that previously described [10], however two additional developments were used for this experiment. A method to accurately check the deviation of the helium level from the half filling position in the sample cell and an approach to measure at very low excitation microwave powers. We will describe these two aspects in more detail.

For transport experiments, electrons are trapped on the liquid helium surface placed midway between two circular electrodes of a parallel-plate capacitor inside a leaktight experimental cell cooled below 1 K. The distance between top and bottom electrodes was h = 2.6mm. An electric field  $E_z$  normal to the surface from the voltages on the electrodes confine electrons and provide shifts of the quantized energies of the electron out-of-plane motion due to the Stark effect. This allows us to bring the transition frequency  $f_{21} = \varepsilon_{21}/h$  in resonance with the external microwave radiation at a fixed frequency f = 150 GHz.

Resonant microwave absorption is observed via the change in the electron magnetoresistance. This change results from the irradiation-induced occupation of higher Rydberg levels in which the matrix elements of the electron-ripplon coupling are different. We employ the capacitive-coupling (Sommer-Tanner [11]) method using a pair of concentric electrodes which comprise the top plate of the parallel-plate capacitor. An ac (1 kHz) voltage signal V is applied to one of the electrodes, thus inducing an ac current I in the surface electrons which are capacitively coupled to the other electrode. The inphase and quadrature components of the cell admittance Y = I/V are measured using a lock-in amplifier.

A deviation of the free helium surface from the midway position between top and bottom electrodes leads to a shift of the transition frequency  $f_{21}$  proportional to the electron density. This is due to the Stark shift from the uncompensated attraction to top/bottom positive image charges. Solving the plane capacitor electrostatics we find that this additional perpendicular electric field  $E_{z,He}$ is given by:

$$E_{z,He} = -\frac{en_e}{2\epsilon_0}(2x-1) \tag{27}$$

where x is the helium filling fraction (the complete expression can be found in [12]). The induced field  $E_{z,He}$  cancels when the helium cell is half full, x = 1/2. Equation (27) thus provides a very accurate measurement of the helium filling of the cell. For our experiments the control of the Helium filling level allowed us to minimize the inhomgeneous broadening which may arise due to a combination of the induced field effect [Eq. (27)] and density gradients in the system. It also allowed us to study the resonance position as function of the electron density as



FIG. 1. Adjustment of the helium level close to half filling. The resonance position was measured as function of the electron density  $n_e$  by sweeping  $E_z$  at f = 144GHz $B_z = 0.73$  Tesla,  $B_x = 0$ . The helium level position was brought closer to half filling by small injections of helium gas using a needle valve.

otherwise such studies are complicated by a strong shift of the resonance position.

After an approximate first filling of the helium cell where the position of the half filling was estimated by the change in the capacitance between the top and bottom electrodes, we measured the shift of the resonance position with electron density finding initial x = 38%. Then we added small quantities of helium to the cell with the needle valve monitoring the change of the resonance position. This allowed us to come to within 3% of the half filling (see Fig. 1). We stopped at this value, because xmay continue to increase over time due to an influx of liquid helium trapped in the feeding capillary. However we did not observe any further noticeable increase in xafter the needle valve was closed. This leads us to expect that values very close to x = 50% can be achieved with this method in the future experiments.

In order to enhance sensitivity, the microwave power was pulse modulated using a low-frequency (17 Hz) square waveform, and a double-demodulation method similar to [10] was used. To find the regime where the linewidth of the resonance becomes independent of the microwave power, the signal from the mm-wave multiplier had to be attenuated significantly. A problem we faced was that our room temperature mm-wave power detector was not sensitive enough for the power readout after the attenuation. We thus used the coupling scheme shown in Fig. 2, where 90% of the signal from the generator was sent directly to the power detector, with 10% only used for sample excitation. The microwave multiplier (Virginia diode WR6.5AMC-I) allowed us to change



FIG. 2. Setup for power dependent experiments and the dependence of the photo-induced change in admittance on the microwave power. The plot shows the change of the real part of the admittance for  $B_z = 0.5$  Tesla,  $B_x = 0.75$  Tesla, T = 350 mK, f = 150 GHz.

- [1] Here and below, as well as in the main text, we are using CGS units. To switch to the SI units, one should set c = 1 in the expressions we are providing, replace  $e^2 \rightarrow e^2/4\pi\epsilon_0$  and in the expressions for w(t) in Eq. (16) and Eq. (3) of the main text replace  $n_s^{3/2} \rightarrow n_s^{3/2}/4\pi\epsilon_0$ .
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the power by more than two orders of magnitude using electrical analog modulation. However this attenuation was not sufficient for our experiments and we used a mechanically tunable attenuator to introduce an additional 1/130 attenuation before coupling into the dilution refrigerator. The value of the added 1/130 attenuation was deduced by comparing the power dependence of the microwave induced change in the cell admittance between the configuration where the attenuator was fully opened and closed to provide the desired attenuation (see Fig. 2).

We then measured the resonance linewidth as a function of microwave power obtaining the data shown in the inset of Fig. 4 in the main text and chose the microwave power at a level where the resonance linewidth was independent of the radiation intensity (300 nW). We note that this power level corresponds to a relative value compared to the estimated full power multiplier output (10 mW), it is not a quantitative measurement of the power entering the helium cell.

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