Range of existence of optical multistability in a resonator with an optically active medium

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Optical bistability in resonators containing a nonlinear gyrotropic isotropic medium is analyzed by studying the dependence of the bifurcation intensity of the incident radiation on its ellipticity. It is found that if the gyrotropy is weak, this dependence may have symmetry-induced singular points or turning points. The stable radiation states in the resonator formed near a singular or turning point differ in respect of the polarization and are due to the evolution of the polarization instability. Bifurcation curves are plotted for a ring resonator containing a medium with a cubic nonlinearity. The form of these curves changes appreciably as the gyrotropy is increased. The change in the resonator transmission accompanying changes in the gyrotropy is studied.

Changes in the polarization of light propagating in optical nonlinear media inside a resonator substantially enrich the pattern of optical bistability, resulting in the formation of new branches of the dependences of the amplitude and polarization parameters of the transmitted radiation on the parameters of the incident radiation.[1-3] The transmission pattern becomes even more interesting for optically active media[4] or media in a magnetic field[5,6] (a model of a gyrotropic medium with a special locally multivalued susceptibility was analyzed in Ref. 7). The calculations[4] show that even a weak gyrotropy of a nonlinear medium in a resonator can alter appreciably the transmission of an optical system if the radiation propagates along the optic axis.

The gyrotropy has a strong influence on optical bistability since it lifts the polarization degeneracy: radiation states differing only in respect of the sign of the polarization parameters (which are the direction of rotation of the vector **E** and the orientation of the polarization ellipse relative to the plane of symmetry containing the optic axis) become inequivalent.

A complete analysis of optical bistability taking into account changes in the polarization of the radiation is extremely cumbersome since, under steady-state conditions, it involves analyzing the dependence of the intensity and two polarization parameters of the transmitted radiation on the corresponding characteristics of the incident radiation. However, it is possible to identify the range of parameters of an optical system in which bistability and multistability may occur, and also to show qualitatively how optical bistability arises[8] by analyzing the form of the bifurcation surface, i.e., the surface in the space of the parameters of the incident radiation on which the number of steady radiation states in a nonlinear resonator changes.

When light propagates along the optic axis in the absence of a gyrotropy, the bifurcation intensity of the incident radiation I_b is a symmetric function of the angle ψ between the major axis of the polarization ellipse and the plane of symmetry of the system, and also of the ellipticity parameter ε:

$$I_b(\psi, \varepsilon) = I_b(-\psi, -\varepsilon),$$

where

$$\varepsilon=(|E_+|^2-|E_-|^2)/2I; \quad I=(|E_+|^2+|E_-|^2)/2; \tag{1}$$

$E_\pm$ are the circularly polarized field components. We can see that if the polarization is symmetric ($\psi=\varepsilon=0$), the function $I_b(\psi,\varepsilon)$ has an extremum (or intersects itself).[8] In the presence of a gyrotropy, an extremum of $I_b(\psi,\varepsilon)$ is shifted from the point $\psi=\varepsilon=0$ and the surface $I_b(\psi,\varepsilon)$ as a whole becomes more complex. It should be noted that for the case of an isotropic medium in a resonator, when the physical properties of the system do not depend on the orientation ψ of the polarization ellipse of the incident radiation, I_b is only a function of ε and an analysis of the bifurcation surface reduces to a study of the curve $I_b(\varepsilon)$.

The bifurcation approach described above is applied in the present paper to optical bistability in resonators containing a nonlinear gyrotropic medium. Analytic expressions describing the function $I_b(\varepsilon)$ near the symmetry-induced extremum are derived for the case of a weak gyrotropy. A ring resonator containing an isotropic optically active medium is used as an example to analyze the bifurcation curves over a wide range of parameters of the incident radiation and their evolution induced by changes in the parameters of the linear and nonlinear gyration.

I. BIFURCATION CURVE NEAR A SYMMETRY-INDUCED EXTREMUM IN A WEAKLY GYROTROPIC ISOTROPIC MEDIUM

The general condition for the change in the number of steady states of transmitted radiation, including the occur

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rence or disappearance of optical bistability for a medium inserted in a resonator regardless of the mechanism of optical nonlinearity, has the form

$$\left|\frac{\partial(I,\varepsilon)}{\partial(I_t,\varepsilon_t)}\right|=0, \tag{2}$$

where I_t and ε_t are the intensity and degree of ellipticity of the transmitted radiation, which in this case determine uniquely the rotation of the polarization ellipse. Two steady states merge on the curve of $I_b(\varepsilon)$ defined by Eq. (2). One of these may be stable, corresponding to optical bistability. However, in some parts of the curve both states may be unstable. The problem of stability can be solved by analyzing the time equations which depend on the relaxation of the nonlinearity and on the resonator round-trip time. In the absence of a gyrotropy, the function $I_b(\varepsilon)$, being a solution of Eq. (2), is even and thus its extremum is found at the point $\varepsilon=0$. In the presence of a weak gyrotropy, it is slightly shifted, the shift being proportional to the gyration parameter g. Near the bifurcation values of the parameters $I\approx I_b(0), |\varepsilon|\ll 1$, the dependences of I and ε on I_t and ε_t are single-valued and may be expressed in the form

$$\delta I=A_I\delta I_t+{}^1/_2A_{II}(\delta I_t)^2+A_{\varepsilon\varepsilon}\varepsilon_t^2+g\Lambda_\varepsilon\varepsilon_t; \tag{3a}$$

$$\varepsilon=B_\varepsilon\varepsilon_t+B_{I\varepsilon}\delta I_t\varepsilon_t+B_{\varepsilon\varepsilon\varepsilon}\varepsilon_t^3+g\Lambda+g\Lambda_I\delta I_t+g\Lambda_{\varepsilon\varepsilon}\varepsilon_t^2; \tag{3b}$$

$$|\varepsilon_t|\ll 1;\ |\delta I/I_b(0)|\ll 1;\ |\delta I_t/I_b(0)|\ll 1. \tag{3c}$$

Here, $\delta I=I-I_b(g\Lambda)$, and $\delta I_t=I_t-I_t(0)$ are the intensity increments of the incident and transmitted radiation with respect to their bifurcation values $I_b(g\Lambda)$ and $I_t(0)$ for linearly polarized transmitted radiation ($\varepsilon_t=0$). Bearing in mind the symmetry of the medium, the odd term with respect to ε_t in the expansion for δI and the even terms with respect to ε_t in the expansion of ε appear only due to the gyration (compare with Ref. 8; in the absence of the gyration, the incident radiation intensity does not vary with changing sign of ε_t and the sign of its ellipticity is evidently not reversed). The expansion given by Eq. (3) is written assuming that the bifurcation curve does not self-intersect near the point $I_b(0)$. The coefficients A, B, and Λ depend on the intensity (the parameters Λ take into account the linear and nonlinear gyration).

Substituting Eq. (3) into Eq. (2), we obtain the threshold condition for optical bistability for linearly polarized transmitted radiation ($\delta I_t\to 0, \varepsilon_t\to 0,$) in the form

$$A_IB_\varepsilon-g^2\Lambda_\varepsilon\Lambda_I=0. \tag{4}$$

Equation (4) determines $I_b(g\Lambda)$. In the limit $g\to 0$, condition (4) is broken down into two: $A_I=0$ or $B_\varepsilon=0$. If g is finite but small, optical bistability occurs in the range of small ε for finite but generally strongly differing values of I_I, B_ε, i.e., condition (4) implies that either

$$|A_I|\sim g^2\ll|B_\varepsilon|;\ A_I=g^2\Lambda_\varepsilon\Lambda_IB_\varepsilon^{-1}, \tag{5}$$

or

$$|B_\varepsilon|\sim g^2\ll|A_I|;\ B_\varepsilon=g^2\Lambda_\varepsilon\Lambda_IA_I^{-1}. \tag{6}$$

These conditions determine the thresholds for intensity and polarization optical bistability, respectively. If condition (5) is satisfied, in the prethreshold range the system is "soft" with respect to fluctuations of the radiation intensity in the resonator: small fluctuations of the incident radiation parameters result in large fluctuations of the intensity I_t.

It can be seen from Eqs. (2) and (3) that the bifurcation curve for intensity optical bistability near the extremum is described by a parabola: $I_b(\varepsilon)-I_b(g\Lambda)\approx A_{\varepsilon\varepsilon}B_\varepsilon^{-2}(\delta\varepsilon)^2+g\Lambda_\varepsilon B_\varepsilon^{-1}\delta\varepsilon$, $\varepsilon=\varepsilon-g\Lambda$. The extremum of $I_b(\varepsilon)$ is situated at the point $\varepsilon^{(0)}=g(\Lambda-\Lambda_\varepsilon B_\varepsilon/2A_{\varepsilon\varepsilon})$. When the bifurcation curve is intersected near its extremum (as a result of changes in I or ε) under conditions of intensity optical bistability given by Eq. (5), the resultant states of the transmitted radiation differ in terms of the intensity by approximately $[A_{II}(I-I_b(\varepsilon))]^{1/2}$, as can be seen from Eq. (3), and in terms of polarization, by an appreciably smaller value proportional to g.

It can be seen from Eqs. (3) and (6) that for polarization optical bistability, in the near-threshold range $[I\approx I_b(0), |\varepsilon|\ll 1]$ the system is soft relative to fluctuations of the radiation intensity in the resonator. From Eqs. (3) and (6) it is readily shown that the form of $I_b(\varepsilon)$ near the extremum is given by

$$I_b(\varepsilon)-I_b(g\Lambda)=F[v(\varepsilon)];\ F(v)=-({}^1/_2a_3v^2+g_2v)/a_1; \tag{7a}$$

$${}^1/_3a_3v^3+{}^1/_2(g_2+a_3g_0)v^2+g_0g_2v+(\varepsilon-g\Lambda)=0, \tag{7b}$$

where

$$a_3=6(B_{\varepsilon\varepsilon\varepsilon}-B_{I\varepsilon}A_{\varepsilon\varepsilon}A_I^{-1});\ a_1=A_I^{-1}B_{I\varepsilon}; \tag{8a}$$

$$g_2=2gA_I^{-1}(A_I\Lambda_{\varepsilon\varepsilon}-A_{\varepsilon\varepsilon}\Lambda_I-B_{I\varepsilon}\Lambda_\varepsilon);\ g_0=g\Lambda_I/B_{I\varepsilon}. \tag{8b}$$

It can be seen from Eq. (7) that for polarization optical bistability the function I (ε) is nonanalytic near the extremum: the extremum is reached at a turning point. At this point we have

$$\varepsilon=\varepsilon_c=g\Lambda-{}^1/_6a_3^{-2}(g_2^3-3a_3g_2^2g_0); \tag{9a}$$

$$I_b(\varepsilon_c)=I_c=I_b(g\Lambda)+g_2^2/2a_1a_3. \tag{9b}$$

Near the turning point we have

$$I_b(\varepsilon)-I_c\approx-(a_3/2a_1)\theta(M(\varepsilon-\varepsilon_c))\{M(\varepsilon-\varepsilon_c) \pm|2a_3/(g_2-a_3g_0)| \times[M(\varepsilon-\varepsilon_c)]^{3/2}\};\ M=2(g_2-a_3g_0)^{-1}, \tag{10}$$

where $\theta(x)$ is a step function. At comparatively large values of $|\varepsilon-\varepsilon_c|$, we find

$$I_b(\varepsilon)-I_c\approx(-a_3/2a_1)(3\varepsilon/a_3)^{2/3};\ |\varepsilon-\varepsilon_c|\gg g^3 \tag{11}$$

[in the absence of a gyrotropy Eq. (11) completely describes $I_b(\varepsilon)$ near the extremum, see Ref. 8].

It follows from Eqs. (7)–(11) that a weak gyrotropy does not eliminate the singular point on the bifurcation curve observed at $g=0$ as a result of symmetry.[8] At this point, three steady states of the radiation in the resonator merge. In the range bounded by the curve $I_b(\varepsilon)$, these states split up.

Equations (7)–(11) can be used to follow the evolution of $I_b(\varepsilon)$ near the extremum as the gyrotropy is increased. This is shown schematically in Fig. 1. At $g=0$ the curve $I_b(\varepsilon)$ is symmetric with respect to the $\varepsilon=0$ axis. As g increases, the turning point is shifted along the ε axis (approxi-

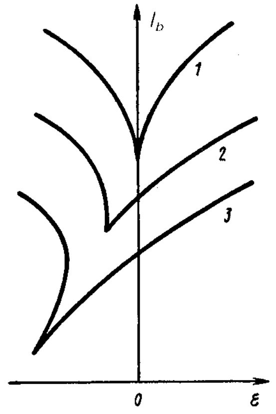

FIG. 1. Evolution of the dependence $I_b(\varepsilon)$ with increasing gyrotropy for the case of polarization optical bistability with small ε; $g=0$ (1), $g>0$ (2), and g (3) $>g$ (2).

mately linearly with g) and along the I axis, proportionally to g^2 when $g \ll 1$. In this case, another singular point $\varepsilon=\varepsilon_s$ is observed on the curve $I_b(\varepsilon)$ where $\varepsilon=\varepsilon_s$, $|dI_b/d\varepsilon|_{\varepsilon=\varepsilon_s} \to \infty$.

The interval $|\varepsilon_c-\varepsilon_s| \sim g^3$ increases rapidly with increasing g. In this range a gyrotropy induces a new effect: by altering the intensity of the field for fixed ε, we intersect the bifurcation curve three times. In this case, as $-a_1a_3I$ increases, bistability is initially observed [when I falls within the range bounded by the curve $I_b(\varepsilon)$] and then disappears and reappears again. The transmission of the system for the parameters I and ε close to the turning point can be analyzed as in Ref. 8. In particular, the dependence $\varepsilon_t(\varepsilon)$ is S-shaped. On the whole, optical bistability is observed most strongly in the polarization of the transmitted radiation.

We note that this method can also be used to analyze the bifurcation surface $I_b(\psi,\varepsilon)$ for a resonator containing a nonlinear crystal. It can be shown in particular that a weak gyrotropy does not eliminate the turning edge found in the case of polarization optical bistability,[8] although the form of this edge becomes slightly more complex.

2. MULTISTABILITY IN A RING RESONATOR CONTAINING GYROTROPIC MEDIUM WITH A CUBIC NONLINEARITY

It is interesting to analyze the influence of the optical activity on multistability in a resonator for the case of an isotropic medium with a cubic nonlinearity frequently encountered experimentally. In the steady-state regime in the absence of self-oscillations,[11] the equations for the field in a resonator neglecting absorption have the form[4]

$$(1-R)|E_\pm|^2=|E_\pm^{(r)}|^2(1+R^2-2R\cos\Phi_\pm), \tag{12}$$

where $E_\pm^{(r)}$ are the circularly polarized components of the field at the front face of the nonlinear medium;

$$\Phi_\pm=\Phi_0\mp\rho_0 l+[(\sigma_1/2\mp\rho_1)|E_\pm^{(r)}|^2+(\sigma_1/2+\sigma_2)|E_\mp^{(r)}|^2]l \tag{13}$$

is the phase shift for these components in the resonator; R is the reflection coefficient of the mirrors; l is the thickness of the medium; $\sigma_{1,2}$ are the cubic nonlinearity parameters of the medium; ρ_0 and ρ_1 are the parameters of the linear and nonlinear optical activity. The intensity and ellipticity of the transmitted radiation are related to $E_\pm^{(r)}$ by

$$I_t=(1-R)(|E_+^{(r)}|^2+|E_-^{(r)}|^2)/2;$$

$$\varepsilon_t=(|E_+^{(r)}|^2-|E_-^{(r)}|^2)/(|E_+^{(r)}|^2+|E_-^{(r)}|^2). \tag{14}$$

It can be seen from Eqs. (12)–(14) that the condition for the occurrence of new steady states is reduced to

$$\left|\frac{\partial(|E_+|^2,|E_-|^2)}{\partial(|E_+^{(r)}|^2,|E_-^{(r)}|^2)}\right|=0. \tag{15}$$

This is a transcendental equation relating $|E_+^{(r)}|^2$ and $|E_-^{(r)}|^2$ on the bifurcation curve. This relationship, together with Eqs. (1) and (12) determines directly the curve $I_b(\varepsilon)$.

The results of a numerical solution of Eqs. (15) and (12) for various values of $\rho_0 l$ and ρ_1/σ_1 are plotted in Figs. 2–4 for $\Phi_0=0$, $\sigma_1=2\sigma_2$ (the last condition implies that the frequency dispersion of the nonlinear susceptibility is neglected), and $R=0.5$. The dimensionless intensity $\tilde{I}_b|\varepsilon|=\sigma_1 lI_b(\varepsilon)$ is plotted on the ordinate. In fact, Figs. 2–4 only show four of an infinite number of bifurcation curves. These correspond to the lowest branches of $I_t(I,\varepsilon)$ and are thus the most interesting. It is clear that the bifurcation curves are extremely complex. These are either closed (curves *1* and *2* in Fig. 2) or reach the boundary $|\varepsilon|=1$. Moreover, they intersect and have singular points or turning points. The curves are separated from each other during plotting utilizing the fact that the bifurcation parameters of the field inside the resonator vary continuously during motion along one bifurcation curve.

In the absence of a gyrotropy ($\rho_{0,1}=0$), the pattern of bifurcation curves is symmetric relative to the $\varepsilon=0$ axis. For a weak gyrotropy the symmetry is slightly disturbed (Fig. 2). It can be seen from Fig. 2 that curve *1* near the $\varepsilon=0$ axis is a parabola, i.e., following the classification of the types of optical bistability put forward above, it describes the occurrence of intensity bistability. Curve *2* has turning points near the axis (describing the occurrence of polarization optical bistability) and curves *3* and *4* intersect. When these intersect, two almost symmetric pairs of steady states of the transmitted radiation with finite ε_t are formed (or merge).

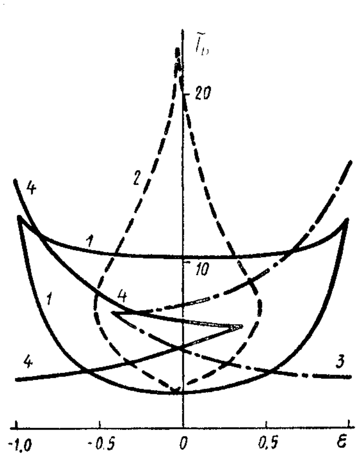

FIG. 2. Bifurcation curves corresponding to the lowest branches of $I_t(I,\varepsilon)$ when $\rho_0 l=0.1$ and $\rho_1=0$.

A characteristic feature of all the curves is the presence of turning points, and for curves *1*, *3*, and *4* the appearance of these points is not related to the symmetry. At a turning point, three steady states of the radiation in the resonator merge. Each bifurcation curve in Fig. 2 corresponds to a particular saddle-point type of unstable state (S state) formed at the point of intersection of this curve and moving continuously on the plane I_t, ε_t as I and ε vary along the bifurcation curve.

For a closed curve this unstable state is unique. On the other hand, other steady states observed on branches of the bifurcation curves converging at a turning point are different. The unstable S state merges with one of these on the appropriate branch. In the case of a comparatively strong gyrotropy, both linear (Fig. 3) and nonlinear (Fig. 4), the pattern of bifurcation curves becomes more complex. It can be seen, that compared with the case of a weak gyrotropy, branch 2 is the least distorted. Instead of curves *1*, *3*, and *4* in Fig. 2, two curves 1 and 3 appear in Figs. 3 and 4 each having three turning points. This change occurs for the following reasons. As the gyrotropy is increased, one of the turning points on curve *1* (the left-hand point in Fig. 2 when $\rho_{0,1} > 0$) approaches the limiting value of ε ($\varepsilon = -1$). For a stronger gyrotropy, each of the three steady states that has merged at this point "generates" its own turning point. Two pairs of radiation states in the resonator occur (or merge) at the points of self-intersection of curves *1* and *3* and at the points of intersection of the various bifurcation curves.

The pattern of the bifurcation curves can be used to study how the number of radiation states in the resonator changes, for example when the intensity of the incident radiation is increased for fixed ε. It can be seen that the number of changes depends strongly on ε. Taking, for example, $\varepsilon = -0.8$ in Fig. 4, it can be seen that as I increases in the range $\tilde{I} = \sigma_1 lI \approx 2.8$, new stable and unstable states appear in the system. Another pair of states is formed when $\tilde{I} \approx 4.7$. When $\tilde{I} \approx 7.1$, the first of the S states merges with a " nonsaddle" state observed at the second point of intersection. Finally, when $\tilde{I} \approx 10.6$ the second S state merges with the steady state found at low intensities.

CONCLUSIONS

This analysis has shown that gyrotropy may influence strongly the range of existence of multistability. This is of considerable interest from the point of view of developing multistable devices such as liquid crystals or semiconductors, where a gyrotropy can be efficiently controlled by altering the temperature or the magnetic field. The approach developed in the present study can be used to determine the ranges of parameters of the pump radiation in which the gyrotropy effects are the strongest. It is clear from the results that the linear and nonlinear gyrotropy effects have a similar influence on optical bistability, since the symmetry is reduced in both cases. On the other hand, the quantitative influence of these mechanisms differs (compare Figs. 3 and 4).

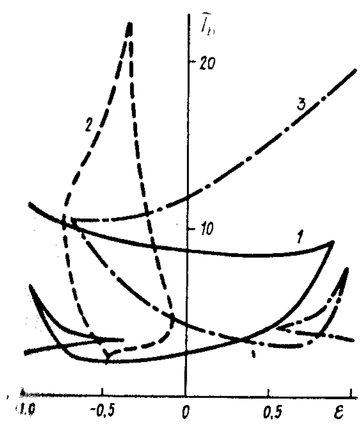

FIG. 3. Bifurcation curves for a fairly strong linear gyrotropy when $\rho_0/\sigma_1 = 1.0$ and $\rho_1 = 0$

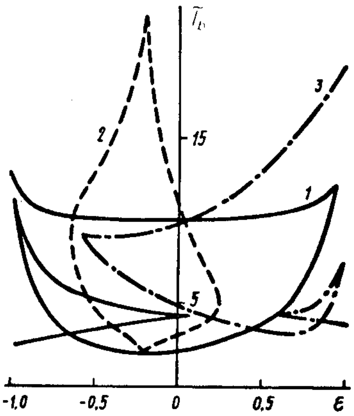

FIG. 4. Bifurcation curves for a strong nonlinear gyrotropy when $\rho_1/\sigma_1 = 0.1$ and $\rho_0 = 0$.

This bifurcation approach to the analysis of optical bistability permits us to examine the results[2,4,12] of calculations of the transmission of a nonlinear system studied in Sec. 2 from a slightly different viewpoint (gyrotropy was neglected in Refs. 2 and 12). These calculations were made for several fixed values of the intensity or ellipticity of the incident radiation. Our approach indicates how and for which values of the parameters the output characteristics obtained in Refs. 2, 4, and 12 can be tuned qualitatively.

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