BISTABILITY AND SELF-OSCILLATIONS IN A CAVITY DUE TO A SELF-INDUCED OPTICAL ANISOTROPY

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It is shown that self-oscillations can arise due to a polarizational self-action in a medium with a cubic nonlinearity in the absence of delay in a cavity. The bifurcation relationships between the parameters of incident radiation are found, for which the number of stationary states of radiation in the cavity is altered or the self-oscillations are excited. The scheme of switching between the stable states is presented.

Propagation of an intense radiation along an optic axis of a nonlinear medium is generally accompanied by a change of its polarization. If the medium is placed into a cavity and optical bistability (OB) occurs, then the coexisting stable states differ not only in intensity of the intracavity field, but also in its polarization. The existence of several stable states is itself due to absorption and/or refraction nonlinearity and to nonlinear coupling between different components of the radiation field [1].

Not only several stable states, but also self-oscillations and optical chaos may arise and coexist in a nonlinear cavity. The non-stationary regimes as a rule are related to a delay in the cavity or to a presence of competing mechanisms of optical nonlinearity [2]. Since a polarizational self-action of radiation can be regarded as a sort of "competition" between the coupled field components, it could be expected to give rise to self-oscillations for a single mechanism of nonlinearity and in the absence of delay.

A complete analysis of the system should result in the determination of the intracavity field and its variation with varying parameters of the incident radiation. However, to describe its main features it is sufficient (i) to find the steady-state regimes available for the system and the ranges of their existence; (ii) to analyze their stability and to find the scheme of switching between them. Solution of the first problem requires investigation of the bifurcation relationships between the parameters of incident radiation, for which the set of regimes is changed, say, the number of stationary states is altered. The second problem is reduced to the study of the cavity transmission in the vicinity of the bifurcation parameter values.

In the present paper the solution of these problems is given for a ring cavity containing a transparent isotropic nongyrotropic medium with cubic optical nonlinearity. The transmission of such cavity for some values of the parameters of the medium and the incident radiation was considered in refs. [3–5]. We suppose that the cavity round trip time L/c is much smaller than the characteristic relaxation time τ of the nonlinear medium response. Although in a nonlinear medium the two circularly polarized field components E_+ and E_- are coupled, their intensities $\propto |E_\pm|^2$ do not change in the absence of absorption. Therefore in the Debye approximation for the relaxation of the nonlinear polarizability of the medium the kinetics of the nonlinear phase gains $\phi_\pm$ of the components $E_\pm$ is given by

$$\tau \;\mathrm{d}\phi_\alpha/\mathrm{d}t = -\phi_\alpha + \lambda_1 |E_\alpha|^2 + \lambda_2 |E|^2, \quad \alpha = \pm\,,$$
$$|E|^2 = |E_+|^2 + |E_-|^2\,, \tag{1}$$

where λ_1, λ_2 characterize the cubic polarizability.

The equations of type (1) refer to media with various mechanisms of nonlinearity (electronic, orientational, etc.). In the absence of frequency dispersion $\lambda_1 = -\lambda_2/2$. Then allowing for the boundary conditions in a ring cavity we obtain

$$\tau \mathrm{d}\phi_\alpha/\mathrm{d}t = G_\alpha(\phi_+ ; \phi_-), \quad \alpha = \pm ,$$

$$G_\alpha = -\phi_\alpha$$

$$+I[(1+\alpha\epsilon)\, g(\phi_\alpha) + 2(1-\alpha\epsilon)\, g(\phi_{-\alpha})],$$

$$g(\phi) = [1 - f\cos(\phi + \phi_0)]^{-1},$$

$$\epsilon = (I_+ - I_-)/(I_+ + I_-), \qquad (2)$$

where I is the properly normalized intensity of the incident radiation, $I_\pm$ are the intensities of its circularly polarized components ($I = I_+ + I_-$; $I_\pm \propto \lambda_2 |E_\pm|^2$), ϕ_0 is the weak-field phase-gain in the cavity, f is its finesse.

The physical systems described by two coupled differential equations of the type (2) can have [6] one or more stationary states and limit cycles for constant input. In our case the presence of more than one stable stationary corresponds to optical bi- or multistability, and a stable limit cycle corresponds to self-oscillations of the field in the cavity.

The ranges of existence of various regimes are bounded by the bifurcation curves on the plane (ϵ, I) (incident radiation intensity versus its ellipticity parameter). The appearance or merging of stationary states occur on the line $I = I_B(\epsilon)$ given by the equation

$$\left[\frac{\partial(I,\epsilon)}{\partial(\phi_+,\phi_-)}\right]_{\mathrm{st}} \propto \left[\frac{\partial(G_+,G_-)}{\partial(\phi_+,\phi_-)}\right]_{\mathrm{st}} = 0 , \qquad (3)$$

where the subscripts "st" indicate that the jacobians are calculated for the stationary values of the phases ϕ_+, ϕ_- at given I and ϵ. The curves $I_B(\epsilon)$ obtained from (2), (3) and corresponding to several lowest branches of the intracavity field are plotted in fig. 1 by thin lines.

Due to the symmetry of the system is $I_B(\epsilon) = I_B(-\epsilon)$. Hence if the branches of $I_B(\epsilon)$ do not intersect at $\epsilon = 0$ they have an extremum here (cf. the curves 1 and 2). The states of the intracavity fields appearing near such extrema ($|\epsilon| \ll 1$, the incident radiation is polarized almost linearly) have small degree of ellipticity. At $\epsilon = 0$, $I_B(\epsilon)$ may be either analytic (curve 1) or having a spinode point, $I_B(\epsilon) - I_B(0) \propto \epsilon^{2/3}$ (curve 2). Such behaviour is a consequence of symmetry and is inherent for the on-

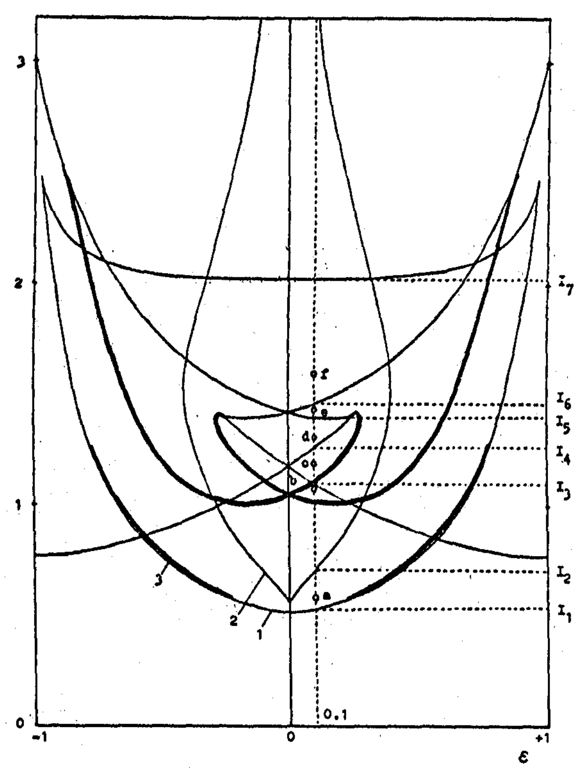

Fig. 1. Bifurcation curves $I_B(\epsilon)$ (thin) and $I_H(\epsilon)$ (thick) corresponding to change of number of stationary states and appearance of a limit cycle, respectively. The branches 1 and 2 describe the onset of refractive and polarizational OB, respectively. I_1, ..., I_7 are numbered in the order of increasing I intersections of bifurcation curves $I_B(\epsilon)$ with the straight line $\epsilon = 0.1$ (see fig. 2).

set of refractive and polarizational OB (polarizational switching), respectively [1]. In a spinode point three stationary states of radiation in the cavity merge. Two of them correspond to a finite degree of ~~intracavity~~ for $\epsilon = 0$ (a spontaneous symmetry breaking).

The conditions for the onset of self-oscillations are [6]:

$$\sum_\alpha \left(\frac{\partial G_\alpha}{\partial \phi_\alpha}\right)_{\mathrm{st}} = 0, \quad \left[\frac{\partial(G_+,G_-)}{\partial(\phi_+,\phi_-)}\right]_{\mathrm{st}} > 0 . \qquad (4)$$

The corresponding bifurcation curves $I = I_H(\epsilon)$ are represented by thick lines in fig. 1 (note that the curve 3 lies above the curve 1 though very close to it). Evidently, $I_H(\epsilon) = I_H(-\epsilon)$. The curves $I_H(\epsilon)$ do not reach the boundaries $\epsilon = \pm 1$ because for circularly polarized radiation set (2) reduces to one equation.

(polarization of the intracavity field

The endpoints of the curves $I_H(\epsilon)$ lie on the curves $I_B(\epsilon)$ and may be shown to be the tangency of these curves. At these points all the characteristic numbers for the respective stationary states vanish. On the opposite sides of the tangency point on the curve $I_B(\epsilon)$ there appear one stable and one unstable or two unstable stationary states. Using fig. 1 one may judge of stability of the states with the aid of simple arguments based on continuity of the intracavity field when the parameters of the incident radiation are varied without crossing the bifurcation curves. E.g., when the branch 1 of $I_B(\epsilon)$ is intersected from below for $\epsilon=0$, one of the appearing states is stable (a standard case of refractive OB), hence the corresponding state is stable for all ϵ (at $I>I_B(\epsilon)$) up to the tangency point of $I_B(\epsilon)$ and the branch 3 of $I_H(\epsilon)$, and then above $I_H(\epsilon)$. When the branch 1 is intersected below the branch 3 two unstable states appear; one of them becomes stable when the curve 3 is intersected, and simultaneously an unstable limit cycle appears. Thus the branch 3 of $I_H(\epsilon)$ in fact "eats away" the range of existence of stable states bounded by the branch 1 of $I_B(\epsilon)$.

To illustrate the switching scheme we consider the behaviour of the system for different I and fixed $\epsilon=0.1$. A sequence of phase portraits of the system (2) is given in fig. 2. For $I<I_1$ (the values I_1, ..., I_7 are the same as in fig. 1) there is only one stable state represented by the stable node P_1. At $I=I_1$ the saddle P_2 and the stable node P_3 (becoming a stable focus for somewhat higher I) appear (fig. 2a). At $I=I_2$ a similar pair P_4, P_5 appears. Then after the first intersection of $I_H(\epsilon)$ the focus P_3 becomes unstable and the stable limit cycle C_1 corresponding to a self-oscillatory regime appears around it (fig. 2b). When the next branch of $I_H(\epsilon)$ is intersected, the same occurs with the focus P_5, and the stable limit cycle C_2 appears. At $I=I_3$ the saddle P_6 and the stable node P_7 appear. Note that with further increase of I the limit cycle C_1 "crawls" to the saddle point P_6 and disappears (fig. 2c). At $I=I_4$ the pair of states P_8, P_9, similar to P_6, P_7 appears, and then the limit cycle C_2 disappears in the same way as C_1 (fig. 2d). For higher I the unstable states merge: P_5 and P_8 at $I=I_5$ (fig. 2e), P_3 and P_6 at $I=I_6$ (fig. 2f). At $I=I_7$ the stable state P_1 merges with P_2 and disappears, the only remaining stable states being P_7 and P_9.

The general rule of appearing and merging of the stationary states is that one saddle point corresponds to each branch of $I_H(\epsilon)$. If there are no spinode points on a part of a branch between its two successive intersections with varying I or ϵ, then the saddle merges with one and the same (that is topologically equivalent) node; otherwise, with non-equivalent nodes.

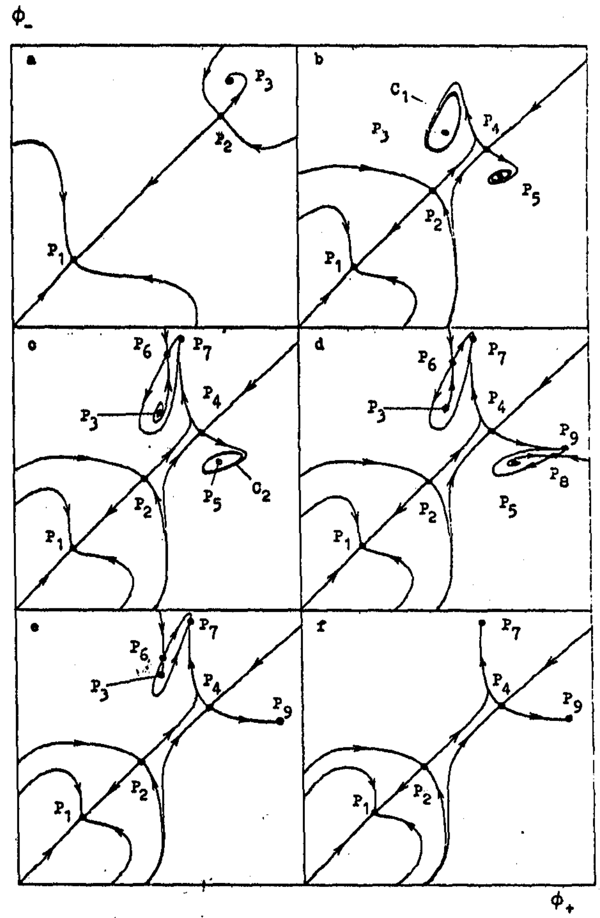

Fig. 2. Trajectories of the dynamic system (2) in the region of the phase plane (ϕ_+, ϕ_-) containing the singular points P_1, ..., P_9. The parameters ϵ, I (a)–(f) are the same as for the points a–f in fig. 1, respectively. (a) $I_1<I<I_2$; (b) $I_2<I<I_3$; (c) $I_3<I<I_4$; (d) $I_4<I<I_5$; (e) $I_5<I<I_6$; (f) $I_6<I<I_7$.

Thus, with constant ϵ and I slowly increasing from zero the system begins with the state labelled P_1; the point representing it in the static characteristics moves along the P_1 branch. At $I=I_7$ this branch disappears and the system is switched to the branch P_7 (see fig. 3). If then I is slowly decreased the system stays on the branch P_7 until it disappears at $I=I_3$.

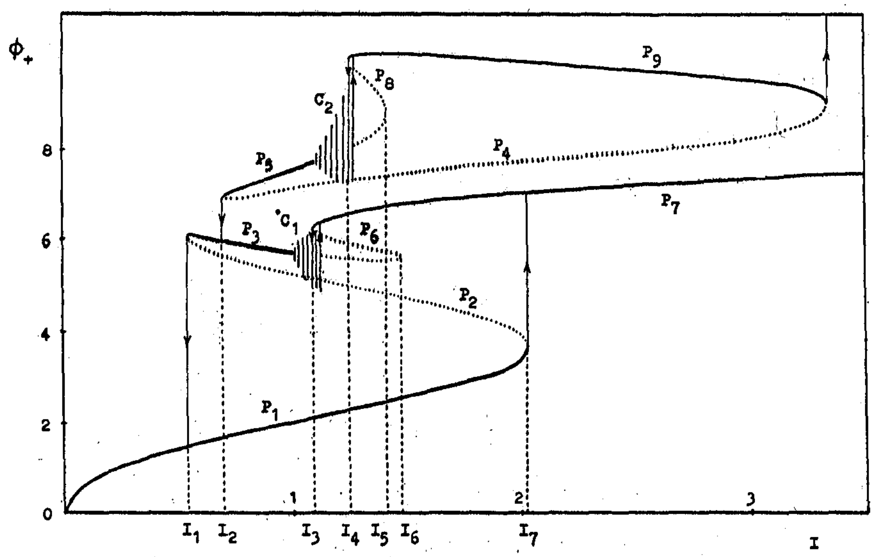

Fig. 3. The lowest branches of the steady steady-state dependence of the nonlinear phase gain $\phi_+ \propto |E_+|^2$ on the intensity of the incident radiation I for $\epsilon = 0.1$. Solid and dotted lines are stable and unstable stationary states. Self-oscillations are shown by dashed areas, whose height corresponds to the spread of the self-oscillations. Switchings are indicated by arrow. The closed loop P_4–P_5–P_8–P_9 can be reached either from similar loops of higher order or by changing ϵ.

Then the system is attracted to the limit cycle C_1, i.e. the self-oscillations of finite amplitude are excited. With further decrease of I the limit cycle shrinks to a point P_3 (a soft quenching of the self-oscillations) and when the branch P_3 disappears at $I=I_1$ the system returns to its initial state P_1 (cf. [7]).

The branches of stable states P_5, P_9 and the self-oscillatory regime C_2 are not switched by this way. To excite them the polarization of the incident radiation should be varied slowly together with its intensity. In particular, for small negative ϵ the first switching with increased I brings the system to the branch P_9 where its stays if the sign of ϵ is changed at fixed I. An escape from this branch may be due to variation not only of ϵ, but of I as well, see fig. 3.

The characteristic period of the considered self-oscillations is determined by the relaxation time of optical Kerr nonlinearity. It can vary over a wide range, depending on the properties of the system, down to the order of picoseconds. As the hard quenching of the self-oscillations is approached, their period increases sharply. Such behaviour under self-induced anisotropy was indeed observed [8]. Adjusting the parameters of incident radiation one can realize both soft (with a smoothly increasing amplitude) and hard excitation of the self-oscillations.

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