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Many-electron magnetotransport in 2D electrons on liquid helium

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Abstract

A many-electron theory of magnetotransport of a nondegenerate 2D electron system is presented along with experimental data in classically strong and quantising magnetic fields. Due to the electron–electron interaction each individual electron is driven by a fluctuating electric field E which converts the discrete Landau levels for non-interacting electrons, spacing $\hbar\omega_c$, into a continuous spectrum. Hence the classical magnetoresistance is very small (compared to the single-electron theory) until $eER_c < \hbar\omega_c$ where R_c is the classical Larmor radius. At high magnetic fields, $\hbar\omega_c \gg kT \gg eEl$ ($l = (\hbar/m\omega_c)^{1/2}$) for $eEl/\hbar$ less than the electron momentum scattering rate, the single-electron theory becomes valid. The onset field B_0 for magnetoresistance lies in the range 0.3–1.0 T for electrons on liquid helium. These many-electron effects have been observed experimentally near 1 K where the electron mobility is high and limited by ^{4}He vapor atoms and below 1 K in the ripplon scattering regime.

1. Introduction

Electrons above the surface of liquid helium provide an example of a nearly ideal nondegenerate two-dimensional electron system (2DES): for temperatures $0.1\,\mathrm{K} < T < 2\,\mathrm{K}$ and electron densities $n \simeq 10^{12}\,\mathrm{m}^{-2}$ as used in many experiments the characteristic kinetic energy kT exceeds the Fermi energy $E_F = kT_F = \pi\hbar^2 n/m$ by two orders of magnitude, whereas the mobility in zero magnetic field, μ_0, can be greater than $10^3\,\mathrm{m}^2/\mathrm{V\,s}$. Therefore the 2DES on helium has been recognized as a good candidate for the investigation of the many-electron effects in 2D systems that are not related to overlapping electron wave functions. Two basic 'traditional' types of such effects have been indeed observed experimentally and described theoretically (see Ref. [1] for a review): mean-field effects (plasma vibrations, including edge magnetoplasmons [2]) and the liquid–solid phase transition in a classical many-electron system (Wigner crystallization [3]). At the same time, data on the conductivity in zero magnetic field, in the range where there is no electron solid, has been interpreted in terms of the single-electron picture of scattering of individual electrons by helium vapor atoms and by capillary waves on the helium surface (ripplons). The matrix elements of coupling are well known in both cases [4]. The electron–ripplon coupling increases with the vertical electric field $E_\perp$ pressing the electrons to the helium surface. The rate of scattering by the vapor atoms can be varied with the temperature, since the vapor pressure depends exponentially on T

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near 1 K. Theory and experiment are in good agreement over a wide range of electron densities and mobilities in zero field.

In fact, the agreement between the single-electron theory and experimental data on the mobility is rather surprising, since the data have been obtained in the range

$$e^2 n^{1/2}/\varepsilon_0 \gg kT \quad (T \gg T_F) \tag{1}$$

where the electron–electron interaction is strong and would be expected to influence the electron kinetics.

In a magnetic field B transverse to the 2D layer the electron energy spectrum in the single-electron approximation consists of discrete Landau levels, energy separation $\hbar\omega_c$ where $\omega_c = eB/m$ is the cyclotron frequency. For such a discrete spectrum the concept of elastic/quasielastic scattering for individual electrons implied in the theory of electron transport for $B = 0$ is inapplicable. Many-electron effects should be particularly important in transport phenomena in classically strong ($\mu_0 B \gg 1$) and quantising ($\hbar\omega_c/kT \gg 1$) magnetic fields. The many-electron magnetotransport for $\hbar\omega_c \gg kT$ was previously considered in Ref. [5]. The theory explained [6] the experimental decrease of the cyclotron resonance halfwidth with increasing pressing field $E_\perp$ [7] despite the associated increase of the electron–ripplon coupling.

In this paper we report the results of theoretical and experimental studies of the magnetoresistance of a 2DES on superfluid helium for a broad range of electron densities, temperatures and magnetic fields. The experimental and theoretical results are in good agreement, the data indicate the many-electron character of the kinetics and the theory resolves the controversy about the applicability of the single-electron picture to transport phenomena in this system and shows where the crossover from many-electron to single-electron kinetics occurs.

2. Many-electron theory

The 'conventional' approach to the theoretical analysis of magnetotransport of a many-electron system would be to find the spectrum of elementary excitations, and then to analyze their scattering by ripplons and vapor atoms. Such a program can be performed for the electron solid [3,6,8]. Here we shall use an alternative approach based on a simple physical picture of electron motion. If the characteristic duration τ_{col} of electron collisions with short-range scatterers (atoms, ripplons) is small compared with the momentum relaxation time (due to the collisions) τ_B, the collisions occur consecutively in time and are nearly independent of each other. To describe the electron transport it then suffices to find the scattering probability for an electron allowing for the fact that it is coupled to other electrons. The effect of this in the strong-coupling range, Eq. (1), is that an electron is driven by a fluctuational (and fluctuating) electric field $\boldsymbol{E}$ from the other electrons. For $B = 0$, if the fluctuational field is weak enough, the characteristic quantum uncertainty of the electron kinetic energy due to the finite thermal wavelength λ_T, $\delta\varepsilon = eE\lambda_T \ll kT$ ($\lambda_T = \hbar/(2mkT)^{1/2}$; $E = \langle \boldsymbol{E}^2\rangle^{1/2}$). The electron motion is then semiclassical, and a quasielastic electron collision with a short-range scatterer is not influenced by many-electron effects: this is why the single-electron theory often works for $B = 0$ for strong-coupling, Eq. (1). But for $\omega_c\tau_0 = \mu_0 B \gg 1$ (a classically strong field) the fluctuational field $\boldsymbol{E}$ can dramatically change the magnetotransport, compared to the single-electron approximation, as the energy spectrum of an electron in crossed E and B fields is continuous (not discrete, as for $E = 0$) and an electron can be therefore scattered quasielastically.

Since, for short-range scatterers, only one electron is immediately involved in a collision (neglecting momentum transfer of order $\hbar n^{1/2}$), the expression for magnetoconductivity in the Born approximation to the lowest order in $\omega_c\tau_B$ is of the same form as in the Drude theory, with the important difference that the electron–electron interaction influences the motion of an electron in the course of a collision and is treated in a nonperturbative way. The resistivity $\rho^* = \rho_{xx}(B)/\rho_0$ normalised to the zero-field value ρ_0 is then given by

$$\rho^* \equiv \rho^*_{\mathrm{m}} = \left(\frac{\lambda\!\!\!^{-}_T}{\lambda\!\!\!^{-}}\right)^2\left(\frac{\tau_0}{\tau_B}\right),$$

$$\tau_B^{-1} = \tfrac{1}{2}\lambda\!\!\!^{-\,2}\hbar^{-2}\sum_q q^2\overline{|V_q|^2}\xi(\boldsymbol{q}),$$

$$\xi(\boldsymbol{q}) = \int_{-\infty}^{\infty} \mathrm{d}t\, \xi(\boldsymbol{q}, t),$$

$$\xi(\boldsymbol{q}, t) = \langle \exp(\mathrm{i}\boldsymbol{q}\boldsymbol{r}_j(t)) \exp(-\mathrm{i}\boldsymbol{q}\boldsymbol{r}_j(0))\rangle . \qquad (2)$$

Here $\overline{|V_q|^2}$ is the mean squared Fourier component of the random quasistationary field that scatters electrons (that of ripplons or helium-vapor atoms), $\boldsymbol{r}_j$ is the position vector of the jth electron (the correlator $\xi(\boldsymbol{q}, t)$ is independent of j) and $\lambda\!\!\!^{-}$ is the characteristic wavelength of an electron: $\lambda\!\!\!^{-} = \lambda\!\!\!^{-}_T$ for $\hbar\omega_c \ll kT$ and $\lambda\!\!\!^{-} = l = (\hbar/eB)^{1/2}$, the magnetic length, for $\hbar\omega_c \gg kT$. The electron–electron coupling determines the dynamics of an individual electron and thus the value of $\xi(\boldsymbol{q}, t)$. In other words, the 'electron community' transfers the momentum to the scatterers via the scattering of individual electrons. It is this transfer that gives rise to the magnetoconductivity.

In the single-electron approximation at $B = 0$, $\xi(q) = (2\pi m/kTq^2)^{1/2}\exp(-\hbar^2q^2/8mkT)$, whereas for finite B the integral over time in Eq. (2) diverges as an electron moves along a closed loop in classical terms (when evaluated in terms of quantum statistics, the correlator $\xi(\boldsymbol{q}, t)$ is the sum of harmonics $\exp(in\omega_c t)$). This is no longer true if the electron is also driven by an electric field from the other electrons. The value of $\xi(\boldsymbol{q})$ and the physics of the scattering depend [9] on the values of the parameters

$$\eta = \omega_c(2mkT)^{1/2}/eE\,, \quad \zeta = \eta\hbar\omega_c/2kT \qquad (3)$$

and on $\hbar\omega_c/kT$. The parameter η gives the ratio of the inter-Landau-level spacing $\hbar\omega_c$ to the uncertainty $eE\lambda\!\!\!^{-}_T$ in the kinetic energy of a classical electron.

The simplest, and most unexpected, results on magnetoconductivity occur in the range of classically strong magnetic fields where $\hbar\omega_c \ll kT$. It follows from Eq. (2) that, for $\zeta \ll \eta \ll 1$, the value of $\xi(q)$ differs from that for $B = 0$ by an extra factor $(1 + F)\exp(F\hbar^2q^2/8mkT)$, $F = (1/48)(\hbar\omega_c/kT)^2$. This factor is close to unity for realistic values of $q^2 < 8mkT/\hbar^2$ and gives a very small magnetoresistance, quadratic in B: $\rho^* - 1 \propto (\hbar\omega_c/kT)^2$. The magnetoresistance, although it arises from the strong effect of the fluctuational field E that 'blows away' electrons from the scatterers and thus prevents multiple scattering, is independent of E itself. The fluctuational field produces a separate term $\propto (eE\lambda\!\!\!^{-}_T)^2/(kT)^2$ in $\xi(q)$; it contains an additional small factor ($\sim 10^{-2}$).

A different situation arises for 'intermediate' classically strong magnetic fields where $\eta \gg 1$ (still $\hbar\omega_c/kT \ll 1$). Here, an electron performs several rotations about a scatterer before the fluctuational field drives it away. Therefore, the probability of scattering increases compared to the case $B = 0$. The correlator $\xi(\boldsymbol{q})$ was evaluated for classical statistics by the steepest descent method, with saddle points lying at $2\pi s/\omega_c - \mathrm{i}\hbar/2kT$ (for $\eta \ll 1$ the only saddle point is $s = 0$),

$$\xi(q) \simeq \left(\frac{2\pi m}{kTq^2}\right)^{1/2} \times \sum_s \exp\left(\frac{-\hbar^2q^2}{8mkT}\right)\left\langle \exp\left(\frac{\mathrm{i}2\pi s e}{m\omega_c^2}\boldsymbol{E}\cdot\boldsymbol{q}\right)\right\rangle . \qquad (4)$$

The values of s that contribute to this sum are limited to $|s| \simeq \zeta$ (larger s values can be significant for ripplon scattering). The parameter ζ increases rapidly with increasing B, as does the magnetoresistance, above $B = B_0$, where $\zeta = 1$. The onset field B_0 is the field above which the electron drift over a time $\sim\omega_c^{-1}$ is less than the thermal wavelength $\lambda\!\!\!^{-}_T$. For a Gaussian many-electron field and scattering by vapor atoms Eqs. (2)–(4) give the following classical magnetoresistance:

$$\rho^*_{\mathrm{mc}} = \sum_{s=-\infty}^{\infty} (1 + 4\pi^2s^2(B_0/B)^4)^{-3/2} . \qquad (5)$$

The fluctuational electric field which determines B_0 can be estimated by assuming short-range order in the electron system (which seems reasonable for $e^2n^{1/2}/\varepsilon_0 \gg kT$) and equating kT to the energy $e^2E^2/m\omega_p^2$ of electron vibrations in the field of other electrons at a characteristic 2D plasma frequency $\omega_p = (e^2 n^{3/2}/2\varepsilon_0 m)^{1/2}$; this gives $E \simeq 0.84(kTn^{3/2}/\varepsilon_0)^{1/2}$ [6], $\eta \simeq \omega_c/\omega_p$ and $B_0 = 1.66 \times 10^{-5} n^{3/8} T^{1/2}$.

Quantum mechanically, the motion of an electron in the crossed $\boldsymbol{E}$ and $\boldsymbol{B}$ fields is a superposition of quantised cyclotron motion and drift of the center of the cyclotron orbit. This picture is valid and the drift is semiclassical if the field $\boldsymbol{E}$ is uniform over the wavelength $\lambda\!\!\!^{-} = l/(\bar{n}+1)^{1/2}$, $\bar{n} = [\exp(\hbar\omega_c/kT) - 1]^{-1}$ and $eE\lambda\!\!\!^{-} \ll \hbar\omega_c$. Under these conditions the correlator $\xi(\boldsymbol{q})$ is

$$\xi(\boldsymbol{q}) = 2(lq)^{-1}\tau_e \exp\{-l^2q^2(2\bar{n}+1)/2\} \times \sum_{m=0}^{\infty} [l^2q^2/2]^{2m}[\bar{n}(\bar{n}+1)]^m (m!)^{-2} \quad (6)$$

where $\tau_e = lB\langle|E|^{-1}\rangle$. The parameter τ_e is the 'time of flight' for a cyclotron orbit in a field E to drift over a distance l equal to the radius of the electron wave function and corresponds to the duration of a collision with a short-range scatterer τ_{col} in the limit $\hbar\omega_c \gg kT$. For large Planck numbers, $\bar{n} \simeq kT/\hbar\omega_c \gg 1$, Eq. (6) reproduces Eq. (4) for $\zeta \gg 1$: in this range the quantum and classical many-electron theories match and give identical results. The reduced resistivity for E given by the estimate above is

$$\rho^*_{mq} = 0.15\,(\omega_c/\omega_p)\,(\hbar\omega_c/kT)^{3/2}\,\Xi \quad (7)$$

where $\Xi = 1$ in the extreme quantum limit $\hbar\omega_c \gg kT$ and Eq. (7) agrees with Dykman and Khazan [5].

Equations (2)–(6) show that for $B \gg B_0$ (i.e. $\zeta \gg 1$) the scattering rate τ_B^{-1} and the magnetoresistance increase rapidly with magnetic field, as does τ_{col}. In strong fields the condition for the Born approximation, $\tau_{col} \ll \tau_B$, no longer holds, many-electron effects become less important, and a crossover to single-electron magnetoresistance should occur.

3. The experiments

The magnetoresistance of the 2DES on superfluid helium was measured using the Sommer–Tanner technique [10] with co-planar electrodes

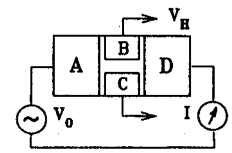

Fig. 1. The electrode geometry used.

(see Fig. 1) in a rectangular geometry situated below the helium surface and the electron sheet. The electrons were produced by a glow discharge and held in place by DC voltages on the electrodes, a surrounding guard ring and a top plate above the helium surface. A small AC voltage V_0 at audio frequencies $f(=\omega/2\pi)$ was applied to electrode A and the AC current I to electrode D was measured with a lock-in amplifier. For a perfectly conducting electron sheet at $B = 0$ the phase of the capacitively coupled current I is $\pi/2$ with respect to V_0. The phase shift $\phi(B)$ away from $\pi/2$ was measured as a function of B for a range of electron densities and temperatures $0.5\,\mathrm{K} < T < 1.3\,\mathrm{K}$. The data cover a wide range of $\mu_0 B$ values and extend into the quantum limit $\hbar\omega_c/kT \gg 1$. The electron density was determined from the linear Hall voltage $V_H \propto B/ne$ as measured between electrodes B and C and calibrated using the transition to the 2D electron solid phase at $T_m = 0.216 \times 10^{-6} n^{1/2}$ K [1,11].

The combination of electrodes and electrons acts as a two-dimensional transmission line where the electrical response of the electron sheet is determined by the magnetoresistivity tensor components ρ_{xx} and ρ_{xy} and the capacitance per unit area, C_s, between the electrons and the electrodes. Numerical analysis of this system in the fully screened limit has shown that the phase shift $\phi(B)$ in a magnetic field for rectangular electrodes is given [12] by

$$\phi(B) = K\omega\rho_{xx}(1 + (\alpha\rho_{xy}/\rho_{xx})^2) = \phi_0\rho^*(1 + (\alpha\mu_0 B/\rho^*)^2) \quad (8)$$

for $\phi < 0.3$ rad where K and α (=0.225 in this case) depend on the electrode geometry. The second expression, for the normalized resistivity $\rho^*(B) = \rho_{xx}(B)/\rho_0$, is obtained by using the Hall resistance $\rho_{xy} = B/ne$ as confirmed experimentally [13] in agreement with theoretical arguments

[14] and ϕ_0 is the phase shift in zero field. Hence we can obtain $\rho(B, T)$ from the measured $\phi(B)$. Equation (8) is only valid if the characteristic decay lengths of a damped voltage wave on the transmission line parallel and perpendicular to the local current flow direction, $\delta_{\parallel} = (2/\omega C_s \rho_{xx})^{1/2}$ and $\delta_{\perp} = (\rho_{xx}/\rho_{xy})\delta_{\parallel}$, are both greater than the sheet dimensions, and hence low frequencies are required. At higher frequencies in a partly screened system, 2D edge magnetoplasmons will propagate round the finite electron sheet [2].

4. Results

The electron scattering rate is strongly temperature dependent. Measurements have been made in both the low-temperature regime below 1 K [15] where the scattering is by thermal ripplons (and also depends on the vertical electric pressing field) and above 1 K [9] where the dominant scattering is by ^{4}He vapor atoms, which act as short-range scattering centers, ideal for comparison with theory. Positive magnetoresistance was observed for all densities and temperatures investigated as shown in the low-field region in Fig. 2. This data was taken at a density $n = 1.7 \times 10^{12}\,\mathrm{m}^{-2}$ at 0.924, 1.02, 1.13 and 1.24 K using a frequency of 3940 Hz. The low-field mobility values at these temperatures were found to be 145, 90, 48 and 30 $\mathrm{m}^2/\mathrm{V\,s}$ from the initial slope of $\phi(B)/\phi_0$ which is proportional to $(\mu_0 B)^2$ in Eq. (8). These values agree with the zero-field values from other experiments [16] and single-electron calculations [17] and hence confirm that $\rho^* \simeq 1$ in low fields. The zero-field resistivity values ρ_0 are shown by solid lines for $B < 0.1$ T. Up to $B = 1$ T, ρ_{xx} increases slowly and quadratically with field. We will now concentrate on the region above 1 K where the scattering by ^{4}He vapor atoms is dominant but the mobility μ_0 is still high.

The shape of the normalised resistivity $\rho^* = \rho_{xx}/\rho_0$ as a function of B was found to depend on both temperature and electron density as shown in Fig. 3(a) where ρ^* is plotted for the data in Fig. 2 and in Fig. 3(b) for 1.00 K. At higher fields the resistivity increases rapidly as shown in Figs. 4, 5 and 6 for $n = 0.6 \times 10^{12}\,\mathrm{m}^{-2}$, $T = 1.003$ K; $n = 2.1 \times 10^{12}\,\mathrm{m}^{-2}$, $T = 1.003$ K and $n = 2.1 \times 10^{12}\,\mathrm{m}^{-2}$, $T = 1.215$ K respectively. This data was measured at 956 Hz; ρ^* was independent of frequency below 5 kHz. In general, ρ^* increases for a given field with decreasing temperature and density.

A remarkable feature of the observed magnetoresistance is that it is relatively small for classically strong magnetic fields: $\rho^* < 1.1$ even for $B = 0.4$ T where $\mu_0 B > 10$ and hence the Landau-level spacing is more than an order of

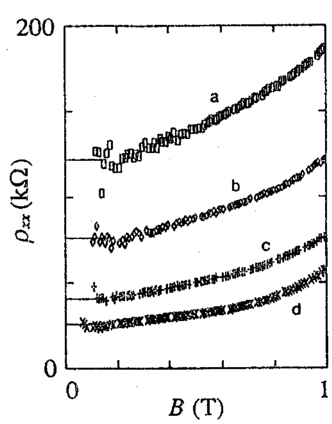

Fig. 2. The resistivity $\rho_{xx}(B)$ versus B for $n = 1.7 \times 10^{12}\,\mathrm{m}^{-2}$ at 1.24 (a), 1.13 (b), 1.02 (c) and 0.924 K (d).

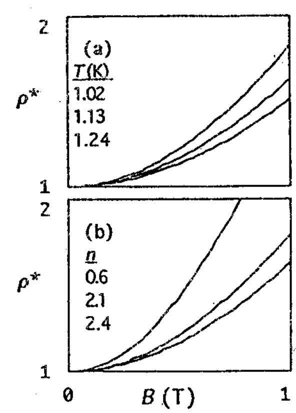

Fig. 3. (a) The normalised resistivity $\rho^*(B)$ versus B for the data in Fig. 2 ($n = 1.7 \times 10^{12}\,\mathrm{m}^{-2}$) at 1.02, 1.13 and 1.24 K; (b) the normalised resistivity $\rho^*(B)$ versus B for $n = 0.6$, 2.1 and $2.4 \times 10^{12}\,\mathrm{m}^{-2}$ at 1.00 K.

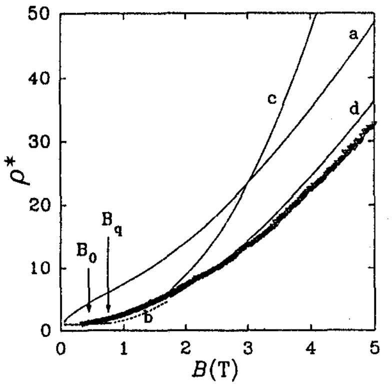

Fig. 4. The normalised resistivity $\rho^*(B)$ versus B for $n = 0.6 \times 10^{12}\ \mathrm{m}^{-2}$, $T = 1.003$ K. The lines show the theory for independent electrons, ρ_s^* (line a), the classical many-electron theory ρ_{mc}^*, Eq. (5) (line b), the quantum many-electron theory ρ_{mq}^*, Eq. (6) (line c) and the total resistivity ρ_t^* (line d). The onset field B_0 and the quantum limit, $\hbar\omega_c/kT = 1$ at B_q, are marked.

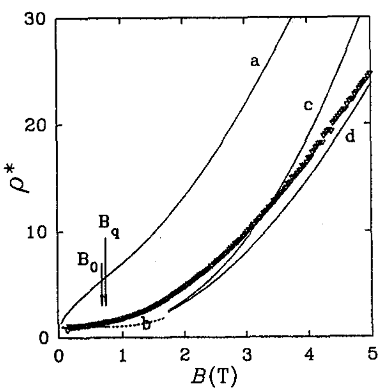

Fig. 5. Same as in Fig. 4 for $n = 2.1 \times 10^{12}\ \mathrm{m}^{-2}$, $T = 1.003$ K.

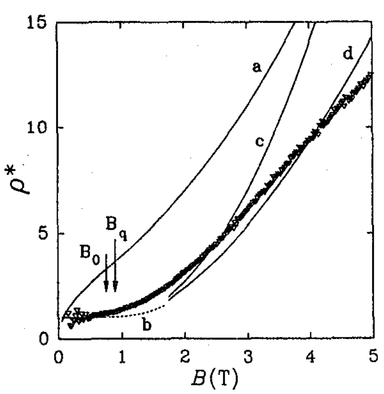

Fig. 6. Same as in Fig. 4 for $n = 2.1 \times 10^{12}\ \mathrm{m}^{-2}$, $T = 1.215$ K.

magnitude larger than the level width as given by $\hbar/\tau_0$. In the conventional single-electron theory based on the SCBA the width of the levels $\hbar/\tau_B$ is due to electron collisions with ^{4}He vapor atoms and, since the density of states increases due to the 'squeezing' of the energy spectrum into Landau levels, $\tau_B^{-1} = (2\mu_0 B/\pi)^{1/2}\tau_0^{-1}$ for $\mu_0 B \gg 1$ and increases sharply with B [13,14]. In the classical limit, $\hbar\omega_c/kT \ll 1$, this leads to $\rho_s^* \simeq (\mu_0 B)^{1/2}$ while in the quantum limit, $\hbar\omega_c/kT \gg 1$, $\rho_s^* \simeq (\mu_0 B)^{1/2}(\hbar\omega_c/kT)$. The full theoretical expression for ρ_s^* using the SCBA for a non-degenerate 2DES has been given by van der Heijden et al. [13] and is shown (line a) in Figs. 4–6. In each case ρ_s^* lies above the data and shows a stronger field dependence. It is this striking observation which indicates the importance of many-electron effects and is the subject of this paper. Moreover, ρ^* displays a density dependence that also indicates the influence of many-electron effects.

We will now compare these results with the many-electron theories presented above. The low-field classical theory for $\eta \ll 1$ (which corresponds to $B \ll 0.4$ T for $n = 2.1 \times 10^{12}\ \mathrm{m}^{-2}$) predicts a very small magnetoresistance and is shown as line b in Figs. 4–6. The onset field B_0 for magnetoresistance from the classical theory, Eq. (5), is 0.44, 0.69 and 0.76 T for the data in Figs. 4, 5 and 6 respectively as shown. But quantum effects are already important in this region as $\hbar\omega_c = kT$ at 0.90 T at 1 K. For $B > 2$ T we can use the quantum magnetoresistance ρ_{mq}^*, calculated from Eq. (7), and plotted as line c in each figure. However, for $\rho^* > 5$ ($B > 3$ T), the results of the many-electron theory and of the single-electron SCBA differ by less than a factor of 2 and the combined total resistivity ρ_t is then calculated from the expression

$$1/\rho_t = 1/\rho_m + \rho_t/\rho_s^2 \tag{9}$$

which is derived from the Einstein diffusion equation in which the scattering rate is proportional to $\hbar\omega_c/\Gamma$, where Γ is the energy uncertainty of each Landau level, and allows for the self-consistent nature of the SCBA. The normalised total resistivity ρ_t^* is plotted as line d in each figure and shows good agreement with the measurements, particularly since there are no adjustable parameters in these calculations. At the highest fields the magnetoresistance increasingly approaches ρ_s^* from the SCBA.

In conclusion, we have observed and explained the many-electron character of the magnetoresistance of a 2DES in classically strong and in quantising magnetic fields.

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