

Dykman *et al.* Reply: The phase shift between the periodic response of a system (the signal) and the periodic driving force that gives rise to it is defined *uniquely* in statistical physics. The ensemble-averaged signal $\langle q(t) \rangle = \sum_n a(n) \cos[n\Omega t + \phi(n)]$, where $2\pi/\Omega$ is the period of the force. All of the phases $\phi(n)$ can be measured experimentally. In the commonly considered case of a cosine force $A \cos(\Omega t)$ and a nearly cosine signal, the term "phase shift" refers to $\phi \equiv \phi(1)$. There is no ambiguity about this; neither are there two different phase shifts [1]. It was ϕ that was investigated both in [2] and in earlier theoretical papers [3,4]. Provided the periodic force is weak, ϕ can be expressed in terms of a linear susceptibility [5]. It was the finiteness of the phase ϕ that Gammaitoni *et al.* claimed [6], wrongly [2], to have been "ruled out as apparently spurious" in stochastic resonance.

The topic of our Letter [2] was phase shifts in *stochastic resonance* (SR), a noise-induced enhancement of the signal-to-noise ratio R that is significant when [4] $\Omega \ll \tau_r^{-1}$, where τ_r^{-1} ($=1$ for the overdamped bistable system of [2]) is the reciprocal intrawell relaxation time (not "librational frequency"). In the range $\Omega \tau_r \gg 1$ [1], on the other hand (actually, $\Omega \tau_r > \sim 0.5$ for the system of [2]), SR does not occur; see Fig. 1, inset. In contrast to the exponentially fast rise of $-\phi(D)$ with increasing D (followed by a slower decrease) observed [2] for small A and Ω , $-\phi(D)$ for *large* Ω (Fig. 1) displays a much shallower maximum (but nonetheless increases, rather than decreases [1], for small D ; we have noted that the signal, too, initially increases with D). The steep initial rise of $-\phi(D)$ for *small* Ω [2] is associated with the onset of the noise-induced interwell transitions that are responsible for SR; moreover, ϕ is evidently a more sensi-

tive indicator of these transitions than R . The monotonic decrease of $-\phi(D)$ in [7] does not contradict this result, because the signal from the experiment had apparently [8] been passed through a two-state filter prior to analysis, thus removing the effect of the intrawell vibrations and mimicking the two-state approximation of earlier theories [3,4].

Applied consistently to the SR problem—which involves more than merely a linearization of the transition probabilities as suggested in [9]—our linear response theory (LRT) approach has been shown [2,10] to yield good agreement with experimental measurements of the amplitude and phase of the signal over a wide range of parameters for small amplitudes of the force.

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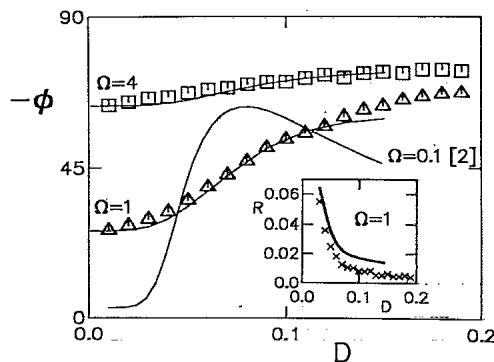


FIG. 1. Phase shift $-\phi$ of the signal induced by a weak periodic force of frequency Ω in the overdamped bistable system (1) of [2], as a function of noise intensity D . Inset: Signal-to-noise ratio R as a function of D at large Ω . The data points are by digital simulation; the curves are LRT based on Eqs. (6)–(10) of [2], to first order in $L_n(\omega)$.

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