Non-Lorentzian power-law absorption on the wings of electron–ripplon resonance peaks

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The peaks in the conductivity of a two-dimensional electron crystal on the surface of helium, arising due to the excitation of capillary waves with wave vectors close to the reciprocal lattice vectors **G**, are examined. It is shown that at finite temperatures the conductivity in the wings of the peaks decreases as a function of the frequency detuning according to an unusual power law. The exponent depends strongly on the parameters of the electron system and on the order of the resonance (i.e., on the value of G/G_{min}). This power law is a result of large long-wavelength fluctuations, which are characteristic for two-dimensional systems. In the case examined, the wings of the absorption peaks are formed due to processes in which several long-wavelength phonons of the electron crystal are created and annihilated.

Two–dimensional (2D) electrons on the surface of liquid helium at very low temperatures T and high densities N become ordered and form a Wigner crystal. As a result of the crystallization, the nature of the interaction of the electronic excitations of the system with excitations of the helium surface (ripplons) changes: The electronic excitations with wave vectors **k** are comparatively strongly coupled with the ripplons, whose wave vectors **q** differ from **k** by a reciprocal lattice vector of the Wigner crystal **G**. As a result, resonance peaks appear in the spectrum of absorption of long–wavelength ($k \ll G_{min}$) electromagnetic radiation by two–dimensional electrons at frequencies corresponding to ripplons ($\mathbf{q} = \mathbf{G} + \mathbf{k} \approx \mathbf{G}$) (Ref. 1). The observation of such peaks[2] gave direct experimental confirmation of Wigner crystallization.

In the interpretation of the experimental results[2] in Refs. 1, 3, and 4, the main attention was directed toward determining the frequencies of electron–ripplon resonances. In this paper we examine the form of the wings of the resonance absorption peaks. The unusual asymptotic behavior of the absorption on the wings is related to the specific nature of ordering in 2D systems. In such systems, as is well known (see, e.g., Ref. 5), at finite temperatures the density–density correlation function in the ordered phase decreases according to a power law. For this reason, the law of conservation of the quasimomentum is not strictly satisfied, so that phonons in the Wigner crystal with momentum $\hbar\mathbf{k}$ interact directly not with individual ripplons, but with entire groups of ripplons with momenta $\hbar\mathbf{q}$ close to $\hbar(\mathbf{k} + \mathbf{G})$. The magnitude of the interaction is determined by the Fourier transformation of the density–density correlation function of the Wigner crystal. This function in the static limit decreases according to a power law with increasing $|\mathbf{q} - \mathbf{k} - \mathbf{G}|$, and the exponent depends on the temperature and on **G**.

Below, we show that the absorption peaks corresponding to electron–ripplon resonances also have a power-law asymptotic behavior in the wings (as a function of frequency). However, in this case, it turns out that due to the slowness of the ripplons the form of the wings is determined by the form of the density–density correlation function of the Wigner crystal in the region of comparatively long times; i.e., processes in which phonons of the Wigner crystal are created and annihilated

make a substantial contribution to the absorption spectrum. Below, the calculation is carried out under the assumption that the electron–ripplon interaction is very weak, so that the width of the resonance peaks and their displacement relative to the frequencies of the ripplons with wave vectors **q** = **G** are small compared to these frequencies.

1. THE GENERAL EXPRESSION FOR THE CONDUCTIVITY OF A WIGNER CRYSTAL

The Hamiltonian of the electron–ripplon system in the case when the electrons form a Wigner crystal has the form

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_i, \tag{1}$$

$$\mathcal{H}_0 = \sum_{kj} \omega_{kj} a_{kj}^+ a_{kj} + \sum_q \omega_q b_q^+ b_q \quad (\hbar = 1);$$

$$\mathcal{H}_i = \sum_q V_q \rho_q c_q, \quad \rho_q = \sum_n \exp(i q r_n), \quad c_q = b_q + b_{-q}^+. \tag{2}$$

Here a_{kj} and b_q are the annihilation operators of a phonon in the Wigner crystal on branch 3 with momentum **k** and of a ripplon with momentum **q**; ω_{kj} and ω_q are the frequencies of the corresponding excitations. The interaction of the phonons with ripplons arises due to the electron–ripplon interaction, which is described by the Hamiltonian $\mathcal{H}_i$ (2). The coordinates of the electrons r_n in (2) are expressed in terms of the phonon operators in the standard manner:

$$r_n = R_n + \sum_{kj} e_{kj} x_{kj} \exp(i k R_n), \quad x_{kj} = A_{kj} a_{kj} + A_{-kj}^* a_{-kj}^+ \tag{3}$$

R_n is the position of a site in the lattice, and $e_{kj} = e_{-kj}$ is the polarization vector of the Wigner crystal phonon. It is evident from (2) and (3) that the phonon–ripplon interaction is significantly nonlinear with respect to the phonon operators, which leads to specific features in the relaxation or excitations in the Wigner crystal as compared to relaxation of excitations of ordinary crystals (see Refs. 6 and 7).

To analyze the absorption of the electromagnetic field it is convenient to express the diagonal component of the conductivity of 2D electrons $\sigma_{\varkappa\varkappa}(\mathbf{k}, \omega)$ ($\varkappa$ = x, y) in terms of the two-time phonon Green's function of the Wigner

crystal (see Ref. 6):

$$\mathrm{Re}\,\sigma_{xx}(\mathbf{k},\omega) = -\omega e^2 N^2 S \sum_j (\varkappa \mathbf{e}_{kj})^2 \,\mathrm{Im}\, Q_{kj}(\omega + i0),$$

$$Q_{kj}(\omega) = \langle\langle x_{kj};\ x_{-kj}\rangle\rangle_\omega;$$

$$\langle\langle \hat{X}_1;\ \hat{X}_2\rangle\rangle_\omega = -i \int_0^\infty dt\, e^{i\omega t} \langle[\hat{X}_1(t),\ \hat{X}_2(0)]\rangle. \tag{4}$$

Here e is the electron charge; N is the density; S is the area of the system; and, $\varkappa$ is a unit vector in the corresponding direction.

In the zeroth order approximation with respect to the interaction the function $Q_{kj}(\omega)$ has a pole (and the conductivity has a δ-like peak) at the characteristic phonon frequency ω_{kj}. When the interaction is taken into account, the peak acquires a finite width Γ_{kj} (see Ref. 6). A finite conductivity also appears far from ω_{kj}, which varies smoothly with frequency. Near the frequencies ω_G of Bragg ($\mathbf{q} = \mathbf{G}$) ripplons peaks arising due to resonance excitation of ripplons (electron-ripplon resonances) are superposed on the smooth frequency dependence of $\sigma(\mathbf{k},\omega)$ ($k \ll G_{min}$).

To anakyze the absorption peaks corresponding to the electron-ripplon resonances and situated far from the characteristic phonon frequencies ($|\omega - \omega_{kj}| \gg \Gamma_{kj}$), it is convenient to put the function $Q_{kj}(\omega)$ in (4) into the form

$$Q_{kj}(\omega) = \frac{2\omega_{kj}|A_{kj}|^2}{\omega^2 - \omega_{kj}^2} - \left(\frac{2\omega_{kj}|A_{kj}|^2}{\omega^2 - \omega_{kj}^2}\right)^2 \left[S\mathscr{F}_{kj}(\omega) + \sum_q (\mathbf{q}\mathbf{e}_{kj})^2 \langle H_q\rangle \right], \tag{5}$$

where

$$\mathscr{F}_{kj}(\omega) = S^{-1} \sum_{qq'} (\mathbf{q}\mathbf{e}_{kj})(\mathbf{q}'\mathbf{e}_{-kj}) \langle\langle H_{q'}(-\mathbf{k});\ H_q(\mathbf{k})\rangle\rangle_\omega,$$

$$H_q(\mathbf{k}) = \sum_n V_q c_q \exp(i\mathbf{q}\mathbf{r}_n + i\mathbf{k}\mathbf{R}_n); \quad H_q \equiv H_q(0) = \sum_n V_q c_q \exp(i\mathbf{q}\mathbf{r}_n) \tag{6}$$

[expressions (5) and (6) were obtained in Ref. 7 using the Fourier transformation (with respect to time) of the equations of motion for x_{kj}, which follow from (1)-(3)]. Taking into account the fact that in the absence of a magnetic field $|A_{kj}|^2 = (2mNS\alpha_{kj})^{-1}$, from (4) and (5) we obtain

$$\mathrm{Re}\,\sigma_{xx}(\mathbf{k},\omega) = \frac{\omega e^2}{m^2} \sum_j (\varkappa \mathbf{e}_{kj})^2 (\omega^2 - \omega_{kj}^2)^{-2}\,\mathrm{Im}\,\mathscr{F}_{kj}(\omega + i0),\quad \omega \neq \omega_{kj}. \tag{7}$$

The function $\mathscr{F}_{kj}(\omega)$ with $k \ll G_{min}$, as will be shown in the next section, has peaks at frequencies $\approx \omega_G$ in the lowest nonvanishing order of perturbation theory, which explains the convenience of expression (7). We note that in obtaining (7) decoupling was not used, and this relation is exact.

2. ELECTRON-RIPPLON RESONANCES

Since the resonance absorption at the frequencies of the Bragg ripplons is due to the electron-ripplon interaction, the integrated intensity of the corresponding peaks varies quadratically as a function of the interaction parameters. This is evident directly from (5) and (7). As already noted, we assume that the interaction is very weak, so that the width of the peaks and their displacement relative to ω_G are very small compared to ω_G (in the experiment of Ref. 2 this condition is obviously well satisfied for the high frequency $G \geq \sqrt{3}G_{min}$ peaks). In this

case, for purposes of revealing the electron-ripplon resonances, it is sufficient to decouple expression (6) for $\mathscr{F}_{kj}(\omega)$ in the zeroth order approximation with respect to $\mathscr{H}_i$.

The result of the decoupling, with (1), (3), and (6) taken into account, has the form

$$\mathscr{F}_{kj}^{(0)}(\omega) = -2N \sum_q (\mathbf{q}\mathbf{e}_{kj})^2 |V_q|^2 \int_0^\infty dt\, e^{i\omega t}\,\mathrm{Im}\,[\zeta_q(\mathbf{k},t)\,\varphi(\omega_q,t)]. \tag{8}$$

Here $\zeta_q(\mathbf{k},t)$ is the density-density correlation function of the Wigner crystal:

$$\zeta_q(\mathbf{k},t) = \sum_n \exp[i(\mathbf{q}-\mathbf{k})\mathbf{R}_n + W_q(\mathbf{R}_n,t)],$$

$$W_q(\mathbf{R}_n,t) = -\sum_{kj} (\mathbf{q}\mathbf{e}_{kj})^2 |A_{kj}|^2 \{(\bar{n}_{kj}+1)[1 - \exp(-i\omega_{kj}t + i\mathbf{k}\mathbf{R}_n)]$$
$$+ \bar{n}_{kj}[1 - \exp(i\omega_{kj}t - i\mathbf{k}\mathbf{R}_n)]\},$$
$$\bar{n}_{kj} \equiv \bar{n}(\omega_{kj}),\ \bar{n}(\omega) = [\exp(\omega/T) - 1]^{-1}; \tag{9}$$

$\varphi(\omega_q,t)$ is the Green's function of the ripplon:

$$\varphi(\omega,t) = [\bar{n}(\omega)+1]\exp(-i\omega t) + \bar{n}(\omega)\exp(i\omega t) \tag{10}$$

[in deriving (8) it was assumed that the system is centrosymmetric.]

The function $\mathscr{F}_{kj}^{(0)}(\omega)$ was actually analyzed in the nonresonant frequency range in Ref. 6 in an investigation of the polarization operator entering into the phonon Green's function of the Wigner crystal. It was shown that both at temperatures exceeding the Debye temperature of the Wigner crystal and at lower temperatures, but at high frequencies ω,

$$\omega \gg \omega_{min}\ (\omega_{min} \equiv \omega_{G_{min}})$$

the term with $\mathbf{R}_n = 0$ and $\zeta_q(\mathbf{k},t)$ ("single-site" scattering) makes the main contribution to $\mathscr{F}_{kj}^{(0)}(\omega)$, while the sum over q encompasses a large region greatly exceeding $N^{1/2}$ ($G_{min} \sim N^{1/2}$).

In the resonance region, where $\omega \approx \omega_G$, terms with large $\mathbf{R}_n$ in (9) make the main contribution to (8), while the values of q lie in a narrow region about the vectors $\mathbf{G}$ (for $k \ll N^{1/2}$). In particular, at $T = 0$ the function $\exp[W_q(\mathbf{R}_n,t)]$ can be expanded in a series with respect to those terms in $W_q(\mathbf{R}_n,t)$ that explicitly depend on time and $\mathbf{R}_n$. The zeroth order term in this expansion gives

$$\zeta_{0,q}(\mathbf{k},t) = NS \exp(-\eta q^2)\delta_{q-k,\,G},\ \eta = \tfrac{1}{2}\sum_{kj}|A_{kj}|^2. \tag{11}$$

Substituting (11) into (8), taking into account (10), we obtain

$$\mathrm{Im}\,\mathscr{F}_{kj}^{(0)}(\omega + i0) = \sum_G D_{kj}(\mathbf{G})\exp(-\eta G^2)\delta(\omega - \omega_{|G+k|}), \tag{12}$$
$$D_{kj}(\mathbf{G}) = \pi N^2 S (\mathbf{G}\mathbf{e}_{kj})^2 |V_G|^2$$

(strictly speaking, the substitution $\mathbf{G} \to \mathbf{G} + \mathbf{k}$ should be made in the expression for D_{kj} and in the exponent, but there and below corrections of order $\sim k/G$ are ignored).

It is evident from (7) and (12) that in the lowest order approximation with respect to V_q at $T = 0$ the peaks in the conductivity of the Wigner crystal are situated at ripplon frequencies $\omega_{|G+k|} \approx \omega_G$ and are δ-like. We note that in this approximation the same result is obtained if the approach in Refs. 1 and 4 is used. The intensity of the peaks, as indicated above from general considerations,

M. I. Dykman

behaves quadratically with respect to the matrix elements $\hat{V}_q$ of the electron–ripplon interaction.

The presence of the time-dependent terms in $W_q(\mathbf{R}_n, t)$ leads to the appearance in $\mathscr{F}_{kj} \leqslant_{kj}^{(0)}(\omega)$ of a background varying smoothly with frequency, corresponding to absorption of light with the creation of a ripplon and several phonons in the Wigner crystal. At $T = 0$ this background is small in the region $\omega \sim \omega_{\min}$, since the Debye frequency of the Wigner crystal

$$\Theta_D \sim \omega_p = (2\pi e^2 N^{3/2}/m)^{1/2}$$

usually greatly exceeds $\omega_{\min}$, so that the density of states of the phonons with frequencies $\lesssim \omega_{\min}$ is small. On the other hand, at high temperatures ($T \gg \Theta_D$), the low-frequency phonon conductivity of the Wigner crystal turns out to be comparatively large: It is close to the conductivity of an ideal electron gas interacting with the ripplons (compare Ref. 6).

3. FORM OF THE WINGS OF CONDUCTIVITY PEAKS AT FINITE TEMPERATURES

For $T \neq 0$, the function $\exp[W_q(\mathbf{R}_n, t)]$ cannot be expanded in a series, because taken separately the terms in $W_q(\mathbf{R}_n, t)$ which depend and do not depend on $\mathbf{R}_n$, t diverge (as in the mean-square displacement of the electron at $T \neq 0$). At the same time the function $W_q(\mathbf{R}_n, t)$, neglecting corrections related to quantum fluctuations, is described by the expression

$$W_q(\mathbf{R}_n, t) = \frac{q^2 T}{4\pi m c_t^2 N} \ln\left[(\lambda^2 R_n^2 + c_t^2 t^2)^{1/2} a_0^{-1}\right],$$

$$(\lambda^2 R_n^2 + c_t^2 t^2)^{1/2} \gg N^{-1/2}, \quad c_t T^{-1}; \quad \lambda \equiv \lambda(c_t t/R_n) \sim 1,$$
$$\lambda \approx 1/2 \quad \text{for} \quad c_t t \ll R_n, \tag{13}$$

where c_t is the velocity of transverse sound in the Wigner crystal: $c_t \sim (e^2 N^{1/2}/m)^{1/2}$; the parameter a_0 is of the order of $N^{-1/2}$ for $T > \Theta_D$ and of the order of c_t/T for $T \ll \Theta_D$. In obtaining (13), only the most "dangerous" contribution of long-wavelength ($\omega_{kt} \equiv c_t k \ll \Theta_D$) transverse ($j = t$) acoustic phonons was included; the longitudinal ($j = l$) phonons are much stiffer than the transverse phonons (in a thick helium film $\omega_{kl} \propto k^{1/2}$ down to very small k), and their contribution is dropped.

The calculation of $\mathscr{F}_{kj}^{(0)}(\omega)$ taking into account (8), (9), and (13) reduces to summation with respect to $\mathbf{R}_n$, $\mathbf{q}$ and integration over t. As a result of the smooth variation of $W_q(\mathbf{R}_n, t)$ as a function of $\mathbf{R}_n$, the function $\zeta_q(\mathbf{k}, t)$ has singularities at $\mathbf{q} - \mathbf{k} = \mathbf{G}$. At the same time, $|V_q|^2$ and ω_q in (8), just as the function $W_q(\mathbf{R}_n, t)$, depend smoothly on $\mathbf{q}$. For this reason, in analyzing the peak in $\mathscr{F}_{kj}^{(0)}(\omega)$ near the frequency of Bragg ripplons with some $\mathbf{G} = \mathbf{G}_0$:

$$\omega \approx \omega_0 \equiv \omega_{G_0}.$$

In these functions we can make the replacement $\mathbf{q} \to \mathbf{G}_0$ ($\mathbf{k} \ll \mathbf{G}_0$), while the summation over $\mathbf{q}$ can be replaced by integration over $\mathbf{Q} = \mathbf{q} - \mathbf{G}_0$ (where $Q \ll G_{\min}$) and summation over the vectors $\mathbf{G}_0$ of equal length. This approximation is justified if the main contribution to $\mathscr{F}_{kj}^{(0)}(\omega)$ indeed comes from the region $Q \ll G_{\min}$.

The actual values of Q and $R_n \sim Q^{-1}$ are determined, as is evident from (13), by the characteristic time t, over which the structure of the absorption peak forms (i.e., the function $\operatorname{Im}\mathscr{F}_{kj}^{(0)}(\omega)$ is generated). The wings of the peak, where $|\omega - \omega_0|$ greatly exceeds the width and displacement of the peak relative to ω_0, form over a time $t \sim |\omega - \omega_0|^{-1}$. For this reason

$$Q^{-1} \sim R_n \sim c_t |\omega - \omega_0|^{-1}. \tag{13a}$$

Since we are examining resonant absorption, the quantity $|\omega - \omega_0|$ is very small, and we assume that the condition

$$c_t |\omega - \omega_0|^{-1} \gg N^{-1/2}. \tag{14}$$

is satisfied. From here it follows that

$$Q^{-1} \sim R_n \gg N^{-1/2}.$$

This inequality not only justifies the assumption made above concerning the smallness of the actual values of $Q/G_{\min}$, but also permits transforming from summation over $\mathbf{R}_n$ to integration. The result of integration with respect to $\mathbf{R}_n$ and $\mathbf{Q}$ in (8) and (9) has the form

$$\operatorname{Im}\mathscr{F}_{kj}^{(0)}(\omega) = \sum_G \delta_{G,\, G_0} D_{kj}(\mathbf{G})\, \psi_G(\omega, \alpha(G)), \quad \omega \approx \omega_0 \equiv \omega_{G_0}, \tag{15}$$

where

$$\psi_G(\omega, \alpha) = \frac{1}{\pi}\left(\frac{a_0}{c_t}\right)^\alpha \int_0^\infty dt\, t^{-\alpha} \cos\left[(\omega - \omega_G)\, t\right] = \psi_G(\alpha)\, |\omega - \omega_G|^{-1+\alpha};$$

$$\psi_G(\alpha) = \frac{1}{\pi}\,\Gamma(1-\alpha) \sin(\pi\alpha/2)(a_0/c_t)^\alpha, \quad \alpha \equiv \alpha(G) = \frac{G^2 T}{4\pi m c_t^2 N} \tag{16}$$

[see Ref. 8; $\Gamma(x)$ is the gamma function].

It is easily verified that the substitution of $\mathbf{G}_0$ for $\mathbf{q}$ in $D_{kj}(\mathbf{q})$ and in $W_q(\mathbf{R}_n, t)$ in making the transformation to (15) is valid if the corrections of order

$$c_t^{-1} |\omega - \omega_0| |\partial \ln D_{kj}(\mathbf{G}_0)/\partial \mathbf{G}_0|;$$

$$\alpha(G_0)\, G_0^{-1} c_t^{-1} |\omega - \omega_0| \ln(a_0^{-1} c_t |\omega - \omega_0|^{-1}).$$

are small. Due to the power-law character of the dependence of D_{kj} on G this reduces to the inequality $c_t G_0 \cdot |\omega - \omega_0|^{-1} \gg 1$, which follows from (14).

Another condition for the applicability of (15) and (16) is related to the possibility of making the substitution $\varphi(\omega_q, t) \to \varphi(\omega_G, t)$ in (8). This is possible if $|Q d\omega_G \cdot dG^{-1}|t \ll 1$, whence it follows, taking into account (13a) and (16), that

$$|d\omega_G/dG| \ll c_t. \tag{17}$$

This inequality in the case of 2D electrons on the surface of helium holds for several lowest G with a reserve of two to three orders of magnitude.

4. DISCUSSION

According to (7), (15), and (16), on the wings of peaks corresponding to electron–ripplon resonances the absorption decreases with frequency according to the power law $|\omega - \omega_G|^{-1+\alpha(G)}$. The factor $\alpha(G)$ in the exponent is proportional to the temperature, and the square of the reciprocal lattice vector G, and is inversely proportional to the square of the velocity of transverse sound c_t.

In contrast to the present work, in Ref. 3 a difference absorption spectrum on the wings was obtained, which in

M. I. Dykman

the limit of weak coupling has a Lorentzian asymptotic form $\text{Im}\,\mathscr{F}_{kj}(\omega) \sim (\omega - \omega_G)^{-2}$. This is related to the use in Ref. 3 of the assumption that translational symmetry exists in the 2D crystal at finite temperatures. On the other hand, formulas (15) and (16) were obtained as a result of a systematic inclusion of long-wavelength fluctuations of the Wigner crystal, which, in particular, lead to suppression of translational symmetry.[5] It is significant that the electron—ripplon resonances themselves, in this case, do not vanish.[1)] As is evident from (13a), the spectrum on the wings is formed due to vibrations of the Wigner crystal with wavelengths $\sim c_t |\omega - \omega_G|^{-1}$ (it is this quantity that should be compared to the dimensions of the system).

It is evident from formulas (8), (9), (13), and (16) that the wings of the resonance peaks are essentially formed by two mechanisms: nonconservation of quasimomentum ($\mathbf{q} \neq \mathbf{k} + \mathbf{G}$) and combined absorption processes, in which together with the creation of a resonant ripplon many ($\sim \alpha(G) \ln (a_0^{-1} c_t |\omega - \omega_G|^{-1})$) phonons of the Wigner crystal are created and annihilated. Combined processes arise as a result of the fact that the resonance absorption spectrum is determined by the dynamic $[\zeta_{\mathbf{q}}(\mathbf{k}, t)]$ and not the static $[\zeta_{\mathbf{q}}(\mathbf{k}, 0)]$ density—density correlation function of the Wigner crystal [see (8) and (9)]. If the combined processes could be neglected and we could set $t = 0$ in (13), then the wing of the peak in $\text{Im}\,\mathscr{F}_{kj}^{(0)}(\omega)$ would form only as a result of the nonconservation of quasimomentum and the frequency dispersion of ripplons. It is not difficult to show that the expressions (15) and (16) in this case would remain valid, and only the form of the coefficient $\psi_G(\alpha)$ would change:

$$\psi_G(\alpha) \to \tilde{\psi}_G(\alpha), \quad \tilde{\psi}_G(\alpha) = \pi^{-1/2} \frac{\Gamma(1/2 - \alpha/2)}{\Gamma(\alpha/2)} a_0^\alpha |d\omega_G/dG|^{-\alpha}. \quad (18)$$

The formulas (15), (16), and (18) describe the wings of the absorption peaks, if the surface vibrations of the substrate with which the two-dimensional electrons interact, have a high group velocity $|d\omega_G/dG| \gg c_t$. The energy of the electromagnetic field in this case is entirely transferred to the surface wave. The power-law decrease of absorption at the frequency $\omega - \omega_G$ reflects the power-law decrease with respect to $Q = |\mathbf{q} - \mathbf{k} - \mathbf{G}|$ of the Fourier transform of the static density—density correlation function of the Wigner crystal. Corrections for processes with creation and annihilation of phonons are small with respect to the parameter $c_t |d\omega_G/dG|^{-1}$. For 2D electrons on the surface of helium, as already indicated, this parameter is very large, the inequality (17) holds, and the wing is formed due to combined processes in which phonons of the Wigner crystal participate.

Experimentally, the sharp power-law frequency dependence of the absorption near the resonance peaks must be manifested quite distinctly; at the same time, we note that the decrease $\sim |\omega - \omega|^{-1+\alpha(G)}$ is smoother than on the wing of the Lorentzian peak (where absorption drops off as the inverse square of the frequency detuning). Since $\alpha(G) \propto G^2$, as G increases the wings of the peaks (and the peaks themselves) must be smoothed. The smearing of the peaks of electron—ripplon resonances with increasing G was observed in Ref. 2. From the results of the present work it is clear that the analysis of the wings of peaks could make it possible to determine directly the important characteristic of the 2D crystal: the velocity of transverse sound c_t.

NOTATION

Here $\mathscr{H}$ is the Hamiltonian; $\mathbf{r}_n$ is the position vector of the n-th electron; ω_{kj} is the frequency of the phonon in the electron crystal; $\mathbf{e}_{kj}$ is the polarization vector of the phonon; ω_q is the ripplon frequency; V_q is the matrix element of the electron—ripplon coupling; $\sigma_{\varkappa\varkappa}(\mathbf{k}, \omega)$ is the electronic conductivity; N is the electron density; $Q_{kj}(\omega)$ is the phonon Green's function; $\mathscr{F}_{kj}(\omega)$ is the auxiliary Green's function describing coupled phonon—ripplon resonances; $\zeta_{\mathbf{q}}(\mathbf{k}, t)$ is the Fourier transform of the time-dependent electron-density correlation function; $\varphi(\omega_q, t)$ is the ripplon Green's function; G is the reciprocal lattice vector of the Wigner crystal; and, c_t is the velocity of transverse sound.

[1)]In Refs. 1 and 4 the broadening of the resonance peaks was not taken into account, and only their position was examined. However, if the imaginary part of the polarization operator for phonons of the Wigner crystal found in Ref. 4 is taken into account, then calculations analogous to those performed in the present work yield the nonstandard asymptotic behavior (15) and (16).

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