Surface Science 170 (1986) 108–111
North-Holland, Amsterdam

CYCLOTRON RESONANCE OF INTERACTING NONDEGENERATE TWO-DIMENSIONAL ELECTRONS

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Received 7 August 1985; accepted for publication 13 September 1985

The electron–electron interaction in two-dimensional systems subjected to quantizing magnetic fields is shown to cause the qualitatively new mechanisms of electron scattering by impurities and by medium vibrations (phonons, ripplons). With allowance for these mechanisms, the kinetics of the electron liquid and Wigner crystal are considered and the halfwidth and shift of the cyclotron resonance peak are obtained in the explicit form.

The effects of interaction between two-dimensional (2D) electrons are especially interesting for the electron systems subjected to quantizing transverse magnetic field. In this case, in the single-electron approximation, the energy spectrum is discrete and presents a set of degenerate Landau levels. The electron–electron interaction lifts the level degeneracy. Thus it may influence extremely strongly both the state and the kinetics of the system.

The continuity of the energy spectrum of the electrons interacting one with another provides the possibility of their elastic scattering by impurities or quasielastic scattering by medium vibrations (for low impurity densities and weak coupling to the vibrations such types of scattering are forbidden in the single-electron approximation because of the spectrum discreteness). Therefore, there may be changed qualitatively as compared to the single-electron case, even the characteristics which are not influenced by the electron–electron interaction directly, in particular, magnetoconductivity and the cyclotron resonance spectrum.

In the range of relatively low electron densities N or strong magnetic fields $H_\perp$,

$$\frac{e^2}{l} \gg T \gg \frac{\omega_p^2}{\omega_c}, \quad \exp\left(\frac{\omega_c}{T}\right) \gg 1, \tag{1}$$

where

$$\omega_p = \left(\frac{2\pi e^2 N^{3/2}}{m}\right)^{1/2}, \quad \omega_c = \frac{eH_\perp}{mc}, \quad l = (m\omega_c)^{-1/2}, \quad \hbar = 1,$$

the kinetics of nondegenerate electrons may be investigated without determina-

tion of the elementary excitation spectrum for the many-electron system itself. The scattering by impurities and by low-frequency ($\omega(q) \ll \omega_c$) medium vibrations (MV) is due here to the electron drift under the electric field resulting from the electron density fluctuations. The fluctuational field $\boldsymbol{E}$ acting on each individual electron is homogeneous over the magnetic length l when (1) is fulfilled, and the drift is quasiclassical (see ref. [1]) The quasielastic scattering by MV takes place when the drift velocity $v_d = cE/H_\perp$ exceeds the MV phase velocity.

The consistent (nonperturbative in the electron–electron interaction) analysis shows that the cyclotron resonance (CR) peak is Lorentzian when electrons are scattered elastically or quasielastically. The hálfwidth of the peak for the case of coupling to MV is

$$\Gamma = 2\pi T \sum_{\boldsymbol{q}} \left(\tfrac{1}{2} l^2 q^2\right)^2 |V_{\boldsymbol{q}}|^2 \omega^{-1}(q) \exp\left(-\tfrac{1}{2} l^2 q^2\right) \times \left\langle \delta\left\{ cH_\perp^{-2} \boldsymbol{q}[\boldsymbol{E} \times \boldsymbol{H}_\perp] - \omega(q) \right\} \right\rangle_{ee}. \tag{2}$$

Here $V_{\boldsymbol{q}}$ is the matrix element of coupling, $\boldsymbol{q}$ is the 2D momentum of the MV, $\langle \cdots \rangle_{ee}$ denotes the averaging over the electron configurations with the weight $\exp(-H_{ee}/T)$,

$$H_{ee} = \tfrac{1}{2} e^2 \sum_{n \neq m} |\boldsymbol{r}_n - \boldsymbol{r}_m|^{-1}, \quad \boldsymbol{E} = \boldsymbol{E}_s = -\frac{1}{e} \frac{\partial H_{ee}}{\partial \boldsymbol{r}_s} \tag{3}$$

(the value of Γ in (2) is evidently independent of the electron number s in (3)).

In the important case of relatively soft MV,

$$\omega(q)\tau_e \ll 1 \quad (q \lesssim l^{-1}), \quad \tau_e = \left\langle (eEl)^{-1} \right\rangle_{ee} \equiv \left\langle l/v_d \right\rangle_{ee}, \tag{4}$$

the expression (2) takes the form

$$\Gamma = \tfrac{1}{2} \tau_e T \sum_{\boldsymbol{q}} |V_{\boldsymbol{q}}|^2 \omega^{-1}(q) (lq)^3 \exp\left(-\tfrac{1}{2} l^2 q^2\right). \tag{5}$$

Eq. (5) holds also in the case of scattering by impurities if one replaces $2T|V_{\boldsymbol{q}}|^2 \omega^{-1}(q)$ by the Fourier component of the static random field correlator.

The parameter τ_e in (4) and (5) determines the characteristic duration of the scattering process. When there is the short-range order in the electron system $\tau_e \sim (T\omega_p^2/\omega_c)^{-1/2}$, and Γ follows the power dependence on density and temperature, $\Gamma \propto N^{-3/4} T^{1/2}$. In the range (1), (4), the magnetoconductivity $\sigma \sim e^2 N \Gamma / m\omega_c T \propto N^{1/4} T^{-1/2}$.

The sharp CR peak narrowing with increasing density, obvious from (5), was observed by Edel'man [2] for 2D electrons above the helium surface. A comparison of the present theory with the experiment [2] is given in fig. 1. It should be noted that the theory does not contain adjustable parameters.

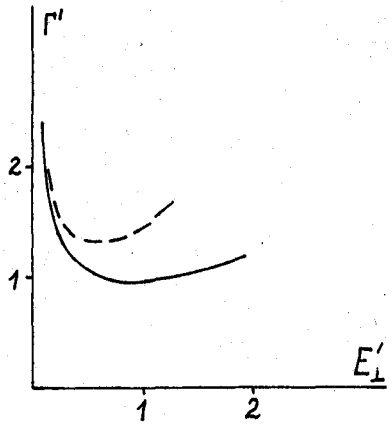

Fig. 1. The dependence of the reduced CR peak halfwidth on the field $E_\perp$ pressing electrons to the liquid helium surface. The full curve is calculated by eq. (5), the broken curve shows Edel'man's experimental data [2] for the range (1) (the inequalities (1) correspond to $E'_\perp \ll 2.2$), $E_\perp = 2\pi eN$, $\Gamma' = \Gamma/\Gamma_H$, $E'_\perp = E_\perp/E_H$, $\Gamma_H \approx 2.4\times10^{-5}\ T^{1/2}(e^2/l)^{3/2}(\sigma l^2)^{-1}$, $E_H \approx 0.69\times 10^{-3}\ e/l^2$ (σ is the surface tension).

In the range of densities where $e^2(\pi N)^{1/2}/T > 137$, the interaction between 2D electrons was found by Grimes and Adams [3] to cause their crystallization. The investigation of the kinetics of the electrons that form a Wigner solid (WS) and interact with MV comes to the investigation of the WS phonon kinetics. The latter differs substantially from the kinetics of "traditional" phonon systems. One of the main features is due to the essential nonlinearity of coupling to MV. This nonlinearity is obvious (cf. ref. [4]) from the Hamiltonian of coupling

$$H_i = \sum_n \sum_q V_q \exp(i\boldsymbol{q}\boldsymbol{r}_n)\, c_q, \quad c_q = b_q + b^\dagger_{-q} \tag{6}$$

(c_q denotes the MV field operator), if one expresses the electron coordinates $\boldsymbol{r}_n$ in terms of the WS phonon creation and annihilation operators $a^\dagger_{k\lambda}$, $a_{k\lambda}$:

$$\boldsymbol{r}_n = \boldsymbol{R}_n + \boldsymbol{u}_n, \quad \boldsymbol{u}_n = \sum_{k\lambda} \boldsymbol{u}_{k\lambda} \exp(i\boldsymbol{k}\boldsymbol{R}_n), \quad \boldsymbol{u}_{k\lambda} = \boldsymbol{A}_{k\lambda} a_{k\lambda} + \boldsymbol{A}^*_{-k\lambda} a^\dagger_{-k\lambda} \tag{7}$$

(λ and $\boldsymbol{k}$ label the phonon branches and wavevectors, $\lambda = \pm 1$). Since the mean-square displacement $\langle \boldsymbol{u}_n^2 \rangle$ diverges for a 2D solid at $T \neq 0$, the nonlinearity of H_i in $\boldsymbol{u}_{k\lambda}$ must be taken into account to all orders of the perturbation theory. It follows also from (6) and (7) that H_i cannot be put into the form of a series in relative displacements $\boldsymbol{u}_n - \boldsymbol{u}_m$. Therefore the dynamic renormalization of WS phonons appears finite even if $k \to 0$. Since the MV wavenumber q may exceed substantially $k_{max} \sim N^{1/2}$, the Umklapp processes may be essential for relaxation.

With the aid of a certain special method, the long-wavelength WS phonon Green function could be found explicitly, including the essential corrections to

the Born approximation. This allowed us to describe the CR spectrum in particular. In the range (1), the results coincide with those obtained above using a quite different approach (but (2) and (3) are valid also in the absence of long-range order). In the range of relatively high densities where $\omega_p \gg \omega_c$, and at $\exp(\omega_c/T) \gg 1$, the halfwidth Γ and shift P of the CR peak are

$$\Gamma \sim \Gamma_1 = \left(16mc_t^2N\right)^{-1}\sum_q u_q q^2\left(2\bar{n}_q+1\right), \quad P \approx P_0 + P_1, \tag{8}$$

$$P_0 = \sum_q u_q\,\omega^{-1}(q), \quad P_1 = \frac{2}{\pi}\Gamma_1 \ln(\omega_c/\tilde{\omega}), \quad u_q = |V_q|^2 l^2 q^2 \exp(-\eta q^2),$$

$$\bar{n}_q = \left[\exp\left(\frac{\omega(q)}{T}\right)-1\right]^{-1}, \quad \eta = \tfrac{1}{2}\sum_{k\lambda}|A_{k\lambda}|^2 \sim (m\omega_p)^{-1},$$

$$\tilde{\omega} = \max\left(T,\ \omega\left(\eta^{-1/2}\right),\Omega\right).$$

Here c_t is the WS transverse sound velocity, Ω is the frequency of those lower-branch ($\lambda = -1$) phonons whose spectrum renormalization due to coupling to MV is of the order of their initial frequency.

The CR peak broadening Γ_1 is determined by the probability of the MV-induced decays of long-wavelength "optical" WS phonons ($\omega_{k1} \approx \omega_c$) into the phonons of the branch $\lambda = -1$ with $k \approx \omega_c/c_t$. At $\omega_p \gg \omega_c$, the CR peak shift $P \gg \Gamma$ (in the range (1), $|P| \ll \Gamma$). The expressions (8) for Γ and P qualitatively and quantitatively agree with Edel'man's data [2] referring to the range $\omega_p \gtrsim 1.5\omega_c$.

At $T \gg \omega_p$, ω_c, the dynamics of WS coupled to MV are analogous to some extent to the dynamics of the crystal formed by Brownian particles, and the CR spectrum is described by the results of the classical single-electron theory.

The good agreement of the present theory with experiment confirms the essential role of the suggested many-electron mechanisms of scattering by impurities and medium vibrations in quantizing magnetic fields. At the same time, the above consideration shows that cyclotron resonance may be used to determine not only kinetic characteristics but also some other properties of the electron system. In particular, it follows from the data given in fig. 1 that in quantizing magnetic fields the short-range order exists up to such densities that $e^2(\pi N)^{1/2}T^{-1} \sim 30$.

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