Reflections from a thin film

- Part of the wave reflects from the top surface and part from the bottom surface.
- The part that reflects from the top surface has a $180^\circ$ phase change while the part that reflects from the bottom does not.
- When will there be constructive interference between the two reflected waves?
Reflections from a thin film

- Wave #2 has to travel further by a distance $2t$ (ignore any angle)
- So you might think that if $2t = m\lambda$ (where $m$ is an integer) that you would get constructive interference
- But...ahh...the phase shift...so I get constructive interference when $2t = (m+1/2)\lambda$
- But...ahh...I remember that the wavelength changes inside the film to $\lambda_n = \lambda/n$
- ...so, finally, I get constructive interference when
  - $2t = (m+1/2)\lambda_n$
  - or $2nt = (m+1/2)\lambda$
Reflections from a thin film

- So I get destructive interference when
  - $2t = m\lambda_n$
  - or $2nt = m\lambda$

- Two things influence whether I have constructive or destructive interference (or somewhere in between)
  - difference in path length traveled
  - any phase changes on reflection

  ▲ in this example, I have one $180^\circ$ phase shift because I’m going from air to a film with an index $n$ back to air
There’s a phase shift of $\pi$ at each of the surfaces (assume that $n_{\text{glass}} > n_{\text{film}}$).

Reflections for a given wavelength from a glass surface can be eliminated by choosing the thickness of the thin film to be one-quarter of the wavelength of the light in the film.

- $t_{\text{film}} = \frac{\lambda_{\text{film}}}{4} = \frac{\lambda_{\text{air}}}{4n}$

<table>
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<tr>
<th>Air</th>
<th>Thin film</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
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<td>1. Incident wave approaches the first surface.</td>
<td></td>
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<tr>
<td>2. Part of the wave reflects back with a phase shift of $\pi$ rad, part continues on into the film.</td>
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<tr>
<td>3. Part of the transmitted wave reflects at the second surface, part continues on into the glass.</td>
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<td>4. The two reflected waves are overlapped and interfere.</td>
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As for example, non-reflective coating for a solar cell.
A very thin oil film \((n=1.25)\) floats on water \((n=1.33)\). What is the thinnest film that produces a strong reflection for green light with a wavelength of 500 nm?
Thin film interference

- Green cast of a cat’s eyes in a flash photograph caused by constructive thin film interference (for ~500 nm)
Newton’s rings

- When a curved piece of glass is placed on a flat piece of glass and illuminated from above, observation reveals rings of color.
- These are known as Newton’s rings (because he studied them).

If the light source is monochromatic, then light and dark rings can be observed.
Newton’s rings

- A curved piece of glass sits on a flat piece of glass
- Both have indices of refraction $n$
- An incident ray of light reflects off of point $P_1$ and point $P_2$
- Ray 2 has to travel an extra distance $2P_1P_2$
- As we get closer to point C, the path length difference decreases
  - how fast depends on the radius of curvature of the glass

FIGURE 38–10 The geometry used to obtain the conditions for constructive interference in Newton’s rings. The two rays moving toward the eye interfere after they reflect from different surfaces.
Newton’s rings

- There’s an 180° phase shift on reflection from $P_2$, but not from $P_1$
- Expect constructive interference for

$$\Delta L = 2P_1P_2 = (m + \frac{1}{2} \lambda)$$

- $m=0, +/-1, +/-2, …$
- Expect to have a dark spot in the middle
  - 180° phase shift and no path length difference

**FIGURE 38–10** The geometry used to obtain the conditions for constructive interference in Newton’s rings. The two rays moving toward the eye interfere after they reflect from different surfaces.
Newton’s rings

- Newton’s rings give a practical way of testing the flatness of a glass surface.
- Place the surface to be tested on top of an optical flat (a glass surface known to be flat to within a fraction of a wavelength) then no regions of constructive interference should appear if the tested piece is flat and parallel to the optical flat.
- If the test piece is flat but not parallel to the optical flat, the interference fringes are straight lines.
- With this test, a surface can be polished until no curved interference fringes appear.

Have to be careful; if the surfaces are too flat, they will bond together.
Example

- Suppose I have two flat glass plates of length $L=10\,\text{cm}$ touching at one end, but separated by a wire of diameter $d=0.01\,\text{mm}$ at the other end.
- Light shines down almost perpendicularly on the glass and is reflected up to the eye.
- What is the distance $x$ between the observed maxima if the incident (blue) light has a wavelength of 420 nm?
Example

- Here both pieces of glass are flat
- Only the ray reflected from the second plate undergoes a phase change of 180°
- If the distance between the plates is given by y, then the phase difference between the two waves is determined by \( \Delta L = 2y \)
- The phase difference is given by
  \[
  \phi_{\Delta L} = 2\pi \frac{2y}{\lambda} = \frac{4\pi y}{\lambda}
  \]
- The total phase difference is
  \[
  \phi = \phi_{\text{ref}} + \phi_{\Delta L} = \pi + \frac{4\pi y}{\lambda}
  \]
- For constructive interference, the total phase difference must be
  \[
  2m\pi = \pi + \frac{4\pi y}{\lambda}
  \]
  - where \( m \) is an integer
Example

- Solve this equation for $y$
  $$y = \frac{\lambda}{4}(2m - 1)$$

- Use geometrical relation between similar triangles
  $$\frac{y}{d} = \frac{x}{L}$$
  $$x = \frac{L}{d} y = \frac{L \lambda}{d 4}(2m - 1)$$

- Difference in $x$ from one maximum to another corresponds to a change in $m$ of 1

$$\Delta x = \frac{L}{d} y = \frac{L \lambda}{d 4} 2$$

$$\Delta x = \left(\frac{10 \times 10^{-2} m}{0.01 \times 10^{-3} m}\right) \left(\frac{420 \times 10^{-9} m}{4}\right) = 2 mm$$

For glass plates 10 cm long, there will be about 50 bands of constructive interference.
Optical interferometers are devices that utilize the interference between light waves to measure quantities such as wavelength, small path-length differences, wave speeds and indices of refraction.

A light source is split by a beam splitter (a half-silvered mirror) into two coherent waves that may travel different distances or through different media before they rejoin and interfere.

1. The wave is divided at this point.

2. The returning waves recombine at this point.

3. The detector measures the superposition of the two waves that have traveled different paths.
Michaelson interferometer

- We’ve already encountered the Michaelson interferometer, intended to measure the ether drift.
Michaelson interferometer

- Mirror $M_2$ can be moved forward or backward by turning a precision screw.
- The waves travel distances $r_1=2L_1$ and $r_2=2L_2$.
- The path length difference is $\Delta r=2L_2-2L_1$.
- Constructive interference for $\Delta r=m\lambda$.
- Destructive interference for $\Delta r=(m+1/2)\lambda$.
The technology of wavelength measurements using a Michelson interferometer became so good that in 1960 an international scientific committee decided to use it as the definition of a meter.

They defined the meter to be exactly 1,650,763.73 wavelengths of a particular orange color of light emitted by the Krypton isotope $^{86}$Kr.

Aim this light at a Michelson interferometer, then move the mirror $M_2$ while counting out a pre-determined number of bright spots (corresponding to constructive interference).

This then is defined to be a given distance (some fraction of a m).
Gravitational waves are ripples in the fabric of space-time. When they pass through LIGO's L-shaped detector they will decrease the distance between the test masses in one arm of the L, while increasing it in the other. These changes are minute: just $10^{-16}$ centimeters, or one-hundred-millionth the diameter of a hydrogen atom over the 4 kilometer length of the arm. Such tiny changes can be detected only by isolating the test masses from all other disturbances, such as seismic vibrations of the earth and gas molecules in the air. The measurement is performed by bouncing high-power laser light beams back and forth between the test masses in each arm, and then interfering the two arms' beams with each other. The slight changes in test-mass distances throw the two arms' laser beams out of phase with each other, thereby disturbing their interference and revealing the form of the passing gravitational wave.
2 LIGO sites

- One in Washington, one in Louisiana

black hole merger detected by LIGO this year
Neutron star collisions

- Kind of event that LIGO is looking for
- Where did all of the gold (and other heavy elements) in the solar system originate?
- Too much for production in early universe, stars, supernova explosions
- Maybe in the collision of neutron stars
According to Albert Einstein, we live in a universe full of gravitational waves. He suggested that the movements of heavy objects, such as black holes and dense stars, create waves that change space and time. We have a chance to detect these waves, but they need your help to do it! Einstein@Home uses computer time donated by computer owners all over the world to process data from gravitational wave detectors. Participants in Einstein@Home download software to their computers, which process gravitational wave data when not being used for other computer applications, like word processors or games or PHY294H homework.
Measuring the index of refraction

- A Michelson interferometer can be used to measure indices of refraction, especially of gases
- First gas is pumped out of cell
- Number of wavelengths inside the cell is
  \[ m_1 = \frac{2d}{\lambda_{\text{vac}}} \]
- Then cell is filled with gas to be measured at 1 atm. Number of wavelengths inside the cell is
  \[ m_2 = \frac{2d}{\lambda} = \frac{2d}{\lambda_{\text{vac}} / n} \]
- Physical distance has not changed but number of wavelengths along the path has
  \[ \Delta m = m_2 - m_1 = (n - 1) \frac{2d}{\lambda_{\text{vac}}} \]
- Each increase of one wavelength causes one bright-dark-bright fringe shift at the output, so index of refraction can be determined by counting fringe shifts as cell is filled
Example

- A Michelson interferometer uses a helium-neon laser with a wavelength (in vacuum) of 633.0 nm
- As a 4.0 cm thick cell is slowly filled with a gas, 43 bright-dark-bright fringes shifts are seen and counted. What is the index of refraction of the gas (at this wavelength)?

\[ \Delta m = m_2 - m_1 = (n - 1) \frac{2d}{\lambda_{vac}} \]

\[ n = 1 + \frac{\lambda_{vac} \Delta m}{2d} = 1 + \frac{(6.330 \times 10^{-7} \text{m})(43)}{2(0.0400 \text{m})} = 1.00034 \]
Examples of interference: Holography

- A beam splitter divides a laser beam into two waves
- One wave illuminates the object of interest
- The light scattered by object is a complex wave but it is the wave you would see if you looked at the object from the position of the film
- The other wave is called the reference beam and is reflected directly towards the film
- The scattered light and reference light meet at the film and interfere with the film recording the interference pattern
- The interference pattern will be quite complex
Holography

- The hologram is played by sending just the reference beam through it.
- The reference beam diffracts through the transparent parts of a hologram.
- The diffracted wave is exactly the same as the light wave that had been scattered by the object.
- The diffracted reference beam reconstructs the original scattered wave.
- The view is 3-dimensional.

(c) Playing a hologram
The diffraction of the laser beam through the light and dark patches of the film reconstructs the original scattered wave. An observer “sees” the object as if it were here.
Diffraction

- Diffraction occurs when a wave passes through a small opening not so different in size from the wavelength of the wave.
- The wave spreads out as we saw previously.
- So instead of a bright spot just in the middle we see a spread-out distribution of light:
  - but with some structure to it.
- Type of diffraction we’re studying is called Fraunhofer diffraction:
  - screen is far away from slit,
  - …or there’s a converging lens just after the slit
  - Demo

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Where are the dark spots?

- Here’s where Huygen’s principle comes in handy.
- As the wave travels through the slit, treat each point in the slit as a source of waves.
- Light from one part of the slit can interfere with light from another part.
- Let’s divide the slit into halves and consider the wavelets coming from point 1 and from point 3.
- Wavelet 1 has to travel further.
- IF the additional distance, \( a/2 \sin \theta \) is equal to \( \lambda/2 \), then the wavelets from points 1 and 3 are exactly half of a wavelength out of phase.
  - destructive interference.
- Also true for 3 and 5, 2 and 4, any two points in the top and bottom of the slit separated by \( a/2 \).

Can go through the same exercise dividing the screen in 4 parts, 6 parts,…