The future linear collider

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The future linear collider is an accelerator that is currently being proposed to collide electrons and positrons at energies ranging from \( \sim 90 \text{ GeV} \) up to \( \sim 1 \text{ TeV} \). The physics potential of such a machine is described and the main features of the accelerator are outlined.

1. Introduction

Particle physics is on the verge of a revolution. The path to a deeper understanding of nature at the most fundamental level is about to be illuminated by new experimental data, throwing light on the origin of mass, probing the space-time in which we live on ever finer scales and extending our potential to discover new particles with ever higher masses. In this way we will understand better how the particles that make up our everyday world fit into a grander pattern, how the widely different forces interacting between them may be different aspects of a single unified force, or perhaps that the Universe in which we live possesses symmetries much deeper than we have yet imagined.

This experimental data will be provided by new accelerators operating at higher energies and intensities. The next such accelerator is called the Large Hadron Collider (LHC) and is currently being constructed at CERN, the European laboratory for particle physics in Geneva. The LHC, currently expected to start running in 2007, will collide protons together at very high energies and will give us the first pictures of nature at these new energy scales. As previously in the history of particle physics, this picture could be refined and completed by another type of accelerator that collides electrons (\( e^- \)) with their anti-particles, positrons (\( e^+ \)). Such a machine is now being considered as the accelerator to be completed a few years after the startup of the LHC and several candidate accelerating technologies exist to provide the required energies. Projects are currently being proposed in America, Asia and Europe. The NLC (American), JLC (Japanese) and TESLA (German-based) projects all aim at exploring the TeV scale whereas CLIC (CERN-based) aims longer-term at multi-TeV energies. An excellent starting point for a more detailed exploration of the worldwide LC projects and workshops can be found at [1]. In March 2001 the TESLA project published its Technical Design Report [2] and the studies there, together with those in the US and Japan [3], provide much more detail to the linear collider physics case and the technical challenges than can be presented here. In this article the next high energy \( e^+ e^- \) accelerator is referred to as the Linear Collider (LC), to encompass all the projects presently being proposed.

1.1. Motivation for a high energy linear collider

The famous Einstein relation \( E = mc^2 \) implies that to produce a particle of mass \( m \), requires an energy \( E \) of at least \( mc^2 \), where \( c \) is the speed of light. In fact, because conservation laws generally require a particle to be produced only together with its anti-particle, an energy of at least \( 2mc^2 \) is often required. This motivates the need for the highest possible energies in order to produce directly any new states that in turn may reveal new symmetries or forces of nature.

In high energy physics, the most convenient fundamental unit of energy is the electron volt, eV, where \( 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \) and for the rest of this article we will deal mostly with larger units of \( 1 \text{ TeV} = 10^{12} \text{ eV} \), or \( 1 \text{ GeV} = 10^9 \text{ eV} \). The first linear collider, the SLC built at SLAC in California, ran at \( \sim 91 \text{ GeV} \) in order to study the properties of the \( Z^0 \) boson. The highest energy \( e^+ e^- \) collisions achieved to date were obtained at the LEP collider at CERN, where total centre of mass (com) energies of order 0.2 TeV were eventually reached. In order to achieve the LC physics programme discussed
below, com energies of order 1 TeV (the so-called ‘TeV scale’) will be necessary.

A particle with the electronic charge and mass $m$, moving relativistically with energy $E$ in a circle of radius $R$ radiates via synchrotron radiation a fraction of its energy which, per turn, is given by

$$\left(\frac{\Delta E}{E}\right)_{\text{synch}} = 88.5 \times \frac{[E(\text{TeV})]^3}{R(\text{km})^3} \left(\frac{m_e}{m}\right)^4,$$

(1)

where $m_e$ is the mass of the electron [4]. The maximum energy at a circular machine such as LEP is a balance between the maximum radio frequency (RF) power available and the energy losses due to synchrotron radiation.

While synchrotron radiation has many practical uses in the study of materials and reactions [5], the effect is problematic for high energy circular colliders. The LEP tunnel has a radius $\sim 4$ km (corresponding to a total tunnel length/circumference of 27 km) and so at its highest energy of 0.2 TeV the electrons were radiating 2% of their mean energy each turn, which had to be replaced continuously by accelerating structures located around the ring. A similar fractional energy loss at the TeV scale would require a ring of radius 500 km, which is clearly not practical. The only realistic approach to achieving very high energy collisions between electrons and positrons is effectively to turn off synchrotron radiation by accelerating the electrons in a straight line, equivalent to setting $R \to \infty$ in equation (1). This is the practical motivation to build the LC. It should be noted, however, that although the losses due to radiation at the LC are minimized, the power used to accelerate the particles is eventually lost because the particles are dumped after collision. The power requirements at the LC are still large, of order 200 MW, about equivalent to that used by a city of 200 000 people.

An alternative route to higher energies is that adopted for the LHC, which will use the same ring as that used previously for LEP. The LHC will accelerate protons in opposite directions around the ring and collide them with a total com energy of 14 TeV. This is possible because protons are $1.8 \times 10^3$ times heavier than electrons and so, due to the mass factor in equation (1), synchrotron radiation will still not be a problem. However, protons have their disadvantages because they are composite objects, being made up of quarks and gluons (a good introduction can be found in [6]), so not all the com energy is available for the fundamental collisions; the LHC can be considered as a quark-gluon collider rather than a proton one and it turns out that on average only about a tenth of the total energy is useful in collisions between protons. Occasionally, however, collisions with higher energy fractions do occur, but longer accelerator running times are needed to see them.

In contrast to protons, electrons are point-like objects and so all the beam energy is available in each event to produce new particles and interactions. As a result, the energy scales accessible to a TeV LC can approach those of the LHC. There are additional relative advantages of an LC; the particles emerging from an $e^+ e^-$ collision (collectively called an ‘event’) essentially all emerge from the fundamental underlying interaction whereas in proton–proton collisions there is usually a multitude of particles whose origins lie in the left-over proton constituents. These ‘spectator’ particles are not involved in the interesting underlying event and they tend to blur the picture. In addition, the backgrounds associated with proton colliders are higher than those at the LC so the LC is altogether a cleaner environment in which to observe complex events. More fundamentally, electrons and positrons couple only to electroweak forces whereas the quarks and gluons couple dominantly to the strong force so the two types of collider have complementary advantages in obtaining a complete picture of physics at the TeV scale. Further advantages of a LC are discussed in section 3.2.

In summary, the LHC should provide us with our first glimpses of any new physics that may exist at TeV energies, the LC will then allow us to home in on this new physics with very high precision studies of new states and interactions and to discover any remaining states which would otherwise remain hidden.

2. The accelerator
The task of the accelerator is to produce beams of electrons and positrons and to increase their energy from rest up to about 0.5 TeV in such a way that the beams can be focused down to very small sizes and be collided head-on with corresponding beams of positrons. As will be discussed in section 2.1, the small beam sizes are crucial in order to deliver sufficient numbers of interesting collisions to the detector. The main components required for this task, illustrated schematically in figure 1, are the particle sources, the damping rings, the main accelerator, and the collimation and final focus sections. Each of these systems is technically challenging and vital to the performance of the LC and they will now be discussed in some more detail.

2.1. Luminosity
The LC must deliver not only high energy, but also a large number of interesting physics events seen in the detector. Some processes of interest will be rare and so many $e^+ e^-$ collisions will be required in order that a sufficient number of them occur and are observed in the detector. The probability of occurrence is quantified in particle physics by a ‘cross-section’, $\sigma$, which has the physical dimensions of area, although it is not a real ‘area’ in the everyday sense of
the word. It can be thought of as the ‘effective’ area associated with each electron that gives rise to a given physical process during collisions with positrons, however, it really only quantifies conveniently the probability for that process to occur.

The LC does not accelerate continuous beams of electrons (or positrons) but rather accelerates collections of them, called ‘bunches’ and the bunches themselves are delivered one after the other in discrete sets called ‘trains’. The goal of the accelerator is to deliver lots of ‘luminosity’, $\mathcal{L}$, which as the name suggests is a measure of the intensity of the beams. If the efficiency of detection of a process in the detector is $\eta$ and the cross-section of that process is $\sigma$ then the number of events $N_{\text{obs}}$ observed in time $T$ is

$$N_{\text{obs}} = \sigma \eta \mathcal{L} T$$

(2)

Nature provides the value of $\sigma$ for any process, so this is a quantity to be measured. The task of the detector designers is to maximize the efficiency $\eta$ across the range of interesting processes, as described in section 4. The task of the accelerator builders is to maximize both $\mathcal{L}$ and $T$, the latter being the amount of time that the accelerator is actually running, which depends on its reliability. The useful lifetime of the accelerator, detector and experimental collaborations means that integrated $T$ is limited to something like half a decade. The critical quantity $\mathcal{L}$ is given by

$$\mathcal{L} = \frac{r}{4\pi} \frac{N_{e^+} N_{e^-}}{x_{\text{rms}} y_{\text{rms}}},$$

(3)

where $r$ is the rate at which the electron and positrons bunches collide, $N_{e^\pm}$ are the numbers of particles per bunch and $x(y)_{\text{rms}}$ is the rms width of the bunch in the $x(y)$ transverse direction. Traditionally, a right-handed coordinate system is assumed where $z$ is along the direction of motion of the bunch and $y$ is vertically upwards.

All the currently proposed LC projects aim at luminosities $\mathcal{L}$ of order a few $\times 10^{34}$ cm$^{-2}$ s$^{-1}$, or in units more convenient for particle physicists, a few hundred fb$^{-1}$ per year (1 fb$^{-1} = 10^{43}$ m$^{-2}$). This compares to the nominal LHC luminosity of $1 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ and is a factor 100 times greater than that at LEP, when it ran at 200 GeV com energy.

The accelerators can accommodate bunch occupancies $N_{e^\pm}$ of order a few $\times 10^9$ to a few $\times 10^{10}$ and average bunch crossing rates $r$ of order few $\times 10^4$. There are, however, significant differences between the various proposed projects as to exactly how the bunches are divided into pulse ‘trains’ and the technology used to achieve this.

The maximum luminosity must be extracted from each bunch before it is subsequently dumped and so equation (3) requires that the product of $x_{\text{rms}}$ and $y_{\text{rms}}$ must be made as small as possible. For the reasons discussed in section 2.5 it is highly beneficial to have a very flat transverse bunch profile so that $x_{\text{rms}}/y_{\text{rms}}$ is of order 100. As a result, the vertical bunch spot size $y_{\text{rms}}$ is only a few nanometres (or the width of a few tens of atoms!) in size.

2.2 Particle sources and damping rings

Polarized electrons are produced by firing a circularly polarized high-power laser onto a semiconductor photocathode. The electrons are ejected by the photo-electric effect and gain their polarization from the fact the original laser beam is polarized. The electrons are then rapidly separated from the photocathode by a high-voltage dc gun (the use of RF fields is also being explored). The timing of the ejected electrons is determined by when the laser is fired onto the photocathode, so this can be well timed-in to the accelerator bunch structure.

Positrons do not occur naturally in the laboratory because, being anti-particles to electrons, they immediately annihilate when they come into contact with any electrons via the process $e^+ e^- \rightarrow \gamma \gamma$. However, this process can be effectively reversed in the presence of matter to the pair-production process

$$\gamma + (\text{nucleus} + \gamma^*) \rightarrow e^+ e^- + (\text{recoiling nucleus}),$$

(4)

where the role of the nucleus is to provide a virtual photon $\gamma^*$ and to allow conservation of both energy and
momentum. For this to occur, the energy of the incident photon must be at least \(2mc^2\), or 1.02 MeV. After production, the positrons must be separated from their associated electrons by magnetic fields and they can then be accelerated and focused exactly as for electrons (but with opposite charge). The positrons do not annihilate inside the beam pipe because there is a very high vacuum there. The production of polarized positrons is much harder and is the subject of intense research and development worldwide.

Once a bunch of electrons (or positrons) has been thus produced, they will have a momentum spread which is random with respect to their mean value. This random internal motion makes it harder to focus the particle bunch down to very small sizes and a measure of this internal motion is called the ‘emittance’. The emittance associated with any transverse direction is the product of the spatial size of the bunch in that direction and the angular spread due to the components of random motion in that direction. According to Liouville’s famous theorem, in the absence of irreversible effects such as synchrotron radiation or nonlinear effects, the emittance will be constant along the length of the accelerator. It is thus important to start with a low bunch emittance and to keep it low all the way along the 15 km to the interaction point.

The initial bunch has its emittance reduced, or is ‘cooled’, by using a continuous combination of synchrotron radiation and re-acceleration. The synchrotron radiation is produced both by accelerating the bunches in a ring, called the ‘damping ring’ (see figure 1) and by the use of dedicated devices within the ring, called ‘wigglers’, that wiggle the particles about their forward direction using magnetic fields. The synchrotron radiation is emitted essentially parallel to the particles’ motion and so they slow down in this direction by conservation of energy. They are then re-accelerated in the forward direction by RF fields so as to regain their original energy but now their angular spread, and hence their emittance, is reduced. These steps are illustrated in figure 2. The purpose of the damping rings is thus to cool the beams so that they can be kept in tidy bunches throughout the main accelerator and beyond and subsequently focused to the very small sizes required to produce a high luminosity at the interaction point.

2.3. The main linac
The main linac does the vital job of accelerating the particles to their final collision energy which, for a given rate of acceleration, is determined primarily by the linac length. As a result, the main linac is the longest accelerator component and is one of the major cost drivers of the LC project. The JLC and NLC projects are both developing room-temperature normal conducting copper accelerating RF systems. The TESLA project is developing low temperature superconducting RF systems. The CLIC project is developing a novel two-beam approach where the electromagnetic fields originating from a set of lower-energy, higher intensity beams are used to accelerate a single high-energy, lower current beam; both beams using normal conducting technology. This idea is analogous to the operation of an electrical transformer where high current, low voltage can be transformed to low current, high voltage. The advantage of this route is that it could eventually lead to multi-TeV energies.

The normally conducting systems have the advantage that they do not need superconducting technology plus cryogenic systems and they can in principle achieve higher accelerating gradients. However, their smaller beam element sizes imply both the need for very tight tolerances on magnet positions and the possibility of large ‘wake fields’. Wake fields are electromagnetic fields which follow a charged particle bunch when it is travelling close to a conductor and these fields can disrupt the following bunches in the beam. The superconducting technology has the advantage that it is efficient in delivering power from the ‘wall-socket’ to the beam, has larger apertures (and hence smaller wake fields) and the lower RF frequency means that particle bunches are further apart and so can be steered into each other more easily using feedback techniques. The result of these advantages is the attractive prospect of very high luminosity. However, the maximum accelerating gradient is limited by the properties of the superconductor (pure niobium), because its superconductivity will be destroyed if the magnetic fields present in the accelerating structures are too strong.

Both super and normal conducting technologies are being actively and seriously considered and both would yield energies which could access the TeV scale. There is a great deal of collaboration between the various laboratories involved in the LC projects. Many of the problems are common to all the machine proposals and new technolo-
2.4. **Collimation and final focus**

Once the electron (and positron) bunches have been accelerated in the main accelerator there remains the task of cleaning up any stray particles that have for one reason or another left their parent bunch and may later head towards the detector. Such particles, if left unchecked, would cause unwanted signals in the detector and, in the worst cases, could even prevent the detector from turning on. These errant particles are scraped off by dedicated thin collimators placed very close to the path of the main bunch and are then stopped further downstream by thick absorbers.

Finally, the clean beam with low emittance bunches is focused down to the tiny spot sizes mentioned in section 2.1 by very high power quadrupole magnets that are situated very close to (or even inside, as shown in figure 4, section 2.5) the detector. The collimation and ‘final focus’ regions are also indicated schematically in figure 1. Achieving the small spot sizes and getting the nanometre-size bunches to collide is a technological challenge that requires very tight control of beamline component positions along the length of the accelerator and high speed feedback systems to steer the beams if they start to go astray. Even the tiny effects due to natural ground motion are very important and need to be corrected for in real time if the necessary luminosity is to be achieved. For example, the CLIC accelerator will need to have its final focusing magnets actively stabilized to less than a nanometre.

2.5. **Bunch–bunch interactions**

Although the nominal beam energies will be measured to a precision of one part in $10^{-4}$, there are additional effects influencing the energy of individual events. One process, already familiar at LEP and SLC energies, is initial state radiation. In this process, a photon is radiated off one of the incoming electrons or positrons before the fundamental $e^+e^-$ interaction occurs. This process leads to corrections to cross-sections that are calculable to high accuracy within the theory of quantum electrodynamics. The effect can be accounted for by replacing the beam energy by a spectrum of energies, peaked at the nominal beam energy, but extending also to lower energies.

In addition to the interactions between individual $e^+e^-$ pairs, the bunches as a whole will exert strong forces on each other due to the large total electric and magnetic fields from the constituent $10^{10}$ particles in the bunch. In the case of electron–positron collisions, these ‘bulk’ forces will be mostly attractive and result in a mutual contraction of the bunches as they pass through each other. A detailed simulation of the dynamics of the colliding bunches is shown in figure 3. This effect is important because the resulting contraction in cross-sectional area of the bunches leads to an increase in the total luminosity, according to equation (3), which can be as high as a factor $\sim 2$. This effect can, however, lead to a decrease in luminosity for the case of electron–electron collisions where the net interaction is repulsive.

The increase in luminosity from the bunch–bunch interactions does not come without a price. During the collision of the beams the forces on the electrons and positrons due to the electromagnetic fields in the bunch are very strong and result in them emitting electromagnetic radiation, called ‘beamstrahlung’ in analogy to the bremsstrahlung emitted when electrons pass through matter. One way to reduce the level of beamstrahlung is to use very flat bunches and this is the reason why the $y$ dimension of the bunches is of order 1 nm whereas the $x$ dimension is of order 100 nm (section 2.1).

There are two main effects of beamstrahlung. First, the energy remaining for the fundamental $e^+e^-$ interaction is lowered (in addition to initial state radiation effects) and so beamstrahlung results in a further widening of the effective electron/positron energy spectrum typically at the level of about 4% for the TeV-scale machines (the effect is much stronger for the multi-TeV CLIC parameters). The luminosity spectrum can however be reconstructed by looking at the process: $e^+e^-\rightarrow e^+e^-\gamma$beamstrahlung (similar to the radiative Bhabha process). Because of the presence of the beamstrahlung photon, the final state $e^+$ and $e^-$ will not be moving back to back with the same energy as they would in the absence of any radiative effects. By looking carefully at the spectrum of these ‘acollinear’ Bhabha events, the initial luminosity spectrum can be inferred.

Secondly, some of the beamstrahlung radiation appears in the form of $e^+e^-$ pairs which emerge in copious numbers from the interaction point and are a potential source of background in the detector, especially at low angles to the beam direction. This is displayed in figure 4 which shows the tracks of the pairs emerging from the interaction point. They move in helical paths due to the presence of a solenoidal magnetic field (the solenoid is shown within a candidate detector in figure 8, section 4) and the radial extent of these tracks sets the lowest angle at which sensitive detectors can be located. This issues are discussed further in section 4.

3. **The linear collider physics programme**

3.1. **Introduction**

The physics programme at the LC is very rich and wide ranging and there exist several comprehensive overviews [2, 3]. In this article attention is restricted to how the LC
will help our understanding of the mechanism by which the Standard Model (SM) particles get their mass and how the electroweak symmetry is broken (see [6] for an introduction to the SM). The key to this new understanding will be high precision measurements and the advantages of the LC in performing these measurements are first outlined in section 3.2. Understanding the mechanism of electroweak symmetry breaking is central to the LC physics programme and is one of the main motivations for building such a machine. A few key measurements related to electroweak symmetry breaking are outlined in section 3.3. Finally, a candidate theory for physics beyond the SM, namely supersymmetry,
is discussed in section 3.4, where some of the many possibilities for discoveries and precision measurements at the LC are described.

3.2. Precision measurements

As discussed in section 1.1, the low-background environment of the LC has several advantages over the LHC for observing and understanding complex events. The ability to set accurately the com energy of the event by varying the beam energy leads to the technique of threshold scans (section 3.2.1) where very accurate particle mass measurements can be made. Another very important property of the LC is that the beams can be polarized (section 3.2.2), allowing detailed investigation of the spin structure of the forces and the properties of new particles.

3.2.1. Threshold scans. An important feature of the LC hinges on the fact that the com energy of the collisional particles is tunable because it is given by twice the energy of the colliding beams. This allows very high precision measurements of particle masses by measuring the shape of the excitation curve for particle/anti-particle pair production. This is done by increasing the energy in steps across the pair production threshold and by measuring the magnitude of the cross-section as a function of com energy.

An important example of this is given by the measurement of the mass \( m_t \), of the top quark (or \( t \)-quark) which, with a measured mass of \( 174.3 \pm 5.1 \) GeV/c\(^2\) [7], is the heaviest of the quarks in the SM. Being the heaviest quark, its mass is expected to be the most sensitive to any deviations from the SM (section 3.3) and so a precision mass measurement is particularly important. For example, using precision measurements at LEP and SLC as input into calculations of radiative corrections leads to a prediction of the mass of the top quark of \( 170.6^{+11.4}_{-9.0} \) GeV/c\(^2\) [8]. The consistency of this value with the measured one places limits on any new physics that may contribute additional terms to the radiative corrections.

When the com energy is approximately \( 2m_t \), then top–anti-top pairs will be created with a rate which is sensitive to the com energy, as illustrated in figure 5. By fitting the theoretical excitation curve to the data points in this plot, the value of the top mass can be extracted. The expected precision is \( \delta m_t \sim 100 \) MeV/c\(^2\) and, at this level, is dominated by theoretical uncertainties. Constraining the strong coupling \( \alpha_s \) to the world average in the fit would give a top mass precision of order 40 MeV/c\(^2\), an order of magnitude improvement on the LHC value.

More information can be obtained from the shape of a threshold curve. For instance if the pair production mechanism proceeds via the exchange of a spin-1 boson (such as the photon or \( Z^0 \) boson in the case of \( t \bar{t} \) production) then, if additional orbital angular momentum is required in the final state in order to conserve angular momentum, there will be a suppression of the cross-section close to threshold. If the final state particles are spin-\( \frac{1}{2} \) then the cross-section will increase at threshold proportionally to \( \beta \), where \( \beta \) is the speed of the final state particles divided by the speed of light. Alternatively, if the particles are spin-0 (as are some of the supersymmetric particles discussed in section 3.4) then the excitation function increases as \( \beta^3 \), which is much less
steeply than for the spin-$\frac{3}{2}$ production (remember that $\beta$ is always less than one). This technique thus also gives information on the spin of the final state particles.

### 3.2.2. Polarization

The LC has a significant advantage in that it can provide polarized beams. Both the electron and the positron are spin-$\frac{3}{2}$ fermions and so can have their spins polarized with respect to their direction of motion. It is already well known that the SM treats fermions spinning in a left-handed sense differently from those spinning in a right-handed sense. In other words, the SM is a 'chiral' theory. At the LC, electron polarization of $\sim 80\%$ is planned and positron polarization possibly up to $\sim 60\%$ is also envisaged (there will, however, be some trade-off between luminosity and degree of positron polarization). The polarization can be measured to a precision of about 0.5%.

The ability at the LC to polarize the beams and to measure this polarization accurately can be used to explore the spin structure of fundamental couplings, to reduce systematic errors in precision measurements, to provide an effective luminosity gain in some channels such as the production of chiral supersymmetric particles (discussed in section 3.4) and, importantly, to reduce dramatically the backgrounds to some processes, particularly the production of pairs of $W$-bosons, which can only be produced by left-handed electrons (and right-handed positrons). The $W$-boson has a mass of 80 GeV/c$^2$ and is the particle responsible for the weak interaction, for instance the classic $\beta$-decay process that turns neutrons into protons: $n \rightarrow p e^- \bar{\nu}_e$. The $W^+ W^-$ pair production cross-section is large, being of order 1 pb at energies of interest to the LC. Because of decay channels containing neutrinos, such as $W^{-} \rightarrow \mu^- \bar{\nu}_\mu$, $W$-pair events often contain missing energy and momentum, attributable to the neutrino escaping the detector unseen. These cases can mimic those expected in new physics channels, such as supersymmetry (section 3.4) where missing energy and momentum is a typical signature. By polarizing the electrons in a right-handed sense (and as far as possible,
3.3. Electroweak symmetry breaking

Understanding electroweak symmetry breaking lies at the heart of the LC physics programme. In the SM, a particle gains its mass by interacting with a spin-zero field called the Higgs field $H(x)$. Taking as an example a fermionic field $f(x)$ (one such field is needed for every lepton and quark) then the part $L$ of the SM lagrangian, which eventually gives rise to the mass of the fermion, is

$$L = L_{\text{Higgs}} + g_l H(x) \bar{f}(x) f(x),$$

where $L_{\text{Higgs}}$ is the lagrangian relating to the Higgs field alone, $f(x)$ is the value of the field at position $x$ and $f(x)$ is related to its hermitian conjugate. The term $g_l$ is constant, called a Yukawa coupling, whose value is unique to each fermion type.

As it stands, equation (5) possesses all the symmetries of the SM and, in this manifestly symmetric form, all the particles are massless; there are no mass terms allowed by the electroweak (so-called ‘gauge’) symmetries of the form $m f(x)f(x)$, where $m_f$ is the mass of the particle whose field is $f$. At very high energies this is expected to be the case, but in the low energy world in which we live, such mass terms must clearly be present. A pre-requisite for these mass terms must therefore be that the gauge symmetries of equation (5) are broken. This is what is meant by the term electroweak symmetry breaking.

The mechanism of electroweak symmetry breaking is not yet experimentally tested. The SM picture is that the breaking is performed by the Higgs lagrangian, $L_{\text{Higgs}}$, in equation (5). In this picture the potential energy $V_H$ of the Higgs has the form

$$V_H = \frac{1}{2} \mu^2 H^2 + \frac{1}{4} \lambda H^4,$$  

where $\mu$ and $\lambda$ are constants at any given energy scale, but whose value will in fact depend on the energy scale. $\lambda$ must be positive so that the $V_H$ cannot get arbitrarily large and negative for large values of $H$. However, this constraint does not apply to $\mu$ and, if $\mu$ is negative, equation (6) can be rewritten as

$$V_H = \frac{\lambda}{4} \left( H^2 - \frac{\mu^2}{\lambda} \right)^2 + \text{const.}.$$  

The overall constant is neglected for this discussion. In this form, it is clear that the minimum of $V_H$ occurs, not at $H=0$ but rather at $H=v$, where

$$v = \left( \frac{\mu^2}{\lambda} \right)^{1/2}.$$  

This means that when $\mu^2$ is negative, the vacuum will contain everywhere a non-zero value, $v$, of the Higgs field. The vacuum is known to have no preferred direction and so the Higgs field itself can carry no spatial information, in other words, it has to be a scalar (spin-0) bosonic field.

To take account of this non-zero ‘vacuum expectation value’, $v$, we can rewrite $H(x) = v + h(x)$, where $h(x)$ represents small fluctuations of the total field $H(x)$. Inserting this into equation (7) we get

$$V_H \rightarrow V_h = \frac{\lambda}{4} (2vh + h^2)^2 + \text{const} \quad (9)$$

$$= \lambda v^2 h^2 + \lambda vh^3 + \frac{\lambda}{4} h^4 + \text{const}; \quad (10)$$

the first term of equation (10) can be identified as a mass term $\frac{1}{2} m_h^2 h^2$ for the new Higgs field $h$, where $m_h = (2\lambda v^2)^{1/2} = (2|\mu|^2)^{1/2}$.

The value of $v$ is known from the measured values of the $W^\pm$ and $Z^0$ boson masses to be $\sim 246$ GeV so the combination of $\mu$ and $\lambda$ in equation (8) is known. However, this is not enough to predict the mass of $m_h$, so this is still a free parameter of the SM that has to be measured.

As shown above, the Higgs particle couples to the masses of leptons and quarks and so will contribute, as a virtual particle, to the radiative corrections (introduced in section 3.2.1) of all the SM processes via so-called ‘loop diagrams’. The Higgs radiative corrections will be strongest in those processes involving the most massive particle, which in the SM is the top quark. By comparing precision electroweak measurements [10] to these theoretical calculations, an estimate of the Higgs mass can be obtained. As a result, the Higgs mass is expected to be less than about 200 GeV/c². Direct searches for the Higgs boson at LEP have failed to find it and instead have placed a lower limit on its mass of 113 GeV/c² [10]. So, if the Higgs exists, we are already close to finding it and we should be able to explore its properties very accurately at the LC.

An additional possibility for the LC, the so-called ‘GigaZ’ option, is to run it for a limited time at the lower com energy of 91 GeV/c² and to study in detail the $Z^0$-boson. This will require some modifications to the layout of the accelerator, which can, however, be built in from the beginning. While this energy region has already been explored at LEP and SLC, the very high luminosity (section 2.1) at the LC will provide measurements of the key electroweak parameters to typically an order of magnitude better precision. Some of these parameters are listed in table 1 and from such precision measurements the Higgs mass can be constrained indirectly, via its calculable quantum corrections to electroweak processes, to the level of 5%. Comparing this result to that obtained from direct mass measurements will provide a very powerful test of the Higgs sector.
Table 1. Precisions on electroweak parameters from running the LC at lower energies [2]. \( \sin^2 \theta_{\text{eff}} \) is a very important parameter that quantifies the mixing between the component fields making up the photon and the \( Z \)-boson. It also determines the relative masses \( M_Z \) and \( M_{\nu} \) of the \( Z \) and \( W \) bosons. \( \Delta \nu (M_Z) \) quantifies the strength of the strong interaction and \( N_C \) is the number of light neutrinos, which can also be taken as a measure of the number of particle generations under the assumption that all neutrinos are light. The remaining parameters in the table are also very important in detecting physics beyond the standard model via self-consistency tests and the reader is referred to [2] for further details.

<table>
<thead>
<tr>
<th>LEP/SLC/Tevatron</th>
<th>TESLA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin^2 \theta_{\text{eff}} )</td>
<td>0.23146 ± 0.00017</td>
</tr>
<tr>
<td>Line-shape observables</td>
<td></td>
</tr>
<tr>
<td>( M_Z )</td>
<td>91.1875 ± 0.0021 GeV/c(^2)</td>
</tr>
<tr>
<td>( \Delta \nu (M_Z) )</td>
<td>0.1183 ± 0.0027</td>
</tr>
<tr>
<td>( \Delta \nu \ell )</td>
<td>(0.55 ± 0.10) \times 10^{-2}</td>
</tr>
<tr>
<td>( N_C )</td>
<td>2.984 ± 0.008</td>
</tr>
<tr>
<td>Heavy flavours</td>
<td></td>
</tr>
<tr>
<td>( A_h )</td>
<td>0.898 ± 0.015</td>
</tr>
<tr>
<td>( R_h )</td>
<td>0.2165 ± 0.00069</td>
</tr>
<tr>
<td>( M_W )</td>
<td>80.436 ± 0.036 GeV/c(^2)</td>
</tr>
</tbody>
</table>

Turning to equation (5), we see that it can now be rewritten as

\[
L = L_{\text{Higgs}} + g_f v \bar{f}(x)f(x) + g_f h(x) \bar{f}(x)f(x) \tag{11}
\]

and so if we identify \( g_{\nu \nu} = m_f \), we see that a fermion mass term has been generated by the second term and subsequently that the last term can be rewritten as \( (m_f/v) h(x)f(x)f(x) \). In other words, the SM predicts that the strength of coupling of fermions \( f \) to the Higgs boson \( h \) is proportional to the mass of the fermion. So, as discussed in section 3.2.1, the top-quark will have the largest coupling and as a result may be most sensitive to the fine details of electroweak symmetry breaking.

Similarly discovering a candidate Higgs particle is not sufficient to prove this mechanism of electroweak symmetry breaking. It requires proving that the Higgs boson has spin-0 (from threshold scans) and has couplings to the standard model fermions that are proportional to the mass of the fermion. This can be done by measuring the relative proportions ('branching ratios') of Higgs decay channels for which the prospects are shown in figure 7 [2].

The Higgs boson can be extensively studied at the LC. Its mass can be measured to high precision, for example a 120 GeV/c\(^2\) standard model Higgs mass can be measured to a precision of 40 MeV/c\(^2\) by combining all channels. Even if the Higgs decays invisibly, for instance to the invisible lightest supersymmetric particles (section 3.4), then a method unique to the LC can be applied to measure its mass. This is because the Higgs can be produced by the reaction \( e^+e^- \to h^0Z^0 \) and, given that the com energy is known precisely, the \( h^0 \) mass can also be inferred by measuring the energy of the recoiling \( Z^0 \). A mass precision of 70 MeV/c\(^2\) can be achieved from using the \( Z^0 \)-recoil alone [2].

In addition, the Higgs tri-linear self-coupling given by \( \lambda_V = m_h^2/(2v) \) in equation (10) can be determined by measuring the cross-section for the process \( e^+e^- \to Z^0h^0h^0 \). This can result in a statistical precision on the self-coupling of about 22% if \( m_h = 120 \text{ GeV/c}^2 \)[2]. If the coupling turns out to be different from \( \lambda_V \) then this will indicate that new physics is present in defining the Higgs potential.

If the Higgs exists then it should be discovered at the LHC (or, if its mass is low enough, it may be possible to discover it sooner at the Fermilab Tevatron). As we discussed in this section, discovering the Higgs boson is only the first step to exploring the complete mechanism of electroweak symmetry breaking. It is not certain that the Higgs boson exists, but some mechanism that at least mimics its behaviour is required in order to explain the origin of mass for the SM particles. The LC will provide crucial precision measurements and discovery potential, complementary to that of the LHC, in this exciting field.

3.4. Supersymmetry

Supersymmetry (SUSY) has been proposed as a theoretical framework to provide answers to some of the fundamental questions that are not explained by the SM. For instance in the context of unified theories, SUSY can explain naturally why \( \mu^2 \) in equation (6) becomes negative at low energies, which is essential to the mechanism of electroweak symmetry breaking [11]. SUSY is a symmetry relating every particle state to a partner whose spin differs by half a unit and so every fermion has a bosonic sibling and vice versa. For example, the spin-\( \frac{1}{2} \) left-handed electron \( e_L \) will have a spin-0 partner the left-handed ‘selectron’ \( \tilde{e}_L \). Similarly the spin-1 gauge bosons will have spin-\( \frac{3}{2} \) partners, the ‘gauginos’. The theory thus immediately predicts a whole set of new scalar and fermionic states and provides a rich hunting ground for the LC. In addition to these new states, SUSY requires a set of five Higgs-like bosons, the lightest of which will be very similar to the \( h^0 \) of the SM, but the others may be much heavier. Two of these Higgs bosons are charged, \( H^\pm \), providing a rich and varied spectrum of Higgs states to explore. To date, no SUSY particles have been observed which means either that SUSY does not exist, or that the symmetry is in fact ‘broken’ so that the SUSY particles are heavier and have just not yet been seen. As discussed in section 1.1 the search for such new particles is one of the motivations for higher energy.

Fundamental scalars play key roles in our current description of the vacuum, ranging from the generation
of mass for the SM particles (section 3.3) to providing the mechanism for inflation in the early universe. However, to date Nature has not revealed any such scalars and whether they exist or not is a fundamental question for our generation to answer. Currently the most sought after fundamental scalar is the Higgs boson (section 3.3) whose mass, as explained above, is expected to be less than about 200 GeV/$c^2$ and greater than 113 GeV/$c^2$. In the absence of SUSY there is no symmetry to keep the Higgs light and so quantum effects (radiative corrections via ‘loop diagrams’) would tend to result in it being very heavy; in fact its mass would tend towards the scale where new physics appears, possibly at the GUT scale, of order $10^{16}$ GeV. There are already fermionic chiral symmetries present in the SM that naturally protect the fermions from such quantum corrections and so SUSY, by introducing a symmetry between the chiral fermions and their bosonic partners, also protects the bosonic masses. Put another way, the quantum corrections arising from the fermions are cancelled by those from their bosonic partners. In this manner, the lightness of the Higgs can be explained naturally by SUSY and the upper bound on its mass within SUSY models lies at about 135 GeV/$c^2$. Discovering a light Higgs would thus be the first necessary (but not sufficient) condition for the existence of SUSY.

In order for this mechanism to work, however, the SUSY particles (‘sparticles’) must not be very much heavier (less than a TeV/$c^2$ or so) than their SM partners. In the case that SUSY is realized in nature we can expect a whole new set of sparticles to appear at the LC and the LHC. The LHC is particularly suited to studying the SUSY partners of the quarks and gluons, whereas the LC is ideally suited for the sleptons and electroweak gauginos. If SUSY is present, it will dominate the physics programmes at both machines and the interplay between LC and LHC analyses provides a strong argument for some overlap in their running schedules.

3.4.1. Verifying SUSY. In the simplest SUSY theories, the lightest SUSY particle (LSP) is expected to be stable and may provide much of the dark matter in our universe. If the mass scale of SUSY is light enough for it to be observed at the LC then the LSP mass can be measured there typically to better than 1%, providing a crucial fundamental constant for cosmology. Such a mass measurement would also benefit greatly the analysis of the data from the LHC because the LSP is always present in SUSY events and its mass cannot be measured with such precision at the LHC alone.

Typical signatures for SUSY, at least in those models where the LSP is stable, involve missing energy and momentum in the final state due to the weakly interacting LSPs escaping the detector. An example is provided by smuon ($\tilde{\mu}$) production, where smuons are the scalar partners of the muon. There are two such states $\tilde{\mu}_R$ and $\tilde{\mu}_L$; one for each muon chirality. Consider the production process $e^+ e^- \rightarrow \tilde{\mu}_R \tilde{\mu}_R$ where the smuons subsequently decay to muon plus LSP. The final state thus consists of two muons which are not moving back-to-back plus missing energy. The appearance of such events would provide strong circumstantial evidence for the existence of SUSY, however, proving the existence of SUSY requires more than this. It requires measuring the spins of the sparticles and demonstrating that they differ by half a unit from those of their SM partners. It also requires measuring the couplings of the sparticles to the SM gauge bosons and demonstrating that they are identical, up to quantum corrections, to the corresponding SM couplings.

The spin of a sparticle can be inferred from the angular distributions of the particles into which they decay. For instance spin-0 particles have no preferred axis and so they can only decay isotropically. In addition the spin can also be determined from the shape of the threshold cross-section curve (section 3.2.1). However, one of the main motivations for a threshold scan is that it would provide a measurement of the smuon mass to a precision of 0.1%. Unfortunately it is not guaranteed that the masses of any of the sparticles will be low enough for their pair production at LC energies, the mass limit for charged sparticles being given essentially by half the com energy of the accelerator—or half a TeV/$c^2$. However, an interesting range of SUSY models predict that at least some of the sparticles will be light enough so that these measurements can be made. Such high precision measurements, combined with those from the LHC, would provide an excellent determination of the sparticle spectrum at the TeV scale. The LHC is ideally suited to detect the supersymmetric partners of the quarks and masses in excess of 1 TeV/$c^2$ should be accessible. Armed with this combined knowledge, very accurate extrapolations of SUSY theories could then be made to ultra high energy scales ($\sim 10^{16}$ GeV) where gravity may play an important role in particle physics.

At the start of LC running, the machine would run at the highest available energy to produce as many new states as possible and to determine their masses using ‘end-point’ kinematics. This method uses as a constraint the com energy determined from the known energies of the incoming beams (a major advantage of an $e^+ e^-$ collider) and then determines the masses of both the parent and daughter particles from the end-points of the visible particle spectrum.

The end-point measurement is possible because the energy $E$ of each of the parent particles is one half the (known) total com energy and so their relativistic boost $\gamma_{rel} = E/(m_{parent}c^2)$ is a function of their (unknown) mass only. In the rest frame of the parent particle, the total energy is given by $m_{parent}c^2$ and so the energy of the daughter particles in this frame is a function of the daughter masses and the mass of the parent only. Given
the additional fact that the spin-0 smuon will decay isotropically in its own rest frame, the distribution of the energy of the final state particles must be flat, with endpoints being a function of the parent and daughter masses only. An example of this measurement is shown in figure 6. Using this method an accuracy of typically 1% (or better) can be achieved.

After the initial running at maximum energy, threshold scans would then be performed on selected sparticles to increase their mass measurement accuracy to the 0.1% level. Such scans are particularly appropriate for the spin- \( \frac{3}{2} \) gauginos because their threshold excitation curves turn on as \( \beta \) (defined in section 3.2.1), a function which is much steeper than the \( \beta^2 \) dependence of the scalar thresholds. As we saw above, these selective threshold scans will form a major part of the LC programme, even in the absence of SUSY, for example to determine the top quark mass to high precision or to measure the spin of the Higgs boson.

It will also be necessary to run the LC with different polarizations of the electrons (and if possible the positrons too). This is important in order to measure the spin-dependent couplings, which are at the heart of the structure of the theory.

4. The detector

Excellent detector performance is vital to the LC physics programme and much study is going on worldwide [1] to explore various detector concepts and to test the performance with detailed simulations. One such concept, that proposed in the TESLA TDR [2], is shown in figure 8. This detector is approximately 15 m high by 15 m long, weighs 10 kt and has \( 9 \times 10^8 \) electronic readout channels.

Unlike the case of an accelerator ring such as LEP, the electron and positron bunches cannot be used after collision because at the LC the bunches are so disrupted by the collision process (figure 3) that they can no longer be re-focused afterwards. Instead the bunches must be simply dumped after collision and this means that no luminosity gain can be made by having more than one collision point. At LEP four detectors were placed around the ring, which gave an effective multiplication in the total collected luminosity as well as the scientific benefit of cross-checking and competition between four separate collaborations of scientists. While the first gain is not to be had at the LC, there may indeed be two collision points placed side by side at the end of the accelerator, with detectors specialized for somewhat different physics processes, for instance one may specialize in two-photon collisions as discussed briefly in section 5.2. In figure 1 only one such detector is shown.

The detector must be able to reconstruct events with large numbers of particle tracks and also to measure precisely high-energy single particles. The detector must surround the interaction point as completely as possible to avoid letting any particles escape undetected. The TESLA design [2] allows energy measurements down to about 27 mrad.

The low angle region, illustrated in figure 4, is important for a range of physics processes; these include multi-particle events where some particles will be in the forward direction, SUSY processes (especially when the mass differences between the particles are small), and also for measuring (or vetoing) the large numbers of ‘\( \gamma \gamma \)’ events. \( \gamma \gamma \) events are collisions between the photons that are associated with the incoming electrons and positrons. The total com energy in these events is not known precisely and the events themselves may be boosted along the beamline axis because the two photons do not generally have the same momentum. As a result, the \( \gamma \gamma \) events appear to have missing energy and momentum and so can mimic some of the signatures for SUSY. The TESLA detector will allow high energy electrons/positrons to be detected without accurate measurement (or ‘tagged’) down to about 4 mrad, where beam-related backgrounds are becoming very high. Such tagging is important because it helps classify the event; for example a tagged \( \gamma \gamma \) event can be distinguished from a similar SUSY event, where no tagged electron or positron would be expected.

As discussed in section 2.5 tracking at low angles is also important in order to measure the com energy spectrum using acollinear Bhabha events. The detector design at low angles, and in the vertex detector region, is constrained by the presence of large numbers of \( e^+ e^- \) pairs (shown red in figure 4) produced by the beamstrahlung photons. A higher magnetic field is important to contain these particles at low-angle trajectories and a 4T detector field is presently being considered [2].

In a typical multi-particle event, the individual particle energies are typically of order 1–2 GeV, so the quality of event reconstruction hinges on the ability to measure very precisely the positions, directions and energies of such lower-energy particles. The process whereby individual particles are reconstructed by combining information optimally from both tracking and calorimetry is called ‘energy flow’ and the goal performance is set at an energy flow resolution \( \delta E(\text{GeV}) \approx 0.3(E(\text{GeV}))^{1/2} \), a factor of two improvement on that achieved at LEP/SLC. This performance will require very high resolution and high granularity detectors.

In addition to measuring well the lower momentum tracks, it is also important to be able to make an excellent measurement of high momentum tracks, for instance to reconstruct the Higgs mass from recoil against the \( Z^0 \) boson (section 3.3). Tracks and particle decay vertices need to be reconstructed very well so that the particle types can be identified. This is essential to a wide range of physics processes, including measuring the branching ratios of the Higgs boson(s) and reconstructing the complicated decay.
chains of SUSY events. A high performance ‘vertex detector’ (which can be seen surrounding the interaction point in figure 4) is also essential to help with pattern recognition in multi-particle events and to provide additional high-precision spatial measurements which allow the curvature of the tracks to be determined accurately. The tracks of charged particles curve due to the presence of a magnetic field supplied by the large superconducting solenoid shown in figure 8 and the radius of curvature is inversely proportional to the particle (transverse) momentum.

The demands on the detector for the LC are stringent, but seem to be readily achievable on the time scales required for the LC programme. Worldwide, groups are already forming to study in detail the various detector options and to ensure that the excellent physics potential at the LC is fulfilled.

5. Other LC modes of operation

The majority of the luminosity at the LC will be spent in the $e^+e^-$ mode. However, other modes of operation offer distinct advantages in a variety of physics processes. The additional modes are $e^-e^-$, $e^-\gamma$ and $\gamma\gamma$ and these are now discussed in turn.

5.1. Electron-electron collisions

Running in $e^-e^-$ mode requires relatively little modification of the accelerator and in this sense comes ‘for free’. One cost is a reduction in the total available luminosity because the two beams have the same charge and so do not benefit from the bunch–bunch interaction described in section 2.5. Despite this loss, there are some distinct advantages of running in this mode, especially in the field of searches for new physics, such as SUSY.

As an example, this mode will allow an excellent measurement of the mass of the SUSY partner to the left-handed electron, $\tilde{e}_L$, via a threshold scan because its threshold varies as $\beta$ as opposed to the $\beta^2$ of the $e^+e^-$ mode. This production process also allows the interesting search for mixing in the slepton sector by searching for muons in the final state, indicating the decay $\tilde{e}^-\rightarrow\mu^-+\text{LSP}$.

5.2. Photon-photon and photon-electron collisions

Another set of options for the LC involves the production of very high energy photons produced by the Compton process $\gamma e^-\rightarrow\gamma e^-$ where the initial photon is produced by a high-power laser beam focused onto the incoming electron beam. After the Compton collision, the outgoing photon will have an energy belonging to a spectrum that extends up to $\sim 80\%$ of the original electron beam energy.

Operating in the $e\gamma$ mode could open up the process $e\gamma\rightarrow\tilde{e}^-+\text{LSP}$ and so extend the energy reach of the machine to heavier selectrons, should they be too heavy to produce in pair-production mode.

The LC $\gamma\gamma$ mode allows for single Higgs production which would be useful to measure the $h^0\gamma\gamma$ coupling and also possibly allow the production of the heavier SUSY Higgs particles, which may be too heavy to produce in the $e^-e^-$ mode, where in general more than one particle has to be created. In addition the $\gamma\gamma$ mode would allow the pair production of sparticles as a pure quantum electromagnetic process, leading to simplifications in the extraction of relevant SUSY parameters. The $\gamma\gamma$ mode would also allow precision measurements of the structure of the photon, which is of high interest to the study of the strong interaction in its own right as well as providing a means to quantify more accurately the $\gamma\gamma$ backgrounds to other processes.

The production of these high energy photons requires very sophisticated, high power laser systems. The laser beams have to be brought through the detector and focused onto the incoming electron beam. Both the layouts of the input beams and the extracted beams have to be carefully designed. These issues are now being looked at actively and much research and development will be required before an engineering design will be ready.

6. Summary

Both the LHC and the LC will be needed in order to explore fully the TeV scale. Some states that are invisible at the LHC, such as an invisible Higgs, would be both discovered and have their properties measured at the LC. Other states that may be discovered at the LHC will, if their mass is not too high, be studied with extremely high precision at the LC. The benefits apply in both directions; for instance if SUSY is realized in nature then the LHC will almost certainly discover it and will tell the LC where to concentrate its threshold scans. This complementarity would clearly be greatly beneficial to both programmes and is a strong argument for a few years of concurrent running of the LHC and the LC.

The LHC will provide us with our first glimpses of any new physics at the TeV scale. The LC would complete the picture with higher resolution and complementary discovery channels. Worldwide, the race is on to build the LC and it will necessarily be an internationally funded project (costing several billions of dollars) with all the regions playing significant roles. The next decade will be a very exciting period in which results from the LHC start to emerge and construction of the LC approaches completion; textbooks with new visions of physics at the TeV scale and beyond will soon be written.
References

[1] A central website that contains links to all the various worldwide studies is http://lcwws.physics.yale.edu/lc/.


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