PHYS851 Quantum Mechanics I, Fall 2009
HOMEWORK ASSIGNMENT 12

Topics Covered: Motion in a central potential, spherical harmonic oscillator, hydrogen atom, orbital electric and magnetic dipole moments

1. [20 pts] A particle of mass M and charge q is constrained to move in a circle of radius r_0 in the $x-y$ plane.

   (a) If no forces other than the forces of constraint act on the particle, what are the energy levels and corresponding wavefunctions?

   (b) A uniform, weak magnetic field of amplitude B_0 is applied along the z-axis. What are the new energy eigenvalues and corresponding wavefunctions?

   (c) Instead of a weak magnetic field along the z-axis, a uniform electric field of magnitude E_0 is applied along the x-axis. Find an approximation for the low-lying energy levels that is valid in the limit $qr_0E_0 \gg \hbar^2/Mr_0^2$.
   Hint: try expanding around the potential about a stable equilibrium point.

2. [10 pts] Write out the fully-normalized hydrogen wavefunctions for all of the 3p orbitals. Expand out any special functions in terms of elementary functions. You can look these up in a book or on-line, but keep in mind that you will be penalized if your expression is not properly normalized.

3. [20 pts] Numerically compute the matrix elements of the z-component of the orbital electric and magnetic dipole moments for the $|200\rangle \to |100\rangle$, $|210\rangle \to |100\rangle$, and $|211\rangle \to |100\rangle$ transitions in hydrogen. Be sure to show your work.

4. [15 pts] Based on the classical relation $E = T+V$, where E is the total energy, T is the kinetic energy, and V is the potential energy, what is the probability that the velocity of the relative coordinate exceeds the speed of light for a hydrogen atom in the 1s state? What about the 2s state? Based on these answers, which of the two energy levels would you expect to have a larger relativistic correction?

5. [10 pts] Consider the Earth-Moon system as a gravitational analog to the hydrogen atom. What is the effective Bohr radius (give both the formula and the numerical value). Based on the classical energy and angular momentum, estimate the n and m quantum numbers for the relative motion (take the z-axis as perpendicular to the orbital plane).