PHYS851 Quantum Mechanics I, Fall 2009

HOMEWORK ASSIGNMENT 3:
Fundamentals of Quantum Mechanics

1. [10pts] The trace of an operator is defined as $Tr\{A\} = \sum_m \langle m|A|m\rangle$, where $\{|m\rangle\}$ is a suitable basis set.

   (a) Prove that the trace is independent of the choice of basis.

   (b) Prove the linearity of the trace operation by proving $Tr\{aA + bB\} = aTr\{A\} + bTr\{B\}$.

   (c) Prove the cyclic property of the trace by proving $Tr\{ABC\} = Tr\{BCA\} = Tr\{CAB\}$.

2. Consider the system with three physical states $\{|1\rangle, |2\rangle, |3\rangle\}$. In this basis, the Hamiltonian matrix is:

$$H = \begin{pmatrix} 1 & 2i & 1 \\ -2i & 2 & -2i \\ 1 & 2i & 1 \end{pmatrix}$$

   Find the eigenvalues $\{\omega_1, \omega_2, \omega_3\}$ and eigenvectors $\{|\omega_1\rangle, |\omega_2\rangle, |\omega_3\rangle\}$ of H. Assume that the initial state of the system is $|\psi(0)\rangle = |1\rangle$. Find the three components $\langle 1|\psi(t)\rangle$, $\langle 2|\psi(t)\rangle$, and $\langle 3|\psi(t)\rangle$. Give all of your answers in proper Dirac notation.

3. Cohen-Tannoudji: pp 203-206: problems 2.2, 2.6, 2.7

4. Cohen-Tannoudji ;pp341-350: problem 3.14