PHYS851 Quantum Mechanics I, Fall 2009
HOMEWORK ASSIGNMENT 9

Topics Covered: parity operator, coherent states, tensor product spaces.

Some Key Concepts: unitary transformations, even/odd functions, creation/annihilation operators, displaced vacuum states, displacement operator, tensor-product states, tensor-product operators, Schmidt decomposition, configuration space.

1. **The Parity Operator:** [20 pts] Determine the matrix element $\langle x|\Pi|x'\rangle$ and use it to simplify the identity $\Pi = \int dx\, dx'\, |x\rangle\langle x|\Pi|x'\rangle\langle x'|$, then use this identity to compute Π^2, Π^3, and Π^n.

   From these results find an expression for $S(u) = \frac{\exp[\Pi u]}{\cosh u}$ in the form $f(u) + g(u)\Pi$.

   What is $\langle x|S(u)|\psi\rangle$? Express your answer in terms of $\psi_{even}(x) = \frac{1}{2}(\psi(x) + \psi(-x))$ and $\psi_{odd}(x) = \frac{1}{2}(\psi(x) - \psi(-x))$.

   Compute $\langle x|S(0)|\psi\rangle$, $\lim_{u\to\infty}\langle x|S(u)|\psi\rangle$, and $\lim_{u\to-\infty}\langle x|S(u)|\psi\rangle$.

2. [15 pts]The coherent state $|\alpha\rangle$ is defined by $|\alpha\rangle = e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$, where the states $\{|n\rangle\}$ are the harmonic oscillator energy eigenstates.

   First, show that for $\alpha = 0$, the coherent state $|\alpha{=}0\rangle$ is exactly equal to the harmonic oscillator ground-state, $|n{=}0\rangle$.

   Then show that any other coherent state can be created by acting on the ground-state, $|0\rangle$, with the 'displacement operator' $D(\alpha)$, i.e. show that $|\alpha\rangle = D(\alpha)|0\rangle$, where

   $$D(\alpha) := e^{\alpha A^\dagger - \alpha^* A} \tag{1}$$

   You may need the Zassenhaus formula $e^{B+C} = e^B e^C e^{-[B,C]/2}$, which is valid only when $[B,[B,C]] = [C,[B,C]] = 0$.

   What is $D(\alpha_2)|\alpha_1\rangle$?

3. [15 pts] Consider a system described by the Hamiltonian $H = \hbar\kappa(A + A^\dagger)$. Use your results from the previous problem to determine $|\psi(t)\rangle$ for a system initially in the ground-state, $|\psi(0)\rangle = |0\rangle$.

4. [10pts each] Cohen Tannoudji, pp341-350: problems 3.6, 3.7, 3.11