

PHYS851 Quantum Mechanics I, Fall 2009  
Semester Outline

1. Dirac Notation

- (a) Bras and Kets
  - i. Hermitian conjugation
- (b) Operators
  - i. Hermitian conjugation of operators
  - ii. Properties of Hermitian operators
  - iii. Properties of Unitary operators
- (c) c-numbers
  - i. The inner product
  - ii. Matrix elements
- (d) Hilbert Spaces
  - i. Discrete indices
    - A. Summation over indices
    - B. Orthonormality via Kronecker delta
    - C. Vector/Matrix and Tensor representations
  - ii. Continuous indices
    - A. Integration over indices
    - B. Orthonormality via Dirac delta
    - C. Single- and multi-variable function representations

2. Quantum Postulates

- (a) Closed system described by state vector in Hilbert space
- (b) Observables described by Hermitian operators
- (c) Evolution of state vector governed by Schrödinger's equation
- (d) Measurement Postulates
  - i. Possible results of a single measurement
  - ii. Probabilities associated with possible results
  - iii. Wave function 'collaps' occurs immediately after result obtained

3. Rabi Model
  - (a) Two-level approximation
  - (b) Generic Rabi Hamiltonian
    - i. Mapping for sinusoidally driven systems
    - ii. Eigenvalues and eigenvectors
  - (c) The avoided crossing
    - i. Energy gap
    - ii. Adiabatic Theorem
    - iii. Sudden Approximation
4. Motion in 1D
  - (a) X and P
  - (b) Wavepacket dynamics
    - i. Delay time and spreading velocity
    - ii. as classical limit of QM
    - iii. Heisenberg equations for expectation values
  - (c) Infinite square well
  - (d) Scattering in 1d
    - i. Boundary conditions
    - ii. Probability current
      - A. Dependence of  $T$  on  $t$
    - iii. Potential step scattering
    - iv. delta-function scattering
      - A. Trick to derive boundary condition on  $\psi'$
5. Quantum Harmonic Oscillator
  - (a) Dimensionless variables
    - i. Natural Units
    - ii. Energy scale
    - iii. Oscillator Length
  - (b) Raising and lowering operators
  - (c) Energy eigenstates
    - i. Number operator
    - ii. Energy eigenvalues
    - iii. Eigenstate wavefunctions
6. Random Topics
  - (a) Parity Operator
  - (b) Uncertainty Principle

7. Quantum Theory of Angular Momentum
  - (a) Angular and radial Hilbert spaces
  - (b) Angular momentum commutation relations
  - (c)  $L_+$  and  $L_-$
  - (d) Simultaneous eigenstates of  $L^2$  and  $L_z$
  - (e) Allowed values of  $\ell$  and  $m$
  - (f) Motion on a ring:  $L_z^2$
  - (g) Motion on a sphere:  $L^2$
8. Motion in a Central Potential
  - (a) Conservation of angular momentum
  - (b) Radial wave equation
  - (c) Principle quantum number
  - (d) Spherical harmonic oscillator
9. Quantum two-body problem
  - (a) Separation of relative and center-of-mass motions
  - (b) Hydrogen atom
  - (c) Energy levels
  - (d) Energy level diagram
  - (e) Orbitals: 1s, 2p, etc...
  - (f) Allowed values of  $n$ ,  $\ell$ , and  $m_\ell$
  - (g) degeneracies
10. Hamiltonian of a charged particle
  - (a) Minimal coupling Hamiltonian
  - (b) Uniform B field
    - i. Orbital magnetic dipole moment
    - ii. Paramagnetic term
    - iii. Diamagnetic term
    - iv. Weak-field approximation
  - (c) Hydrogen atom in uniform magnetic field
  - (d) Electric dipole transitions in Hydrogen
    - i. Electric dipole Hamiltonian
    - ii. Selection rules
11. Spin
  - (a) Simultaneous eigenstates of  $S^2$  and  $S_z$
  - (b) Allowed values of  $s$  and  $m_s$
  - (c) Spin magnetic dipole Hamiltonian
  - (d) Effects on Hydrogen atom level splittings and degeneracies