```
In[2]:=  (* This is how you make a comment in Mathematica *)

In[3]:=  (* In mathematica, you type a formula,
          then hit [shift]+[enter] to have mathematica evaluate the expression  *)

In[4]:=  2 + 2 (* [shift] + [enter] *)

Out[4]=  4

In[5]:=  (* you can enter several expressions, and then evaluate them all at once *)
         (* use [return] to start a new line for each command *)
         a = 3
          b = 7
          c = a + b

Out[5]=  210

In[6]:=  (* mathematica will evaluate an
           expression quietly if you put a semicolon at the end *)
         a = 4;
         b = 9;
         c = a + b

Out[8]=  13

In[9]:=  (* to unset the value of a variable use "Clear" *)

In[10]:=  a

Out[10]=  4

In[11]:=  Clear[a]

In[12]:=  a

Out[12]=  a

In[13]:=  b
          c

Out[13]=  9

Out[14]=  13

In[15]:=  Clear[b, c]

In[16]:=  b
          c

Out[16]=  b

Out[17]=  c
```

```
In[18]:= (* Built-in functions start with a capital letter. Square
            brackets enclose the arguments passed to a function *)
         Sin[2.0]
         Exp[8.1]

Out[18]= 0.909297

Out[19]= 3294.47

In[20]:= (* Defining a function *)
         f[x_] := Exp[-x^2]
         g[x_, y_] := Sin[Sqrt[x^2 + y^2]]

In[22]:= f[x]
         g[a, b]
```

Out[22]= $e^{-x^2}$

Out[23]= $\mathrm{Sin}\left[\sqrt{a^2+b^2}\right]$

```
In[24]:= f[2]
         g[3, z]
```

Out[24]= $\frac{1}{e^4}$

Out[25]= $\mathrm{Sin}\left[\sqrt{9+z^2}\right]$

```
In[26]:= (* Use a decimal point on at least one argument to get a numerical output *)
         f[2.]
         g[3., 2]

Out[26]= 0.0183156

Out[27]= -0.447492

In[28]:= (* Some characters have special meanings *)
         Log[E]
         I^2
         Sin[π]

Out[28]= 1

Out[29]= -1

Out[30]= 0

In[31]:= (* Mathematica is case-sensitive *)
         Log[e]
         i^2
         Sin[Π]

Out[31]= Log[e]
```

Out[32]= $i^2$

```
Out[33]= Sin[Π]
```

In[34]:=
```
(* For powers use [^] *)
3^2
3.^-1
```

Out[34]= 9

Out[35]= 0.333333

In[36]:=
```
(* For multiplication use [*] or a space *)
2*3
2 3
```

Out[36]= 6

Out[37]= 6

In[38]:=
```
(* For division use [/] or [ctrl]+[/] *)
6/2
```
$\frac{6}{2}$

Out[38]= 3

Out[39]= 3

In[40]:=
```
(* Differentiation *)
```

In[41]:=
```
D[f[x]]
D[f[x], {x, 2}]
D[g[x, y], {y, 1}]
D[g[x, y], {x, 1}, {y, 1}]
```

Out[41]= $e^{-x^2}$

Out[42]= $-2 e^{-x^2} + 4 e^{-x^2} x^2$

Out[43]= $\frac{y \operatorname{Cos}\left[\sqrt{x^2 + y^2}\right]}{\sqrt{x^2 + y^2}}$

Out[44]= $-\frac{x\, y \operatorname{Cos}\left[\sqrt{x^2 + y^2}\right]}{(x^2 + y^2)^{3/2}} - \frac{x\, y \operatorname{Sin}\left[\sqrt{x^2 + y^2}\right]}{x^2 + y^2}$

In[45]:=
```
(* Integration *)
```

In[46]:=
```
Integrate[f[x], x]
Integrate[f[x], {x, -∞, ∞}]
```

Out[46]= $\frac{1}{2} \sqrt{\pi} \operatorname{Erf}[x]$

Out[47]= $\sqrt{\pi}$

In[48]:=
```
(* to make special characters, use [Esc] then [text] then [Esc] *)
(* [Esc][p][Esc] gives π *)
(* [Esc][Y][Esc] gives Ψ *)
(* [Esc][inf][Esc] gives ∞ *)
```

```
In[49]:= (* Define a 2x2 matrix *)
         m = {{1, 2}, {3, 4}}

Out[49]= {{1, 2}, {3, 4}}

In[50]:= MatrixForm[m]

Out[50]//MatrixForm=
```

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

```
In[51]:= (* extracting elements *)

In[52]:= m[[1]]

Out[52]= {1, 2}

In[53]:= m[[2]]

Out[53]= {3, 4}

In[54]:= m[[1, 1]]

Out[54]= 1

In[55]:= m[[2, 1]]

Out[55]= 3

In[56]:= (* define a 2-component vector *)
         x = {a, b}

Out[56]= {a, b}

In[57]:= MatrixForm[x]

Out[57]//MatrixForm=
```

$$\begin{pmatrix} a \\ b \end{pmatrix}$$

```
In[58]:= (* MatrixMultiplication *)
         y = m.x;
         MatrixForm[y]

Out[59]//MatrixForm=
```

$$\begin{pmatrix} a + 2 b \\ 3 a + 4 b \end{pmatrix}$$

```
In[60]:= (* Finding eigenvalues and eigenvectors *)
         y = Eigensystem[m]
```

Out[60]= $\left\{\left\{\frac{1}{2}\left(5+\sqrt{33}\right), \frac{1}{2}\left(5-\sqrt{33}\right)\right\}, \left\{\left\{-\frac{4}{3}+\frac{1}{6}\left(5+\sqrt{33}\right), 1\right\}, \left\{-\frac{4}{3}+\frac{1}{6}\left(5-\sqrt{33}\right), 1\right\}\right\}\right\}$

```
In[61]:= (* to extract the eigenvalues *)
         ω1 = y[[1, 1]]
         ω2 = y[[1, 2]]
```

Out[61]= $\frac{1}{2}\,(5+\sqrt{33})$

Out[62]= $\frac{1}{2}\,(5-\sqrt{33})$

```
In[63]:= (* to extract the eigenvectors *)
         v1 = y[[2, 1]]
         v2 = y[[2, 2]]
```

Out[63]= $\left\{-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33}),\ 1\right\}$

Out[64]= $\left\{-\frac{4}{3}+\frac{1}{6}\,(5-\sqrt{33}),\ 1\right\}$

```
In[65]:= (* normalize these vectors *)
```

In[66]:= $\mathbf{e1} = \frac{\mathbf{v1}}{\mathbf{Sqrt[v1.v1]}}$

$\mathbf{e2} = \frac{\mathbf{v2}}{\mathbf{Sqrt[v2.v2]}}$

Out[66]= $\left\{\frac{-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33})}{\sqrt{1+(-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33}))^2}},\ \frac{1}{\sqrt{1+(-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33}))^2}}\right\}$

Out[67]= $\left\{\frac{-\frac{4}{3}+\frac{1}{6}\,(5-\sqrt{33})}{\sqrt{1+(-\frac{4}{3}+\frac{1}{6}\,(5-\sqrt{33}))^2}},\ \frac{1}{\sqrt{1+(-\frac{4}{3}+\frac{1}{6}\,(5-\sqrt{33}))^2}}\right\}$

```
In[68]:= (* to evaluate numerically *)
         N[e1]
```

Out[68]= {0.415974, 0.909377}

```
In[69]:= (* verify results *)
         (m - ω1 IdentityMatrix[2]).v1
```

Out[69]= $\left\{2+\left(1+\frac{1}{2}\,(-5-\sqrt{33})\right)\left(-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33})\right),\ 4+\frac{1}{2}\,(-5-\sqrt{33})+3\left(-\frac{4}{3}+\frac{1}{6}\,(5+\sqrt{33})\right)\right\}$

```
In[70]:= (* we wanted to get zero, let' s try asking Mathematica to simplify the expression *)

In[71]:= FullSimplify[(m - ω1 IdentityMatrix[2]).v1]
```

Out[71]= {0, 0}

```
In[72]:= FullSimplify[(m - ω2 IdentityMatrix[2]).v2]
```

Out[72]= {0, 0}

```
In[73]:= (* Invert a matrix *)
         Inverse[m]
```

Out[73]= $\left\{\{-2,\ 1\},\ \left\{\frac{3}{2},\ -\frac{1}{2}\right\}\right\}$

In[74]:= **m.Inverse[m]**

Out[74]= {{1, 0}, {0, 1}}

In[75]:=
```
(* to exponentiate a matrix *)
MatrixExp[m]
```

Out[75]= $\left\{\left\{-\frac{1}{22}\,e^{\frac{5}{2}-\frac{\sqrt{33}}{2}}\left(-11-\sqrt{33}-11\,e^{\sqrt{33}}+\sqrt{33}\,e^{\sqrt{33}}\right),\ \frac{2\,e^{\frac{5}{2}-\frac{\sqrt{33}}{2}}\left(-1+e^{\sqrt{33}}\right)}{\sqrt{33}}\right\},\ \left\{\sqrt{\frac{3}{11}}\,e^{\frac{5}{2}-\frac{\sqrt{33}}{2}}\left(-1+e^{\sqrt{33}}\right),\ \frac{1}{22}\,e^{\frac{5}{2}-\frac{\sqrt{33}}{2}}\left(11-\sqrt{33}+11\,e^{\sqrt{33}}+\sqrt{33}\,e^{\sqrt{33}}\right)\right\}\right\}$

In[76]:=
```
(* Exp[m] will just exponentiate each element *)
w = {{a, b}, {c, d}};
Exp[w]
```

Out[77]= $\{\{e^a, e^b\}, \{e^c, e^d\}\}$

In[78]:=
```
(* Raising a matrix to a power, e.g. w^2*)
MatrixPower[w, 2]
MatrixPower[m, 2.]
m.m
```

Out[78]= $\{\{a^2 + b\,c, a\,b + b\,d\}, \{a\,c + c\,d, b\,c + d^2\}\}$

Out[79]= {{7., 10.}, {15., 22.}}

Out[80]= {{7, 10}, {15, 22}}

In[81]:=
```
(* w^2 just squares each element *)
w^2
w.w
```

Out[81]= $\{\{a^2, b^2\}, \{c^2, d^2\}\}$

Out[82]= $\{\{a^2 + b\,c, a\,b + b\,d\}, \{a\,c + c\,d, b\,c + d^2\}\}$

In[83]:=

In[84]:=

In[85]:=

In[86]:=

In[87]:=