1. [6 pts] A simple pendulum consists of a point mass $M$ hanging from a massless string of length $R$ and swinging in a vertical plane. Its maximum angle is $90^\circ$, i.e., it was released from rest from a point where the string was horizontal. Let $\theta$ be its angle with respect to the vertical, so $\theta = 0$ corresponds to the lowest point of its arc.

(a) Find the equation of motion that relates $\ddot{\theta}$ to $\sin \theta$ by writing “F = ma” in the tangential ($\hat{\theta}$) direction.

(b) Integrate the equation of motion numerically using Mathematica, including the initial conditions $\theta = \pi/2$ and $\dot{\theta} = 0$ at $t = 0$, to find the time it takes for the pendulum to travel from $\theta = 90^\circ$ to $\theta = 45^\circ$.

(c) Integrate the equation of motion numerically using Mathematica, including the initial conditions $\theta = \pi/2$ and $\dot{\theta} = 0$ at $t = 0$, to find the time it takes for the pendulum to travel from $\theta = 45^\circ$ to $\theta = 0^\circ$. Perhaps you will want to do this by finding the time it takes to travel from $\pi/2$ to 0 and then subtracting the time calculated in part (b).

(Note that you calculated the same two times in HW02, using a method based on energy conservation.)

2. [6 pts] A particle of mass $M$ is moving in a plane, with its Cartesian coordinates $(x, y)$ given by

$$
x = A [Bt - \sin(Bt)]
$$

$$
y = A [1 - \cos(Bt)]
$$

where $A$ and $B$ are positive constants.

(a) Find the times at which the speed is a maximum.

(b) Find the tangential component of acceleration, i.e., the component of acceleration in the direction of motion, as a function of the time $t$.

(c) Find the “radial” component of acceleration, i.e., the magnitude of the component of acceleration that is perpendicular to the direction of motion. (You can do this by first finding a unit vector that is perpendicular to the velocity direction; or you can calculate it from the magnitude of the acceleration vector and its tangential component.)

3. [8 pts] A particle with electric charge $Q$ and mass $M$ is traveling in a region where there is a constant electric field of magnitude $E$ and a constant magnetic field of magnitude $B$. Both the electric and the magnetic field point in the $z$ direction. Assume the initial conditions at $t = 0$ are given by $x = y = z = 0$, $v_x = v_x^0$, $v_y = 0$, and $v_z = v_z^0$.

(a) Write the $x$, $y$, and $z$ components of the equation of motion.

(b) Solve the equations of motion to find the velocity as a function of time.

(c) Find the position of the particle as a function of time.