A 36 kg block is attached to a spring of constant $k = 600 \text{ N/m}$. The block is pulled 3.5 cm away from its equilibrium position and is pushed so that it has an initial velocity of 5.0 cm/s at $t = 0$.

What is the position of the block at $t = 0.75$ seconds?

Answer:

The formula for position as a function of time is: $x(t) = A \cos(\omega t - \phi)$.

We will also need velocity as a function of time: $v(t) = -\omega A \sin(\omega t - \phi)$.

We are given: $m = 36 \text{ kg}$, $k = 600 \text{ N/m}$, $x(0) = 3.5 \text{ cm}$, $v(0) = 5.0 \text{ cm/s}$.

Note that the amplitude is not 3.5 cm, because it does not start from rest. The initial push will make the amplitude larger than 3.5 cm.

First, we can find the angular frequency from:

$$\omega = \sqrt{\frac{k}{m}} = 4.0825 \text{ rad/s}.$$ 

Next, we must find the amplitude and the phase. In general, $A$ and $\phi$ are always determined by the initial conditions, $x(0)$ and $v(0)$. We plug $t = 0$ into the equations for position and velocity and set them equal to the initial conditions:

$$x(0) = A \cos(-\phi) = A \cos(\phi) = 3.5 \text{ cm}.$$ 

$$v(0) = -\omega A \sin(-\phi) = \omega A \sin(\phi) = 5.0 \text{ cm/s}.$$ 

Note that I used $\cos(-\phi) = \cos(\phi)$ and $\sin(-\phi) = -\sin(\phi)$. We now have 2 equations for 2 unknowns, $A$ and $\phi$. You can solve for $\phi$ by taking the ratio of the second equation over the first equation on each side:

$$\frac{\omega A \sin(\phi)}{A \cos(\phi)} = \frac{5.0 \text{ cm/s}}{3.5 \text{ cm}} \Rightarrow \tan(\phi) = 0.34993.$$ 

Our calculator gives $\phi = 0.3366$. We can then plug this back into either of the initial condition equations to find $A = 3.7081 \text{ cm}$.

Now that we know $A$, $\phi$, and $\omega$, we can get the final answer by plugging $t = 0.75 \text{ s}$ into the formula for $x(t)$. The final answer is $x(0.75 \text{ s}) = -3.39 \text{ cm}$.

One final comment: Make sure your calculator is set for angles in radians!!!
Second final comment: Note that the function \( \tan(\phi) \) is periodic under \( \phi \rightarrow \phi + \pi \), while \( \cos(\phi) \) and \( \sin(\phi) \) are periodic under \( \phi \rightarrow \phi + 2\pi \). This means there are actually two distinct solutions for the equation \( \tan(\phi) = 0.34993 \). Either \( \phi = 0.3366 \) (which is what your calculator gives) or \( \phi = 0.3366 + \pi \). The correct choice is the one which makes \( A \) positive.