Electrostatics

Gauss's Law
Announcements

- Homework set 1 is due 1/20
- Lecture notes: linked from lon-capa, or directly at http://www.pa.msu.edu/~schwier/courses/2014SpringPhy184/
- Section 1 lecture notes:
  http://www.pa.msu.edu/~nagy_t/phy184/lecturenotes.html
- Strosacker learning center schedule – BPS 1248
  - Mo: 10am – noon, 1pm – 9pm
  - Tue: noon – 6pm
  - We: noon – 2pm
  - Th: 10am – 1pm, 2pm – 9pm
Example - Equilibrium Position

- Consider two positive charges located on the x axis

- The charges are described by
  - $q_1 = 0.15 \, \mu\text{C}$  \quad x_1 = 0.0 \, \text{m}
  - $q_2 = 0.35 \, \mu\text{C}$  \quad x_2 = 0.40 \, \text{m}

- Where do we need to put a third charge for that charge to be at an equilibrium point?
  - At the equilibrium point, the force from each of the two charges will cancel
Example - Equilibrium Position (2)

- The equilibrium point must be along the x-axis.
- Three regions along the x-axis where we might place our third charge:
  - \( x_3 < x_1 \)
  - \( x_1 < x_3 < x_2 \)
  - \( x_2 < x_3 \)
Example - Equilibrium Position (3)

- $x_3 < x_1$
  - Here the forces from $q_1$ and $q_2$ will always point in the same direction (to the left for a positive test charge)
    - No equilibrium

- $x_2 < x_3$
  - Here the forces from $q_1$ and $q_2$ will always point in the same direction (to the right for a positive test charge)
    - No equilibrium
Example - Equilibrium Position (4)

- $x_1 < x_3 < x_2$
  - Here the forces from $q_1$ and $q_2$ can balance

$$|\vec{F}_{1 \to 3}| = |\vec{F}_{2 \to 3}| \quad \Rightarrow \quad k \frac{q_1 q_3}{(x_3 - x_1)^2} = k \frac{q_2 q_3}{(x_2 - x_3)^2}$$

$$\frac{q_1}{(x_3 - x_1)^2} = \frac{q_2}{(x_3 - x_2)^2} \quad \Rightarrow \quad q_1 (x_2 - x_3)^2 = q_2 (x_3 - x_1)^2$$

$$\sqrt{q_1} (x_2 - x_3) = \sqrt{q_2} (x_3 - x_1) \quad \Rightarrow \quad \sqrt{q_1} x_2 - \sqrt{q_1} x_3 = \sqrt{q_2} x_3 - \sqrt{q_2} x_1$$

$$\sqrt{q_1} x_3 + \sqrt{q_2} x_3 = \sqrt{q_1} x_2 + \sqrt{q_2} x_1$$

$$x_3 = \frac{\sqrt{q_1} x_2 + \sqrt{q_2} x_1}{\sqrt{q_1} + \sqrt{q_2}} = \frac{\sqrt{0.15 \mu C} \cdot (0.4 \text{ m})}{\sqrt{0.15 \mu C} + \sqrt{0.35 \mu C}} = 0.16 \text{ m}$$
General Charge Distributions

- So far we have dealt with point charges
- Consider the electric force due to a charge distribution
- We divide the charge into differential elements of charge, \( dq \), and find the electric force from each differential charge element as if it were a point charge

\[
\begin{align*}
 dq &= \lambda dx \\
 dq &= \sigma dA \\
 dq &= \rho dV
\end{align*}
\]

for a charge distribution

\[
\begin{align*}
 \text{along a line} \\
 \text{over a surface} \\
 \text{throughout a volume}
\end{align*}
\]

- Total charge \( q_{tot} = \int dq \)
General Charge Distributions

- **Line of charge:** charge $dq$ distributed over length $dx$: charge density
  \[ \lambda = \frac{dq}{dx} \]

- **Sheet of charge:** charge $dq$ distributed over area $dx \times dy = dA$: charge density
  \[ \sigma = \frac{dq}{dA} \]

- **Volume of charge:** charge $dq$ distributed over volume $dx \times dy \times dz = dV$: charge density
  \[ \rho = \frac{dq}{dV} \]
Example: total charge of a cylinder

- What is the total charge of a cylinder with radius 1 cm and height 5 cm and charge density $\rho = 5 \text{ nC/m}^3$?

- Answer: $q_{tot} = \int dq = \int \rho dV$

- The charge density is constant, thus

$$q_{tot} = \rho \int dV = \rho \int dA \, dh = \rho \pi r^2 h$$

$$q_{tot} = \rho \pi r^2 h = 5 \times \pi \times 0.01^2 \times 0.05 \, pC \frac{m^2}{m^2} = 0.79 \times 10^{-4} \, nC = 0.079 \, pC$$
The Electric Field

- So far we have thought of the electric force between two stationary charges.
- Suppose one charge were moving, how would the second charge know that the first charge has moved?
- What if there were other charges, how would one charge know about the extra charges?
- To deal with these situations, we introduce the concept of an electric field, which is defined at any point in space as the net electric force on a charge, divided by that charge:

\[ \vec{E}(r) = \frac{\vec{F}(\vec{r})}{q} \]

- The net force on any charge is then:

\[ \vec{F}(\vec{r}) = q\vec{E}(r) \]
The Electric Field

- The electric force on a charge is parallel or antiparallel to the electric field at that point.

\[ \vec{F} = |q| \vec{E} \]

- The magnitude of the force is \( F = |q| E \)
The Electric Field

- If several sources of electric fields are present, the electric field is given by the superposition of the electric fields from all sources
  - Follows from superposition of forces from mechanics
- The superposition principle for the total electric field at any point in space due to $n$ electric field sources can be written as

$$\vec{E}_{\text{net}}(\vec{r}) = \vec{E}_1(\vec{r}) + \vec{E}_2(\vec{r}) + \cdots + \vec{E}_n(\vec{r})$$
The total electric field at any point is the vector sum of all individual electric fields

\[ \vec{E}_{\text{net}} = \sum_i \vec{E}_i \]

Note that this means adding the x-, y-, and z- components separately

\[ E_{\text{net},X} = \sum_i E_{i,X} \]
\[ E_{\text{net},Y} = \sum_i E_{i,Y} \]
\[ E_{\text{net},Z} = \sum_i E_{i,Z} \]
Field Lines

- An electric field can change as a function of the spatial coordinate

- The changing direction and strength of the electric field can be visualized by means of electric field lines
  - Electric field lines graphically represent the net vector force on a unit positive test charge

- The tangent to the field line give the direction of the field and the density of field lines is proportional to the magnitude of the force

- Electric field lines can be compared to the streamlines of wind direction
Field Lines

- To draw an electric field line, we imagine placing a small positive charge at each point in the electric field.
  - This charge is small enough to not disturb the electric field.
  - We call this charge a test charge.

- We calculate the resultant force on the test charge and the direction of the force gives the direction of the field line.
Field Lines

Test charge $q$

$\vec{F}$
Field Lines

- In a nonuniform field, the electric force at a given point is tangent to the electric field lines at that point.
- A high density of field lines means a strong field.
- A low density of field lines means a weak field.
Field Lines

- Electric field lines point away from sources of positive charge and toward sources of negative charge.
- Each field line starts at a charge and ends at another charge.
- Electric field lines always originate on positive charges and terminate on negative charges.
- Electric fields exist in three dimensions, but we will usually present two-dimensional drawings for simplicity.
Point Charge

- The electric field lines from an isolated point charge emanate in radial directions from the point charge
- For a positive point charge, the field lines point outward
- For a negative point charge, the field lines point inward
Two Point Charges of Opposite Sign

- We can use the superposition principle to determine the electric field from two point charges of opposite sign.
Two Point Charges with Same Sign

- We can use the superposition principle to determine the electric field from two point charges with the same sign.
General Observations

- Field lines originate on positive charges and terminate at negative charges
- Field lines never cross
- If the field lines connect, we have an attractive force
  - Imagine the charges pulling on each other
- If the field lines seem to spread out, we have a repulsive force
  - Imagine the charges pushing each other apart
Demo - Electric Field Lines

- Demo - visualization of electric field lines

The charge of grass seeds is redistributed by induction. The Coulomb force makes the seeds align along the field lines.
Electric Field due to Point Charges

- The magnitude of the electric force on a point charge $q_0$ due to another point charge $q$ is

$$F = \frac{1}{4\pi\epsilon_0} \frac{|qq_0|}{r^2}$$

- We can take $q_0$ as a test charge so can express the magnitude of the electric field due to $q$ as

$$E = \frac{|F|}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{|q|}{r^2}$$

- The direction of this electric field is radial
  - Points outward for positive point charges
  - Points inward for negative point charges