Electric Field and Gauss’s law
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**Electric field from charge distribution**

- The magnitude of the electric field at a point P due to an infinitesimal charge $dq$ is

$$dE = k \frac{|dq|}{r^2} \quad \hat{r}$$

- Where $\hat{r}$ is a vector pointing to P with magnitude 1.

- The total field is given by the integral

$$\vec{E}_{net} = \int d\vec{E} = \int_{all \ points \ \ P} k \frac{|dq_P|}{r_P^2} \hat{r}_P$$

- The integral is solved by transforming from an integral over $dq$ to an integral over the position $P$ (length, area, volume).
Finite Line of Charge

- To find the electric field along a line bisecting a finite length of wire with linear charge density $\lambda$, we integrate the contributions to the electric field from all the charge.
- We assume that the wire lies along the $x$-axis.
Finite Line of Charge

- The electric field is then

\[ E_y = 2k\lambda y \left( \frac{1}{y^2} \frac{a}{\sqrt{y^2 + a^2}} \right) = \frac{2k\lambda}{y} \frac{a}{\sqrt{y^2 + a^2}} \]

- When the wire becomes infinitely long

\[ a \to \infty \implies \frac{a}{\sqrt{y^2 + a^2}} \to 1 \]

- The electric field a distance \( y \) from an infinitely long wire is

\[ E_y = \frac{2k\lambda}{y} \]
Let’s imagine that we put a ring with area $A$ perpendicular to a stream of water flowing with velocity $v$

- The product of area times velocity, $Av$, gives the volume of water passing through the ring per unit time
  - The units are $m^3/s$

- If we tilt the ring at an angle $\theta$, then the projected area is $A\cos\theta$, and the volume of water per unit time flowing through the ring is $Av\cos\theta$
Electric Flux

- We call the amount of water flowing through the ring the “flux of water”
  
  \[
  \Phi = A \nu \cos \theta
  \]

- We can make an analogy with electric field lines from a constant electric field and flowing water

- We call the density of electric field lines through an area \( A \) the electric flux
  
  \[
  \Phi = EA \cos \theta
  \]

  \( \theta \) is the angle between \( \vec{E} \) and \( \vec{A} \)
Electric Flux for Closed Surface

- Assume an electric field and a closed surface, rather than the open surface associated with our ring analogy.
- In this closed-surface case, the total electric flux through the surface is given by an integral over the closed surface.

$$\Phi = \iint \vec{E} \cdot d\vec{A}$$

- The differential area vectors always point out of the closed surface.
Another Example

- In this closed-surface case, the total electric flux through the surface is given by an integral over the closed surface.
- Decompose the surface into elements $\Delta A$. Then:

$$\Phi = \sum \vec{E} \cdot \Delta \vec{A}$$

With: $\Delta \vec{A} \rightarrow d\vec{A}$

$$\Phi = \oiint \vec{E} \cdot d\vec{A}$$

Remember: The differential area vectors always point out of the closed surface.
Gauss’s Law

- Let imagine we have an imaginary box in the form of a cube
- We assume that this box is constructed of a material that does not affect electric fields
- If we bring a positive charge up to any surface of the box, the charge will feel no force
Gauss’s Law

- Now we place a positive charge inside the box and bring the positive test charge up to the surface of the box.
- The positive test charge feels an outward force due to the positive charge in the box.
- Now we place a negative charge inside the box and bring the positive test charge up to the surface of the box.
- The positive test charge feels an inward force due to the negative charge in the box.
- The electric field lines seem to be flowing out of the box containing the positive charge and into the box containing the negative charge.
Gauss’s Law

- Now let’s imagine an empty box in a uniform electric field.
- If we bring the positive test charge up to side 1, it feels an inward force.
- If we bring a positive test charge up to side 2, it feels an outward force.
- The electric field is parallel to the other four sides, so the positive test charge does not feel any force when brought up to those sides.
- When we had a charge inside the box, the electric field lines seemed to be flowing in or out of the box.
- When we had no charge in the box, the net flow of electric field into the box was the same as the net flow of electric field out of the box.
Gauss’s Law

- We can now formulate Gauss’s Law as
  \[ \Phi = \frac{q}{\varepsilon_0} \]

- \( q \) is the charge inside a closed surface
- We call this surface a \textit{Gaussian surface}
- This surface could be our imaginary box
- This surface could be any closed surface
- Usually we choose this closed surface to have symmetries related to the problem we are trying to study
Gauss’s Law

- Gauss’s Law (named for German mathematician and scientist Johann Carl Friedrich Gauss, 1777 - 1855) states
  \[ \Phi = \frac{q}{\varepsilon_0} \]
  where \( q \) is the net charge enclosed by a Gaussian surface.

- If we add the definition of the electric flux we get another expression for Gauss’s Law
  \[ \iiint E \cdot dA = \frac{q}{\varepsilon_0} \]

- Gauss’s Law states that the surface integral of the electric field components perpendicular to the area times the area is proportional to the net charge within the enclosed surface.
Gauss’s Law and Coulomb’s Law

- Let’s derive Coulomb’s Law from Gauss’s Law
- We start with a positive point charge \( q \)
- The electric field from this point charge is radial and pointing outward with a magnitude

\[
E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}
\]

- We construct a spherical surface with radius \( r \) surrounding this charge
- This is our “Gaussian surface”
Gauss’s Law and Coulomb’s Law

- The electric field from a point charge is radial, and thus is perpendicular to the Gaussian surface everywhere.

- The electric field has the same magnitude anywhere on the surface so we can write

\[ \iiint E \cdot dA = \iiint E dA = E \iiint dA \]
Gauss’ Law and Coulomb’s Law

- Now we are left with a simple integral over a spherical surface
  \[ A = \iiint dA = 4\pi r^2 \]
- So for Gauss’s Law related to a point charge we get
  \[ E\iiint dA = E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0} \]
- Which gives
  \[ E = \frac{q}{4\pi \varepsilon_0 r^2} = \frac{1}{4\pi \varepsilon_0} \frac{q}{r^2} \Rightarrow E = k \frac{q}{r^2} \]
- Gauss’s Law can be derived from Coulomb’s Law for a point charge
- It can also be shown that Gauss’s Law holds for any distribution of charge inside any closed surface
Shielding

- Two important consequences of Gauss’s Law
  - The electrostatic field inside any isolated conductor is always zero.
  - Cavities inside conductors are shielded from electric fields.
- To examine these consequences, let’s suppose a net electric field exists at some moment at some point inside an isolated conductor.
- Charges will move to cancel the field inside the conductor.
Conductor in an electric field
Shielding Illustration

- Start with a hollow conductor
- Add charge to the conductor
- The charge will move to the outer surface
- We can define a Gaussian surface that encloses zero charge
  - Flux is 0
  - No electric field!

\[ q=0 \]
\[ E=0 \]
Shielding Demonstration

- We can demonstrate shielding in several ways
- We will place a volunteer in a wire cage and try to fry him/her with large sparks from a Van de Graaff generator
  - Charging the cage
  - Note that the shielding effect does not require a solid conductor
  - A wire mesh will also work, as long as you don’t get too close to the open areas
Faraday Cage Extreme