STELLAR CHROMOSPHERIC AND CORONAL HEATING BY MAGNETOHYDRODYNAMIC WAVES

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ABSTRACT

We investigate how the generation of magneto-hydrodynamic waves by turbulent motions in stellar convection zones depends on the star's effective temperature, surface gravity, and magnetic field strength. We show that the emitted Alfvén wave flux (and acoustic slow wave flux in a very strong magnetic field) is in reasonable agreement with the general trend of observed chromospheric radiative losses in stars, and with the observations of three stars for which magnetic field strength, surface area covered by strong fields, and radiative losses have all been measured.

Subject headings: hydromagnetics — stars: chromospheres — stars: coronae

I. INTRODUCTION

The structure of the solar chromosphere and corona seems to be controlled by magnetic fields. Chromospheric emission occurs predominantly from the network where the magnetic field is strong. The only exceptions are the blue points in Ca II H and K, which are uniformly distributed throughout the cell interiors. The coronal emission is predominantly from closed magnetic "loops." It is likely that magnetic fields control the structure of stellar chromospheres and coronae (Rosner and Vaiana 1980; Linsky 1980).

Acoustic waves generated by turbulence in the convection zone can not be the source of energy to heat coronae. The observed line widths of optically thin transition region lines are too small for a sufficient acoustic flux to heat the solar corona (Athay and White 1978; Brunner 1978). The flux emitted in the Mg II k-line and X-rays decreases with decreasing effective temperature along the main sequence later than about G0 (Basri and Linsky 1979; Rosner and Vaiana 1980) much more slowly than the predicted decrease of acoustic flux from the convection zone (De Loore 1970; Renzini et al. 1977, but see also Schmitz and Ulmschneider 1980a, b). There is little if any observed increase in UV or X-ray flux with decreasing surface gravity, whereas the generated acoustic flux is proportional to \( g^{-1} \) (Basri and Linsky 1979; Rosner and Vaiana 1980), but cooling by mass loss via stellar winds may be important here. Again a crucial role for magnetic fields is indicated.

The question is how magnetic fields control the structure of stellar chromospheres and coronae. There are several possibilities: (1) They may control the generation of the nonthermal energy flux that provides the heat input to the chromosphere. For instance, they permit the generation of additional types of waves, such as Alfvén and slow mode acoustic waves. (2) They control the flow of the nonthermal energy, by channeling and focusing the propagation of waves. (3) They control the dissipation of the nonthermal energy. They allow plasma instabilities to occur, which affects the dissipation of waves, other motions, and currents. (4) They control the cooling effect of mass loss in winds, which only occurs in regions of open magnetic field lines.

In this paper we investigate the first effect—how magnetic fields affect the turbulent generation of waves. We conclude that Alfvén and acoustic slow mode waves generated in a background magnetic field sufficiently strong to dominate the gas motions are prime candidates for heating stellar chromospheres and coronae.

II. TURBULENT WAVE GENERATION

Turbulent motions act as a source of waves. Turbulence can be pictured as a hierarchy of eddies. A turbulent eddy has kinetic energy, and in the presence of a magnetic field also magnetic energy, which it releases when it mixes with its surroundings at the end of its life. Most of this energy is returned to the ambient medium, but a small fraction gets transformed into propagating waves. This process was investigated by Lighthill in his studies of noise emission from jet engines (1952; Crighton 1975; Ffowcs-Williams 1977), and is referred to as the Lighthill mechanism. Wave emission by turbulence in a magnetic field was studied by Kulikov (1955), Parker (1964), and Kato (1968). The radiated power is roughly the energy density, \( \epsilon \), in the turbulent motions, divided by the decay time scale, \( \tau \), for the turbulent motions, multiplied by an efficiency factor depending on the multipole order of the emission and the compactness of the eddy (as measured by the ratio of the size of the eddy, \( l \), to the wave length of the wave, \( \lambda = 2\pi/k \)).
HEATING BY MHD WAVES

For emission of multipole order \( n \), the radiated power is

\[
P \approx \frac{\varepsilon}{\tau} (k l)^{2n+1}.
\]  

(1)

This approach assumes that only a small fraction of the turbulent energy is carried off by waves.

In the absence of magnetic fields, the energy density of the turbulent motions is their kinetic energy,

\[
\varepsilon = \frac{1}{2} \rho u^2.
\]

(2)

In the presence of magnetic fields there is a magnetic contribution as well:

\[
\varepsilon = \frac{1}{2} \rho u^2 + \delta B^2 / 8\pi.
\]

(3)

For a turbulent magnetic field weaker than or equal to equipartition strength,

\[
\varepsilon \approx \rho u^2;
\]

(4)

while if the magnetic field dominates the motions,

\[
\varepsilon = B^2 / 8\pi.
\]

(5)

The turbulence decay time scale is the nonlinear cascade time, which is the eddy turnover time, that is, the eddy size \( l \) divided by its velocity \( u \),

\[
\tau \approx l / u.
\]

(6)

The ratio of eddy size to wavelength depends on the phase velocity \( c \) of the wave. The wave vector of the wave is

\[
k = 2\pi / \lambda = \omega / c,
\]

and the period of the wave is the eddy turnover time, so

\[
\omega \approx \tau^{-1} \approx u / l.
\]

(7)

Hence

\[
k l \approx u / c.
\]

(8)

For acoustic waves (either the fast mode in a weak magnetic field or the slow mode in a strong one) the phase velocity is the sound speed \( s \), so

\[
k l \approx u / s \quad \text{(acoustic waves)}
\]

(9)

For magnetic waves (Alfvén waves, the slow mode in a weak magnetic field, or the fast mode in a strong one) the phase velocity is the Alfvén speed, \( a \), so

\[
k l \approx u / a \quad \text{(magnetic waves)}.
\]

The dominant multipole order depends on the background magnetic field strength and wave type. Multipole emission \( (n=0) \) corresponds to a mass source. In the absence of a magnetic field there are no mass sources in the convection zone. However, a magnetic field channels the Alfvén waves and a strong field channels the slow mode acoustic waves, so that the waves move one-dimensionally along the magnetic field. In these cases turbulence acts as a piston in a tube, and efficient monopole emission dominates. Dipole emission \( (n=1) \) corresponds to a momentum source, that is, an external force. In a uniform medium there is no external force and hence no dipole emission, but in stars there is an external gravitational field, so some dipole emission occurs. Quadrupole emission \( (n=2) \) corresponds to the action of the turbulent Reynolds stresses. This is the dominant process in stellar atmospheres in the absence of magnetic fields, because the net momentum flux from a star is small (Stein 1967).

To summarize: In the absence of magnetic fields, turbulent convection produces acoustic waves by quadrupole emission (Lighthill 1952; Proudman 1952)

\[
P_{AC} \approx \frac{\rho u^3}{l} \left( \frac{u}{s} \right)^5.
\]

(10)

In a strong background magnetic field, Alfvén and acoustic slow mode waves are produced by monopole emission, while fast mode waves are produced by quadrupole emission (Kulsrud 1955; Parker 1964; Kato 1968). If the turbulent magnetic field is less than or equal to equipartition strength with the turbulent motions, the radiated power is:

\[
P_{A} \approx \frac{\rho u^3}{l} \left( \frac{u}{a} \right)
\]

(11)

(Kato 1968),

\[
P_{S} \approx \frac{\rho u^3}{l} \left( \frac{u}{s} \right),
\]

(12)

and

\[
P_{F} \approx \frac{\rho u^3}{l} \left( \frac{u}{a} \right)^5.
\]

(13)

If the turbulent magnetic field dominates the motions, then

\[
P_{A} \approx \frac{B^2 u}{l} \left( \frac{u}{a} \right) \approx \frac{\rho u^2 a}{l}
\]

(14)

(Kato 1968),

\[
P_{S} \approx \frac{B^2 U}{l} \left( \frac{u}{s} \right) \approx \frac{\rho u^2 a}{l} \left( \frac{a}{s} \right).
\]

(15)
and

\[ P_F \approx \frac{B^2 U}{l} \left( \frac{u}{a} \right)^5 \approx \rho a^2 \frac{u}{l} \left( \frac{u}{a} \right)^5. \]  

(16)

The wave energy flux produced is the emitted power times the eddy size

\[ F \approx P l, \]  

(17)

both because the largest velocities occur within one eddy of the top of the convection zone and because waves emitted deeper are scattered by overlying eddies. The total rate of wave energy emission is this flux multiplied by the area of the emitting region \( A \),

\[ L \approx P l A. \]  

(18)

In the case of MHD waves our emission formulae apply to individual magnetic flux tubes, and the area of the emitting region is the surface area pierced by magnetic flux tubes.

III. DEPENDENCE OF WAVE EMISSION ON STELLAR PROPERTIES

The nonthermal wave energy flux produced by turbulence depends on the turbulent velocity amplitude, the density at the top of the convection zone, the magnetic field strength, and the fraction of the stellar surface covered by strong magnetic flux tubes. We will now derive approximate expressions for the turbulent velocity and atmospheric density.

The convective flux, where convection is efficient, is approximately equal to the turbulent energy density times the turbulent velocity, and inside the convection zone the convective flux is nearly equal to the total energy flux of the star. In a weak or equipartition turbulent magnetic field

\[ F \approx \rho u^3 \approx \sigma T_{\text{eff}}^4. \]

Hence the turbulent velocity is

\[ u \approx (F/\rho)^{1/3} \approx (\sigma T_{\text{eff}}^4/\rho)^{1/3}. \]  

(19)

A strong turbulent magnetic field reduces convective instability (Chandrasekhar 1961) and reduces the turbulent velocity. In a strong field the turbulent energy density is dominated by the magnetic field, so the convective flux is

\[ F \approx B^2 u \approx \sigma T_{\text{eff}}^4. \]

Hence the turbulent velocity in a strong magnetic field is

\[ u \approx F/B^2 \approx \sigma T_{\text{eff}}^4/B^2. \]  

(20)

The top of convection zone occurs where photons can escape to space, at optical depth \( \tau \approx 1 \). The temperature at this point is approximately the effective temperature of the star, and the pressure is determined by the condition of hydrostatic equilibrium,

\[ \frac{dP}{dz} = -\rho g, \]

where \( g \) is the surface gravitational acceleration of the star. This can be rewritten by dividing by the opacity as

\[ -\frac{dP}{\rho \kappa dz} = \frac{dP}{dT} = \frac{g}{\kappa}. \]

The pressure at \( \tau \approx 1 \) is approximately

\[ P(\tau = 1) \approx g/\kappa. \]  

(21)

Suppose the opacity can be represented locally by a power law

\[ \kappa = \kappa_0 P^a T^b. \]

Then the pressure at \( \tau = 1 \) will be

\[ P \approx (g/\kappa_0)^{1/(1+a)} T^{-b/(1+a)}, \]

and the density at the top of the convection zone will be

\[ \rho \approx \frac{\mu P}{\kappa_0 T} \approx \frac{u}{\kappa_0} \left( \frac{g}{\kappa_0} \right)^{1/(1+a)} T^{-(1+a+b)/(1+a)} \]  

(22)

where \( \mathcal{R} \) is the gas constant and \( \mu \) the mean molecular weight.

The dependence of the turbulent velocity on stellar surface gravity and effective temperature can be found for a weak or equipartition magnetic field by substituting equation (22) into equation (19). We get

\[ u \approx \left( \frac{\sigma g}{\mu} \right)^{1/3} \left( \frac{\kappa_0}{g} \right)^{1/(3(1+a))} T^{[3(1+a) + b]/[3(1+a)]}. \]  

(23)

For a strong magnetic field the velocity is given by equation (20). The remaining quantities in the expressions (10)–(17) for the emitted wave fluxes are: the sound speed

\[ s = \left( \gamma \mathcal{R} T_{\text{eff}}/\mu \right)^{1/2}, \]

(24)
and the Alfvén speed
\[ a = \frac{B}{(4\pi \rho)^{1/2}} \]
\[ \approx B(\frac{\gamma}{\mu})^{1/2}(\kappa_0/g)^{1/2(1+a)}T_{\text{eff}}^{(1+a+b)/(2(1+a))}. \]

(25)

We are now ready to use the results (22)–(25) to derive the dependence of the emitted wave fluxes (10)–(17) on the stellar surface temperature, gravity, and magnetic field strength. In the absence of any magnetic field, the acoustic flux varies as
\[ F_{AC} \propto g^{5/3(1+a)}T_{\text{eff}}^{5(1+a)+10b)/(6(1+a))}. \]  
(26)

(Renzini et al. 1977). This is in reasonable agreement with their numerical results for $H^-$ opacity, where $a=0.7$ and $b=5$. Encouraged by this agreement between the approximate scaling law and detailed calculations in the acoustic case, we proceed to the magnetic cases. In a weak or equipartition turbulent magnetic field: The Alfvén wave flux varies as
\[ F_A \propto g^{5/6(1+a)}T_{\text{eff}}^{5(1+a)+b}/6(1+a)B^{-1}. \]  
(27)

The acoustic slow mode flux varies as
\[ F_S \propto g^{-1/3(1+a)}T_{\text{eff}}^{3(1+a)+2b}/6(1+a). \]  
(28)

The fast mode flux varies as
\[ F_F \propto g^{5/6(1+a)}T_{\text{eff}}^{5(1+a)+5b}/6(1+a)B^{-5}. \]  
(29)

In very strong turbulent magnetic fields: The Alfvén wave flux varies as
\[ F_A \propto g^{5/2(1+a)}T_{\text{eff}}^{5(1+a)+b)/(2(1+a))B^{-3}. \]  
(30)

The acoustic slow mode flux varies as
\[ F_S \propto T_{\text{eff}}^{2.5}B^{-2}. \]  
(31)

The fast mode flux varies as
\[ F_F \propto g^{5/4(1+a)}T_{\text{eff}}^{4(1+a)+5b}/2(1+a)B^{-15}. \]  
(32)

Table 1 summarizes the dependence of the emitted wave fluxes on surface gravity, magnetic field strength, and effective temperature for cool stars where the opacity is dominated by $H^-$, for which
\[ \kappa_0 \approx 1.4 \times 10^{-23} \text{ cm}^2 \text{ g}^{-1}, \]
\[ a \approx 0.74, \quad b \approx 5.0. \]  
(33)

The results are especially sensitive to the actual dependence of the opacity on temperature and density. We have also neglected the structure of the convection zone (see, e.g., Renzini et al. 1977) and the effect of stratification (see, e.g., Stein 1967), both of which affect the dependence of the emitted flux on stellar parameters. In addition, we have neglected the turbulent correlation properties (see, e.g., Stein 1967; Kato 1968) which primarily affect the overall numerical coefficient. Most important of all, the heating of chromospheres and coronae depends on the total rate of nonthermal wave emissions or the average wave energy flux, not the local wave energy flux emitted from an individual magnetic flux tube. The average wave energy flux is given by the flux expressions (26)–(32) multiplied by the fraction of the stellar surface occupied by magnetic flux tubes, for which we do not yet have a theory.

### IV. COMPARISON WITH OBSERVATIONS

The observations of radiative losses from stellar chromospheres and coronae may be roughly summarized as follows: In late type stars (for which Ca $^+$, Mg $^+$, and H resonance lines are optically thick) the total chromospheric line radiative losses are about three times the flux in Mg $^+$ $h$ and $k$ (Linsky and Ayres 1978). Basri and Linsky (1979) and Stencel et al. (1980) find that for luminosity class III giants the ratio of the flux in the Mg $^+$ $k$-line to the total stellar flux varies with effective

<table>
<thead>
<tr>
<th>Wave</th>
<th>Wave Flux/Total Stellar Flux</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acoustic</td>
<td>$g^{-1}T_{\text{eff}}^{11}$</td>
</tr>
<tr>
<td>Weak or Equipartition</td>
<td></td>
</tr>
<tr>
<td>Turbulent Magnetic Field</td>
<td></td>
</tr>
<tr>
<td>Alfén</td>
<td>$g^{1/10}T_{\text{eff}}^{0.7}B^{-1}$</td>
</tr>
<tr>
<td>Acoustic slow</td>
<td>$g^{-1/5}T_{\text{eff}}^{2.2}$</td>
</tr>
<tr>
<td>Magnetic fast</td>
<td>$g^{1/2}T_{\text{eff}}^{3.4}B^{-5}$</td>
</tr>
<tr>
<td>Strong Turbulent Magnetic Field</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$g^{0.5}T_{\text{eff}}^{2.5}B^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$T_{\text{eff}}^{3.2}B^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$g^{1.3}T_{\text{eff}}^{10}B^{-15}$</td>
</tr>
</tbody>
</table>

**NOTE.** Numerical values for $H^-$ opacity $\kappa_0 \propto P_0^{0.7}T^5$.  

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temperature approximately as

$$F(Mg^+ k)/F_\star \propto T_{\text{eff}}^4. \quad (34)$$

Stencel et al. (1980) also find a slight dependence on surface gravity between giants and supergiants, with

$$F(Mg^+ k)/F_\star \propto g^{-1/4}. \quad (35)$$

The X-ray flux from a star is a measure of the nonthermal heat input to its corona, when its stellar wind is small. Results from the Einstein Observatory stellar survey (Rosner and Vaiana 1980) show that for main sequence stars the X-ray luminosity is roughly constant for spectral types later than G0. This corresponds to an X-ray flux varying as

$$F_\star \propto R_\star^{-2} \propto T_{\text{eff}}^{1.7}. \quad (36)$$

for spectral classes G0-M0 (Allen 1973) and more rapidly for still later spectral types. The X-ray flux from late type giants and supergiants is much less, which may be a result of large energy losses to stellar winds in these stars. It should be noted that for any given effective temperature and surface gravity, there is a large range in the observed UV and X-ray emission. Equations (34)–(36) summarize the general trend of the observed radiative losses.

The observed chromospheric radiative losses are much less sensitive to $T_{\text{eff}}$ and $g$ of a star than predicted for the acoustic flux (Table 1). Part of this discrepancy, but not all, is due to the fact that compressive waves lose about 90% of their energy to radiative damping in the photosphere (Stein and Leibacher 1974; Schmitz and Ulmschneider 1980b). All magnetic wave modes in an equipartition field and Alfvén and slow modes in a strong field vary much more slowly with $T_{\text{eff}}$ and $g$ than acoustic waves (Table 1), in better agreement with the observations of chromospheric UV resonance line losses of Ca" and Mg". This improved agreement is only suggestive, because no account has been taken of the variation in magnetic field strength or the fraction of the stellar surface covered by magnetic flux tubes.

By equipartition arguments, the background magnetic field strength is expected to scale roughly as

$$B \approx P^{1/2} \propto g^{0.3} T^{-3/2}. \quad (37)$$

If we substitute this expression for $B$ into the expressions for the wave fluxes in Table 1, we find that the Alfvén and slow mode behave similarly:

$$\frac{F_A}{F_\star} \propto \frac{F_S}{F_\star} \propto \begin{cases} g^{-1/5} T_{\text{eff}}^{2.2} & \text{(weak or equipartition $\delta B$)} \\ g^{-3/5} T_{\text{eff}}^{6.5} & \text{(strong $\delta B$)} \end{cases} \quad (38)$$

while the fast mode flux is equally or more sensitive to $g$ and $T_{\text{eff}}$ than the nonmagnetic acoustic flux. Note that the Alfvén and slow mode fluxes for these two limiting cases of a turbulent magnetic field dominated by or dominating the turbulent motions bracket the observed dependence of radiative losses on effective temperature and surface gravity (eqs. [34]–[36]). These results, of course, apply only to the general trend of observed radiative losses. The large scatter about this trend is likely due to a large scatter in the surface area occupied by magnetic flux tubes, and also to a scatter in the magnetic field strength in individual stars about the equipartition value. In the solar case there is a correlation between plage strength and magnetic flux. Since the actual situation lies between these limiting cases of weak and strong magnetic field, the fact that they bracket the observed trends fairly narrowly constitutes satisfactory agreement for these approximate expressions. More detailed calculations are now indicated. In particular a theory to predict the magnetic field strength and the fraction of stellar surface covered by magnetic flux tubes in individual stars is needed.

We can go one step further. Robinson, Worden, and Harvey (1980) have actually observed the magnetic field strength and fraction of stellar surface area covered by magnetic flux tubes for two stars for which Mg"h and k radiative losses have also been measured. The observations are listed in Table 2, and compared with the predicted wave fluxes in Table 3. The flux ratios for

| Table 2
| Stellar Properties
<table>
<thead>
<tr>
<th>Star</th>
<th>$T_{\text{eff}}$ (K)</th>
<th>$F(Mg^+ k)/F_\star$</th>
<th>$B$ (kG)</th>
<th>Area (fraction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 Boo A</td>
<td>5300</td>
<td>$5.3 \times 10^{-5}$</td>
<td>2.6</td>
<td>0.3</td>
</tr>
<tr>
<td>70 Oph A</td>
<td>4950</td>
<td>$3.7 \times 10^{-5}$</td>
<td>1.9</td>
<td>0.1</td>
</tr>
<tr>
<td>Sun</td>
<td>5770</td>
<td>$8.0 \times 10^{-6}$</td>
<td>1.5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

*Giampapa et al. 1980; Robinson et al. 1980.
Table 3

<table>
<thead>
<tr>
<th>Observed ...</th>
<th>$F(\xi \text{Boo A})/F(\text{Sun})$</th>
<th>$F(\tau 0 \text{Ph A})/F(\text{Sun})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predicted:</td>
<td>Equipartition $\delta B$</td>
<td>Strong $\delta B$</td>
</tr>
<tr>
<td>Alfvén</td>
<td>16.3</td>
<td>4.9</td>
</tr>
<tr>
<td>Slow</td>
<td>25.0</td>
<td>7.4</td>
</tr>
<tr>
<td>Fast</td>
<td>1.4</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Alfvén and slow mode waves in a strong magnetic field come embarrassingly close to the observations for such a crude theory; and since they are also consistent with the overall observed $g$ and $T_{\text{eff}}$ dependence of chromospheric radiative losses, they are the prime candidates for more detailed calculations. Uchida and Kaburaki (1974), Wentzel (1974), and Ionson (1978) have explored Alfvén wave heating of the solar corona. Beckers and Schneebberger (1977) have failed to detect waves in the corona, but their upper limit is much greater than the required energy flux. Alfvén waves are also thought to play a role in driving the solar and stellar winds (see, e.g., Holweg 1973; Barnes 1974; Hartmann and MacGregor 1980) and are indeed observed in the wind near the Earth. Robinson is continuing his observing program, and while two stars plus the Sun don’t test a theory, a much more definitive test should be possible shortly.

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