Some psychoacoustical experiments with all-pass networks

W. M. Hartmann
Physics Department, [a] *Michigan State University, East Lansing, Michigan 48824 and Laboratory of Psychophysics, Harvard University, Cambridge, Massachusetts 02138*
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It is shown how all-pass filter sytems can be used in labortory experiments in the physics and psychophysics of sound and music. Particular applications include studies of the following: (i) lateralization of sounds in headphone listening; (ii) the spectral modifications produced by phasers, compared with admixing a time-delayed signal; (iii) the pitch that can be obtained by dichotic processing of two spectrally flat noise signals, Huggins effect; and (iv) the perception of the dephasing of harmonics in complex tone. Some of the effects obtained from experiments on lateralization and on the Huggins effect appear to be new results.

I. INTRODUCTION

University courses in introductory acoustics or in the physics of music usually include a generous amount of psychoacoustics, that branch of acoustics that focuses on the human listener as the measure of sounds.[1] Many psychoacoustical effects are quite incredible until the student has had the opportunity to observe them for himself in the laboratory. Often these effects can be demonstrated with modest equipment.[2] This article describes several interesting and useful experiments that can be done with all-pass filters. Because of the universal availability of the integrated circuit operational amplifier, active all-pass filters are easy to build and to use.

In Sec. I the one-pole all-pass filter and its transfer function are introduced. Section II discusses lateralization and localization of sounds with emphasis on phase effects. A variety of experiments on lateralization of signals in headphone listening are suggested; these experiments discover the human sound locating apparatus operating in rather curious ways. Section III discusses phasers, the sound modifiers used by rock musicians to simulate the studio effect of flanging. Interest centers on the degree to which a phaser approximates the true time delay introduced by flanging. A dichotic pitch effect with noise and all-passed noise, the Huggins effect, is discussed in Sec. IV. In Sec. V the all-pass filter is used to demonstrate the insensitivity of the human ear to phase effects.

An all-pass filter is one that has unity gain at all frequencies but produces a phase shift that varies with frequency.[3-5] The mathematical properties of such a filter are discussed in the Appendix. The simplest form of active all-pass filter is shown in Fig. 1. The filter transfer function H is easily derived from the theory of the ideal operational amplifier,

$$H(s) \equiv v_0/v_i = (s\tau - g)/(s\tau + 1), \qquad (1.1)$$

where $g \equiv R_f/R_i$ and $\tau \equiv RC \equiv (2\pi f_0)^{-1}$ and s is the Laplace variable. For an all-pass filter, $g = 1$, so that

$$H(s) = (s\tau - 1)/(s\tau + 1). \qquad (1.2)$$

Exchanging R and C in Fig. 1 only reverses the sign of the right-hand side of Eq. (1.2). The impulse response function of the all-pass filter is the inverse Laplace transform,

$$h(t) = \delta(t) - (2/\tau)\exp(-t/\tau). \qquad (1.3)$$

Because the magnitude of $H(s = i\omega)$ is unity for all ω $(=2\pi f)$ the amplitude response of the filter is flat. The phase shift varies from π at $\omega = 0$ to 0 at $\omega = \infty$ as

$$\phi_1 = 2\cot^{-1}(\omega\tau); \quad 0 < \phi_1 < \pi. \qquad (1.4)$$

If the input is $\sin(\omega t)$, then the output is $\sin(\omega t + \phi_1)$.

II. LATERALIZATION

Our ability to locate the sources of sound and the reflectors of sound is a complicated combination of different processes. These include our ability to separate direct signals from early echos in a room and to ignore the latter (precedence effect) or to include them in a definition of the acoustic environment.[6,7] The sound spectrum is modified by reflection by objects in a room and, above 4000 Hz, by the outer ear permittimg localization in a vertical plane.

The most extensive studies of localization have been concerned with specifically binaural aspects of the process. There is persuasive evidence for a duplex theory, by which low frequency tones ($f \leq 1500$ Hz) are localized by interaural time differences (phase differences) and high frequency tones (≥ 2500 Hz) are localized by interaural intensity differences.[8] Our ability to localize tones with frequencies in the intervening region is relatively poor.[9,10]

When sounds are heared with headphones, rather than in a free field, the sound image seems to be within the head. The image may be *lateralized* to one side of the head or the other.[11] There is good evidence that the same mechanisms responsible for localization of free-field sources are responsible for the lateralization of sounds from headphones. For example, in the low-frequency region one can convert the azimuth angle θ of a free-field source to interaural phase angle difference, ϕ,

$$\phi = 2\pi r(\theta + \sin\theta)/\lambda, \qquad (2.1)$$

where r is the head radius = 8.75 cm and λ is the wave length of the tone in air. Azimuth θ is measured from the forward direction.[8] Mills[12] found that the minimum detectable phase difference from a free-field source agreed with the minimum detectable phase difference between left and right headphones signals[13] for all frequencies from 250 to 1500 Hz. Lateralization produced by an interaural phase difference can be eliminated by an opposing cue from an interaural intensity difference.[14]

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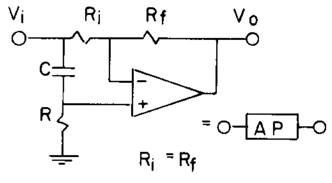

Fig. 1. Schematic diagram of a single one-pole all-pass filter stage.

The effect of interaural phase differences can be demonstrated by including a frequency-dependent phase shift in one channel of a stereo signal using two identical all-pass filters and an inverter, connected as shown in Fig. 2. The phase shift varies from π at $\omega = 0$ to $-\pi$ at $\omega = \infty$,

$$\phi_{2,0} = \pi - 4 \tan^{-1} (\omega\tau), \tag{2.2}$$

where, as usual, $-\pi/2 < \tan^{-1}(x) \leq \pi/2$. The notation $\phi_{2,0}$ means that the overall phase range is 2π (two filters) and when $\omega\tau = 1$ the phase shift is zero. This defines the characteristic frequency of the network, $f_0 = (2\pi\tau)^{-1}$. Consider a sinewave with frequency f_n, which is n semitones above f_0. Suppose that the semitone scale is equitempered, i.e.,

$$f_{-n}/f_0 = f_0/f_n. \tag{2.3}$$

Then equal musical intervals above and below f_0 produce equal and opposite phase shifts because, by the properties of the inverse tangent,

$$\phi_{2,0}(-n) = -\phi_{2,0}(n). \tag{2.4}$$

Only equitempered scales have this property. The values of phase shift are shown in Fig. 3, together with the interchannel time delay. Of course, a given phase shift corresponds to a larger time difference for lower frequencies.

Many experiments can be done with this simple idea. The experiments raise questions that are not yet completely answered. One can probe the limits of discrimination for tones lateralized by interaural phase differences. Is the limit set by a minimum perceptible time delay or by a minimum phase angle? According to the data of Mills[12] the answer is apparently neither and somewhere between the two. Our ability to lateralize images on the basis of phase differences alone diminishes with increasing frequency.

Interaural differences can be created in headphones that are not heard in the real acoustical world. This can be seen in Fig. 3 for frequencies more than six semitones below f_0 where θ becomes greater than 90°. It appears that the ear has no objection to these unnatural sounds. If a tone is becoming increasingly lateralized due to increasing interaural phase differences then the trend continues smoothly into the region of physically impossible phase differences (Ref. 15). (Of course any phase difference is physically possible if one includes the possibility of echos. The above discussion assumes that the precedence effect eliminates all echos from any neural computation of location.)

A simple perceptual view of the lateralization process is one in which the ear identifies some feature(s) of the

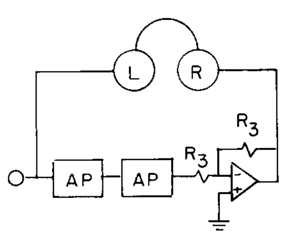

Fig. 2. Block diagram showing the connection of two all-pass filter stages and a unity gain inverter to produce the interaural phase shifting network. The connection to left and right headphones is only schematic. In practice matched power amplifiers must be used in the two channels.

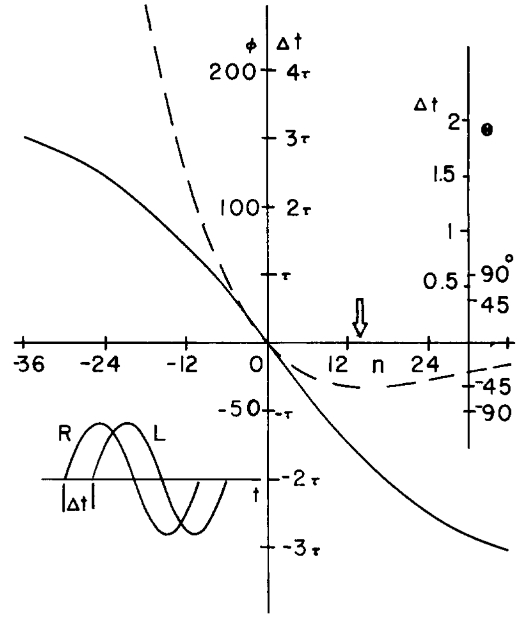

Fig. 3. Two curves show the phase shift, $\phi_{2,0}$ (—) and the time, Δt (- -) by which a sine wave in the left headphone lags a sine wave in the right headphone in the circuit of Fig. 2 when the sine wave frequency is n semitones higher than f_0. The phase shift is expressed in degrees and the time delay is in units of τ. The phase shift for positive n is antisymmetrical with that for negative n. The inset shows the waves in left and right ears. On the right vertical axis the time lead Δt in milliseconds and the corresponding azimuth angle in degrees for localization of a free field source are given for $f_0 = 261$ Hz, equal to middle C, and for a standard human head radius of 8.75 cm, and for room temperature. The arrow shows the maximum left time lead at a frequency ratio of 2.26.

acoustical waveform and compares arrival times at the two ears, $\Delta t = t_R - t_L$. The process is subject to the logical constraint $-\frac{1}{2}f < \Delta t < \frac{1}{2}f$. The operation of such a mechanism on the stimulus created by the circuit of Fig. 2 (0 condition) is shown in the left-hand column of Fig. 4. Suppose that the tones of an equitemperted scale increase in frequency through f_0. The model predicts that the sound image should move from right to left as shown in Fig. 5(a). This behavior is observed experimentally (Ref. 16).

If the inverter is removed from the circuit of Fig. 2 then the phase shift at $f = f_0$ is not 0 but is π (π condition). The operation of the interaural timing mechanism on stimuli created in the π condition is shown in the right-hand column of Fig. 4. The predicted behavior is shown in Fig. 5(b), and the observed behavior is in Fig. 5(c). The model successfully predicts some of the observed changes from the 0 condition. (i) The observed reversal of the positions of individual tones is predicted. (ii) When f is close to f_0 as shown in top and bottom panels of Fig. 4, it is difficult to position the image in the 0 condition but easy to find the lateralized image in the π condition. However, the model fails to predict the nature of the *motion* of the sound image in the π condition. The predicted pattern includes a discontinuity in the position at the characteristic frequency f_0, but the observed motion, shown in Fig. 5(c), is like the reverse of the motion observed in the 0 condition, [Fig. 5(a)] as though the listerner had put on the headphones backwards. Therefore, it seems that lateralization involves more than simple calculation of interaural time differences; it includes a process of logical inference as well. The ear refuses to accept the discontinuity.

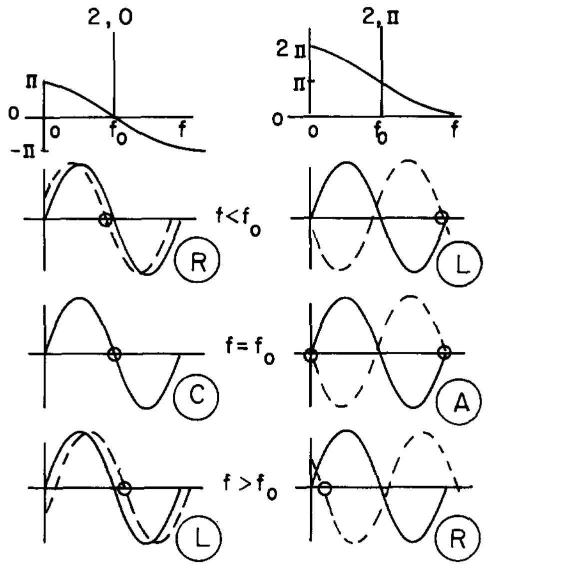

Fig. 4. Two columns show the 2,0 and the 2, π conditions for interaural phase difference experiments. The top row shows the phase differences in radians as a function of sine wave frequency $0 < f < \infty$. The following rows show the waveform in left (—) and right (- - -) ears for different frequencies. One arbitrary feature, a negative going zero crossing is always in the center of the horizontal axis for the left ear and is identified with a (0) for the right ear. The judgments, based upon the order of arrival of such a feature, are shown in the circle, L is left, R is right, C is centered, and A is ambiguous.

Note that two all-pass sections have an overall phase range of 2π. In the π condition they do not produce phase differences of 0 or 2π for finite frequencies. Therefore, the π experiment above does not involve the ear in the paradox of finding that phase differences of both π and 0 lead to a centered sound image. Suppose, however, that a second all-pass network, identical to that of Fig. 2 is cascaded with the first. Then an overall phase shift of 4π is realized, and it is easy to vary the interaural phase difference from 0 to 2π. In Figs. 5(e) and 5(f) are shown the expected and the observed behavior for such a cascade of filters in the π condition. Whereas the 0 condition, shown in Fig. 5(d), produces a strong lateralization, the π condition is somewhat ambiguous. At the lowest frequencies the sound image moves to the left, as predicted, but the high-frequency behavior may cross the medial plane (as shown) or it may remain in the medial plane. Apparently the phase coincidences at 0 and 2π act as references and the ear is forced to some compromise between a phase-shift paradox, 0, π and 2π and a discontinuity in lateralization. One symptom of the resulting ambiguity is that upward-going and downward-going portions of the frequency sweep may lead to different localization patterns, a result that never occurs in the other experiments reported here.

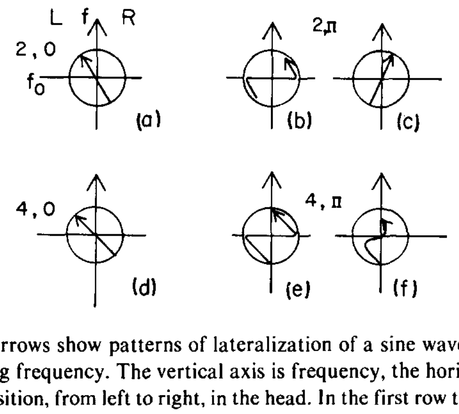

Fig. 5. Arrows show patterns of lateralization of a sine wave tone with increasing frequency. The vertical axis is frequency, the horizontal axis shows position, from left to right, in the head. In the first row the stimulus is made with two all-pass filters (Fig. 2). (a) Shows the pattern that is expected and observed in the 0 condition; (b) shows the pattern that is expected; and (c) shows the pattern that is observed in the π condition. In the second line are shown patterns for a circuit with four all-pass filters; (d) shows the pattern expected and observed in the 0 condition; (e) shows the pattern that is expected; and (f) the pattern that is observed in the π condition.

This kind of experiment acquires added interest in view of one of the auditory illusions of Deutsch.[17] She found a strong tendency for right-handed listeners to hear the higher of two tones lateralized to the right. The experience of one right-handed observer, the author, in a two-tone experiment with $+\phi_n$ and $-\phi_n$ phase shifts, is that the phase effect dominates any tendency to lateralize the higher tone to the right. However, there may be some remnant of the auditory illusion because the lateralization effect is not symmetrically reversed when the headphones are reversed.

One can confound the lateralization effect entirely by adding harmonics to the tone. The effect of harmonics can be viewed in two ways that are physically the same but psychophysically different. In the spectral domain one argues as follows. Because the harmonics of a periodic tone are phase locked the ear perceives the tone as a coherent entity and ascribes a common origin and a common location to all harmonics. However, the phase shift produced by the all-pass filter is not proportional to the frequency of the harmonics; therefore, the various harmonics of an all-passed complex tone provide conflicting lateralization information. Hence, the lateralized image is destroyed and the complex tone is heard in the center of the head.

The other view is to regard the all-pass filtering as propagation through a dispersive medium. A tone with a given complex structure in time is bent out of shape by the frequency dependent velocity. Therefore, when the brain tries to perform a binaural analysis of the shifted and unshifted tones it does not find enough common features in the waveform to measure an interaural time delay. Support for the former (spectral) view comes from the following curious effect. Although there is a tendency to perceive a tone with harmonic partials as coherent, it may sometimes happen that a contrary lateralization cue causes the tone to split. The effect can be observed if the tone moves, as in the equitempered run experiment, up and down through the characteristic frequency of the all-pass filter (π condition). The complex tone may split into a buzz tone that is stationary in the center of the head and a fundamental tone that slides from left to right and back, as for a sine wave.

III. PHASERS AND FLANGING

When a signal is added to a delayed version of itself the result is a comb filtering of the signal, as noted in Ref. 2. The sum signal has a hollow etherial sound that is popular among contemporary rock guitarists. The required time shifting can be done with analog or digital delay lines. The technique is known as "flanging."

A similar effect can be produced by a phaser. A schematic view of the signal path in a phase is shown in Fig. 6.[18]

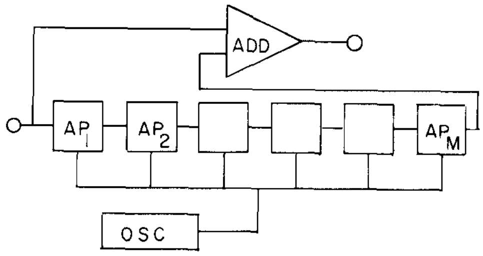

Fig. 6. Block diagram shows the connection of M all-pass stages and an adder to create a phaser. The characteristic frequencies of all the stages are controlled by the voltage from a slow oscillator. Note: in commercial phasers both signal paths to the adder may include phase shifting.

In practice the characteristic frequencies of the M all-pass stages are voltage controlled by replacing the resistor R in Fig. 1 with a field effect transistor. A rolling sound is created by a slow sine wave control voltage and $M = 6$ stages.

The approximation of time delay by a phaser improves as M increases in the following way. Suppose that the number M of identical all-pass filters in series and their common characteristic frequency $f_0 \equiv \tau^{-1}$ both grow so that $T = 2\tau M$ is constant. Then in the limit $M = \infty$ the system of all-pass filters produces a true time delay equal to T. This statement can be proved as follows.

The processed signal $v_0(t)$ is given by the inverse Fourier transform

$$v_0(t) = (2\pi)^{-1} \int_{-\infty}^{\infty} H(\omega) v_i(\omega) \exp(i\,\omega t) d\omega, \quad (3.1)$$

where the input signal v_i is

$$v_i(\omega) = \int_{-\infty}^{\infty} v_i(t) \exp(-i\omega t) dt. \quad (3.2)$$

The transformation $H(\omega)$ may be due to time delay H_T or due to the all-pass network of M stages H_M. Because the impulse response of a system with time delay T is

$$h_T(t) = \delta(t - T), \quad (3.3)$$

the transfer function is the Fourier transform

$$H_T(\omega) = \exp(-i\omega T) \equiv \exp(-i\phi_T). \quad (3.4)$$

The transfer function for M identical all-pass filters is obtained by summing the phase shifts caused by the individual stages. From Eq. (2.2), with M even,

$$H_M(\omega) = \exp[-i2M \arctan(\omega\tau)] \equiv \exp(-i\phi_M). \quad (3.5)$$

(For odd M the same expression applies if the signal is inverted.) In the limit of vanishing τ the arctangent in (3.5) becomes equal to its argument for any finite frequency. If the number of filters grows simultaneously such that $2M\tau = T$ is constant then from Eqs. (3.4) and (3.5),

$$\phi_M = 2M\tau\omega = \omega T = \phi_T, \quad (3.6)$$

i.e., phase shifts produced by M all-pass filters and time delay are the same for all ω.

For finite M the situation is shown in Fig. 7. A finite number of identical all-pass filters approximates time delay for small ωT. If the processed signal is added to the input signal then time delay produces a comb filter with an infinite number of evenly spaced peaks and valleys in the amplitude response function. The response of the system with M all-pass filters has a total number of peaks and valleys equal to $M - 1$ (not counting those at $\omega = 0$ and ∞). They are not equally spaced but the separation increases with increasing frequency because the frequency of the mth peak or valley is proportional to $\tan(m\pi/2M)$ $(0 < m < M)$. An argument has been presented in Ref. 19 to show that for finite M the phaser best approximates a system with time delay if all the all-pass stages have the same characteristic frequency.[19]

The most effective demonstrations of a phaser involve dynamic changes in the filter characteristics while processing music or noise. The familiar sound of a jet aircraft taking off plus the echo from a nearby building is easily simulated with a white noise source and a phaser. Although the phaser simulation is a tonally colored signal it does not produce the sense of pitch associated with noise plus delayed noise.[20]

IV. HUGGINS EFFECT

The Huggins effect[21] is one of the most surprising and dramatic pitch effects in psychoacoustics. The stimulus source for the effect is white noise. As in the case of the lateralization experiments one of two earphones receives the source signal directly, the other earphone receives the signal after processing with an all-pass filter. The all-pass filter produces a frequency-dependent phase shift from 2π radians at 0 Hz to 0 rad at infinite frequency. If the phase shift changes from $3\pi/2$ to $\pi/2$ rad over a frequency difference that is about 10% of the center frequency (frequency for a phase shift of π radians) then a pitch corresponding to the center frequency will be heard.

The experiment is remarkable because the spectral information provided to the listener is coded in a very obscure fashion. There is no experiment that can be performed on the all-pass filtered signal by itself to distinguish it from unprocessed white noise. Both the processed and the unprocessed signals must be simultaneously available, with phase information intact, to some central processor to extract any information from the signals. The autocorrelation functions in the two earphones are both delta functions, but

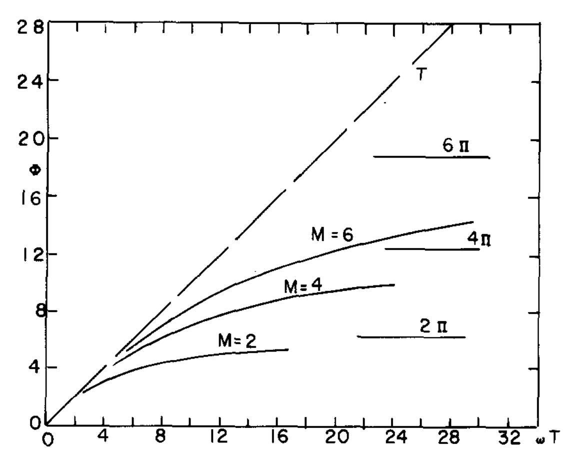

Fig. 7. The figure shows the phase shifts ϕ as a function of relative frequency ωT for time delay (dashed curve T) and for strings of M all-pass filters with $\tau = T/(2M)$, where M is 2, 4, or 6. Asymptotes are at 2π, 4π, or 6π, respectively.

the cross correlation function is the impulse response of the all-pass filter, which oscillates at frequency f_0.

To produce the Huggins effect requires an all-pass filter that is more sharply tuned than the cascade of one-pole all-pass filters in Fig. 2. As before, let the center frequency of the filter be denoted by f_0. Let f_+ be the frequency that gives $\pi/2$ greater phase shift than f_0, and f_- be the frequency that gives $\pi/2$ less phase shift. The "bandwidth" is defined as the difference, $f_- - f_+$.
For the filter of Fig. 2,

$$f_- - f_+ = 2f_0. \tag{4.1}$$

A bandwidth of twice the center frequency is far too wide to get the Huggins effect. To produce a narrower bandwidth requires a two-pole all-pass filter, constructed as follows:

The general transfer function for a two-pole bandpass filter with center frequency ω_0 and given "Q" is

$$B(s) = \frac{\omega_0 s/Q}{s^2 + (\omega_0/Q)s + \omega_0^2}. \tag{4.2}$$

The corresponding all-pass filter has zeros in the right half of the complex s plane, symmetrically across the imaginary axis from the poles, i.e.,

$$H(s) = \frac{s^2 - (\omega_0/Q)s + \omega_0^2}{s^2 + (\omega_0/Q)s + \omega_0^2} \tag{4.3}$$

$$= 1 - 2B(s). \tag{4.4}$$

Therefore to construct an all-pass filter requires only that one add the input to a bandpass filter to the inverse of twice the output of the filter. For this application the active state-variable filter is the correct choice.[22] Both the Q and the center frequency of the state-variable filter may be varied independently over a wide freqeuncy range with good stability at high values of Q. The f_0 and Q adjustments may be made without upsetting the balance described in Eq. (4.4).

Because the Huggins pitch at f_0 occurs at the center frequency of the corresponding bandpass filter, a listener can switch the filter to the band pass mode to compare his perception of the Huggins pitch with the obvious pitch of noise in a sharply tuned pass band. By moving the controls and comparing with the correct answer one quickly learns to hear the effect. Precise measurements of filter parameters may be obtained from Lissajous patterns with a sine wave source.

The Huggins pitch can be heard for center frequencies ranging from 300 to 1 KHz. Unlike some other dichotic pitch effects which are heard only at low intensity levels (40 db) Huggins pitch is best heard at moderate levels, 60–75 db in a 10-KHz band.

The experiments performed in preparing this paper led to results not reported by Cramer and Huggins. These results are based on the experience of three listeners.

(i) A phase shift of zero at f_0 produces a pitch sensation. The Huggins experiment corresponds to the π condition described in Sec. II. If the all-passed signal is inverted than the O-condition results, and this condition, with phase shift varying from $\pi/2$ to $-\pi/2$, at the same rate, produces an equally strong pitch. This result had been discovered earlier by Guttman.[23]

(ii) Among weak pitch effects[24] the Huggins pitch is one of the strongest. It is only slightly weaker than repetition pitch with unfiltered noise. It is considerably stronger than dichotic repetition pitch or the nonspectral pitch of amplitude modulated noise.[25]

(iii) Cramer and Huggins studied the strength of the pitch effect for a few different bandwidths. For their smallest bandwidth, 6% of f_0, the pitch effect was strongest. They predicted that for very small bandwidths the pitch effect must disappear because of vanishing spectral energy in the region of phase variation. With an active state-variable filter it is possible to obtain bandwidths narrower than those used by Cramer and Huggins. The conclusion of the present study is that for center frequencies beteen 450 and 900 Hz the pitch is strongest when the bandwidth is 7% (±2%) of f_0. As the bandwidth is decreased to 1% the pitch does disappear.

(iv) If the noise signal is directly added to the all-pass filtered noise in the π condition then the result has a spectral dip at f_0. This sum signal produces no sense of pitch anywhere near f_0. Therefore, the binaural processing which produces the pitch cannot be a simple addition of the signals. The pitch could arise from a signal difference operation.

V. PERCEPTION OF PHASE

According to Ohm's acoustical law, the auditory system is sensitive to the power spectrum of sound but is insensitive to the phase relationships among the Fourier components. Theoretically the ear cannot detect changes in the shapes of waveforms so long as the spectrum is constant. Changes in timbre produced by changing phase relationships among the harmonics of a periodic tone violate Ohm's law; they can be investigated with all-pass filters. The one-pole all-pass of Eq. (1.1) produces a gentle phase distortion of the waveshape which is not easily heard. The two-pole all-pass of Eq. (4.3) produces more dramatic changes, especially at high Q. The most audible phase changes are those which change the peak levels of the envelope of the waveform.[26]

The big question in these phase studies, for which there is no complete answer, concerns the role played by nonlinear distortion. Unfortunately most studies of phase perception do not attempt to control or measure the distortion. A review of literature is given in Ref. 27. Even if no distortion is present in the *source* of sounds, the essential nonlinearities of the *ear* may account for the perception of phase.[28,29]

When distortion is present, changing the phase relationships among the harmonics of a tone changes the intensities of the harmonics, and this, of course, is plainly audible. For example, consider an input signal v consisting of a fundamental and a second harmonic, each of unit amplitude. Suppose that the processed signal w includes some square law distortion, viz.,

$$w = v + \eta v^2. \tag{5.1}$$

The intensity of the fundamental tone then becomes,

$$w_1^2 = 1 + \eta^2 - 2\eta \sin \phi, \tag{5.2}$$

where ϕ is the relative phase angle between fundamental and second harmonic. The fundamental intensity varies by 1 dB (the approximate limit of discrimination) when the harmonic distortion $\eta/2$ is 3%.

The combined effects of distortion and phase angle variation can be demonstrated with an all-pass filter and a simple nonlinear device such as a diode. One can vary the relative phase angles with the all-pass system of Fig. 2 to increase the waveform envelope peak for some common waveforms. These include the square wave, the sawtooth, the ramp (Wavetek), and the pulse, except for very narrow pulses. For the triangle wave, however, any changes in phase angles must cause the peak amplitude to decrease. If phase shifting is followed by distortion then phase variation in these waves can produce dramatic timbre changes.

Relative phase angles do not seem to be especially important in transient signals. This can be demonstrated by using an all-pass filter and a function generator that produces tone bursts. After spending several hours listening to tone bursts of complex waves with a variety of all-pass characteristic frequencies, the listener is likely to conclude that any phase dependence that can be detected in the tone bursts can be heard at least as well in a steady tone.

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APPENDIX: ANALYTIC PROPERTIES OF FILTERS

This appendix notes certain mathematical properties of filters from a physicist's perspective. Additional details may be gained from texts on operational mathematics and network analysis.[2-4]

For a general linear filter the output v_0 is related to the input v_i by the convolution integral,

$$v_0(t) = \int_{-\infty}^{\infty} h(t-t')\, v_i(t')\, dt', \qquad \text{(A1)}$$

where h is the impulse response, equal to the inverse Fourier transform of the transfer function, i.e.,

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(i\omega t)\, H(\omega)\, d\omega. \qquad \text{(A2)}$$

For a causal filter the present output cannot depend upon the input in the future; therefore, $h(t) = 0$ for $t < 0$.

This zero value of $h(t)$ for negative values of t is obtained from an integral on a contour made from the real axis, Eq. (A2), and the semicircle at infinity around the *lower* half plane. On this semicircle the real part of $i\omega t$ is negative for $t < 0$, so that the integral vanishes on this semicircle for well-behaved $H(\omega)$. If $H(\omega)$ is analytic in the lower half plane, so that the contour encircles no poles, then $h(t)$ is zero for $t < 0$, as required. The Laplace transform $\mathcal{H}(s)$ is then analytic in the right half plane and the causal filter is also stable. The above circumstances are necessary and sufficient to ensure that the real and imaginary parts of H are Hilbert transforms of each other.

The usual analysis of the complex transfer function H, however, is in terms of amplitude A and phase ϕ, the parameters of a Bode plot:

$$V = A + i\phi = \mathrm{Ln}\, H, \qquad \text{(A3)}$$

where the Ln function is the principal value natural logarithm. Parameters A and ϕ are Hilbert transforms of one another if $V(\omega)$ is well behaved and analytic in the lower half plane. But for an all-pass filter $H(\omega)$ has at least one zero in the lower half plane so that $V(\omega)$ has at least one branch point there. Therefore, A and ϕ are not Hilbert transforms of one another for an all-pass filter. Instead, $A = 0$ and $\phi(\omega)$ is proportional to the discontinuity across the branch cut.

[a]Permanent address.

[1]J. G. Roederer, *Introduction to the Physics and Psychophysics of Music*, 2nd ed. (Springer Verlag, Heidelberg, 1975).
[2]W. M. Hartmann, Am. J. Phys. **43**, 755 (1975).
[3]N. Balabanian, *Network Synthesis* (Prentice-Hall, Englewood Cliffs, 1958).
[4]M. E. van Valkenburg, *Introduction to Modern Network Synthesis* (Wiley, New York, 1960).
[5]E. A. Guillemin, *Theory of Linear Physical Systems* (Wiley, New York, 1963).
[6]A variety of mechanisms is presented by A. H. Benade, *Fundamentals of Musical Acoustics* (Oxford University, New York, 1976), pp. 201–222.
[7]A. W. Mills, in *Foundations of Modern Auditory Theory*, edited by J. V. Tobias (Academic, New York, 1972), Vol. II, p. 303.
[8]D. M. Green, *Introduction to Hearing* Lawrence Erlbaum, Hillsdale, NJ, 1976), Chap. 8.
[9]S. S. Stevens and E. Newman, Proc. Nat. Acad. Sci. **20**, 593 (1934).
[10]A. W. Mills, J. Acoust. Soc. Am. **30**, 237 (1958).
[11]See however, N. Sakamoto, T. Gotoh, and Y. Kimbura, J. Audio Engineering Soc. **24**, 710 (1976) who report out-of-head localization in headphone listening given sufficient reverberation.
[12]A. W. Mills, J. Acoust. Soc. Am. **32**, 132 (1960).
[13]J. Zwislocki and R. S. Feldman, J. Acoust. Soc. Am. **28**, 860 (1956).
[14]E. R. Hafter and S. C. Carrier, J. Acoust. Soc. Am. **51**, 1852 (1972).
[15]J. V. Tobias, in Ref. 7, Vol. II, p. 465.
[16]The experiments were done at 75 db with a top frequency of 978 Hz, a bottom frequency of 167 Hz and center frequency, the geometric mean, 404 Hz.
[17]D. Deutsch, Sci. Am. **233** (4), 92 (Oct., 1975).
[18]D. Bohn, *National Audio Handbook* (National Semiconductor, Santa Clara, CA, 1976), p. 5–10 and 11.
[19]W. M. Hartmann, J. Audio Engr. Soc. **26**, 439 (1978).
[20]F. A. Bilsen and R. J. Ritsma, Acustica **22**, 63 (1969–70).
[21]E. M. Cramer and W. H. Huggins, J. Acoust. Soc. Am. **30**, 413 (1958).
[22]B. A. Hutchins, *Musical Engineer's Handbook* (Electronotes, Ithaca, NY, 1975), Chap. 4.
[23]N. Guttman, J. Acoust. Soc. Am. **34**, 1996 (1962).
[24]For a review of pitch effects with noise sources see F. A. Bilsen, J. Acoust. Soc. Am. **61**, 150 (1977).
[25]E. M. Burns and N. F. Viemeister, J. Acoust. Soc. Am. **60**, 863 (1976); also, R. Wicke and A. J. M. Houstma, J. Acoust. Soc. Am. **58**, S83(A) (1975).
[26]M. R. Schroeder, J. Acoust. Soc. Am. **31**, 1579 (1959).
[27]J. Blaert and P. Laws, J. Acoust. Soc. Am. **63**, 1478 (1978).
[28]T. J. F. Buunen, thesis (University of Delft, Holland, 1976) (unpublished).
[29]T. J. F. Buunen, J. M. Festen, and G. van den Brink, J. Acoust. Soc. Am. **55**, 297 (1974).