On the detection of a tone masked by two tones

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We reopen the question of an appropriate representation to describe the masking of a sine tone midway in frequency between two sine maskers. We suggest that when the maskers are closely spaced in frequency signal detection is mediated by differences in the the stimulus envelope caused by the target signal. We show that in previous two-masker experiments detection threshold was limited by stimulus duration. Our experiments for long stimulus durations find thresholds that are approximately independent of the phase angle between the target and the maskers. Because the different phase angles correspond to very different stimulus envelopes the observed invariance is not easy to understand. For a general phase angle the presence of the signal causes both periodic changes in the envelope and periodic frequency modulation. The experimental detection data do not allow one to distinguish between these two effects.

PACS numbers: 43.66.Dc, 43.66.Ba, 43.66.Nm [JH]

INTRODUCTION

In 1954, Zwicker performed a masking experiment in which two sine tones were used to mask a narrow noise band target which had a center frequency midway between the two masker frequencies. Zwicker measured masked threshold as a function of the frequency separation of the two maskers. The results of the experiment contributed to the quantification of the critical band concept.

In 1965, Green performed a similar experiment, in which the target was a third sine tone. Green's experiments were primarily directed towards testing a model of energy detection. In this model peripheral filtering is followed by square-law detection. The output of the square-law detector is averaged by a leaky integrator (one-pole low-pass filter) with a time constant of about 0.1 s. The case of closely spaced maskers is of particular interest. According to Green's argument the beats between the closely spaced maskers produce energy fluctuations in the output of the detector which interfere with the detection of the target. Because these energy fluctuations are subjected to low-pass filtering, they decrease as the beat frequency increases (viz., as the maskers become more widely spaced in frequency). Therefore threshold should decrease with increasing masker spacing.

In 1976, Phipps and Henning repeated Green's experiment under conditions in which the phase relations between maskers and target were rigidly fixed. Their data agreed with the data of Green for large frequency separations between the maskers, but disagreed for small separations. Phipps and Henning found that for small separations, detection performance depended significantly on the phase angle between the maskers and target. They, therefore, concluded that Green's model for the detection process is wrong.

The work of Phipps and Henning represents an advance in that the mathematical analysis included the entire stimulus complex of maskers and target. The mathematical analysis is, however, in a form which is far from transparent. Further, the experiments of Phipps and Henning, as well as those of Green, employed short tone durations, 0.20 s or shorter. The present report has four purposes:

(1) To show that a mathematical analysis of the stimulus by phasors results in an easy-to-understand view of the envelope changes and frequency changes caused by the target.

(2) To suggest that the envelope changes provide the cue for detection of the target for small masker separations.

(3) To show that detection performance in the experiments of Green and of Phipps and Henning was limited by the short duration of the stimulus and that the phase dependence and the dependence on frequency range found by Phipps and Henning is not present for long stimulus durations.

(4) To note that detection performance for long stimulus durations is not readily explainable by the contemporary models of energy detection and FM detection.

I. ANALYSIS OF THE STIMULUS

Consider a target with amplitude a and angular frequency ω_s in the presence of unity amplitude maskers with angular frequencies ω_h and ω_l . With an appropriate choice of the time origin and angle θ , any such stimulus can be described by the form

$$y(t) = \sin(\omega_{t}t - \theta) + a\sin(\omega_{t}t + \phi) + \sin(\omega_{h}t + \theta).$$
(1)

The masker frequencies are equally above and below the target frequency:

$$_{I}=\omega_{s}-\omega_{m}, \qquad (2)$$

$$\omega_h = \omega_s + \omega_m \,, \tag{3}$$

and the maskers themselves are separated by the beat frequency

$$\omega_h - \omega_l = 2\omega_m \,. \tag{4}$$

Equation (1) is a sum of three terms, each with a different frequency. This is an appropriate representation of the stimulus if the auditory system is capable of resolving the three components. However, if ω_1 and ω_h are so close together that they cannot be resolved, the time dependence in Eq. (1) represents two perceptually

ω

different aspects. There is a rapid time variation at audible frequencies. The auditory system deals with this, as usual, by a wholistic transformation resulting in the perception of a tone. There is a slow time variation at subaudible frequencies which produces an explicitly perceived time dependence in the features of the tone. In this event the perceptually relevant representation of the stimulus in Eq. (1) is one in which the two forms of time dependence appear as separate factors in a product.

The most convenient product representation of the stimulus is the complex phasor form, employing the identity,

$$e^{ix} = \cos x + i \sin x \,. \tag{5}$$

The sine target, for example, appears in this notation as $[ae^{i\omega}S^t]$. Taking the absolute value causes the rapid time dependence to disappear, leaving only the time dependence of the envelope, e.g., the absolute value of the target itself is simply a.

In phasor form the stimulus is

$$y(t) = (e^{i\omega_s t})(e^{-i\theta\omega_s t}\omega_m)(e^{-i\omega_m t} + ae^{i\phi} + e^{i\omega_m t}).$$
(6)

The first factor describes the oscillations at the average (the target) frequency. The second factor is a constant phase shift of no interest. The third factor is the modulation factor on which all subsequent attention is focused. The modulation factor is a complex number of the form

$$M = |M| e^{i\phi} = E e^{i\phi}. \tag{7}$$

Generally, the absolute value |M|, which is the envelope E, is a function of time and so is the modulation phase, ψ .

A. Envelope

The envelope is easily calculated from the absolute value of the third factor in Eq. (6) using Eq. (5) and its complex conjugate. The envelope is given by

$$E = [2 + a^{2} + 2\cos(2\omega_{m}t) + 4a\cos\phi\cos(\omega_{m}t)]^{1/2}.$$
 (8)

Especially interesting are the extreme cases in which the target is in phase ($\phi = 0$) or 90° out of phase ($\phi = \pi/2$) with the masker. As noted by Phipps and Henning, the former case corresponds to over-modulated AM, the latter case corresponds to over-modulated quasi FM.

For
$$\phi = 0$$
,
 $E = |a + 2\cos\omega_m t|$. (9)
For $\phi = \pi/2$,

$$E = [2 + a^2 + 2\cos(2\omega_m t)]^{1/2}.$$
 (10)

(11)

These envelopes are shown in Fig. 1.

B. Modulation phase

The modulation phase angle is generally given by

 $\psi = \tan^{-1}[a\sin\phi/(2\cos\omega_m t + a\cos\phi)].$

For the case $\phi = 0$ the phase angle is simply zero.



FIG. 1. The figure shows envelopes for the two-masker stimulus with a target amplitude a=1/2. There are two phase conditions (a) $\phi = 0$ and (b) $\phi = \pi/2$. The envelopes are drawn by plotting both E and -E. Frequency $2f_m = 2\omega_m/2\pi$ is the beat frequency in the absence of the target. In Fig. 1(b) the instantaneous frequency, relative to f_m , is also shown. The dashed lines indicate the stimulus segment heard by Phipps and Henning.

For the case $\phi = \pi/2$ the phase angle varies periodically with time. Therefore the instantaneous frequency of the complex as a whole is given by ω_{s} plus a modulation increment given by

$$\Delta \omega = \frac{d\psi}{dt} = \frac{2a\omega_m \sin \omega_m t}{4\cos^2 \omega_m t + a^2}.$$
 (12)

This is also shown in Fig. 1.

The graphs of envelope and frequency oscillations shown in Fig. 1 are appealing in that they seem to correspond to sensations experienced by subjects when listening to the stimuli for a small modulation frequency and for a target above threshold. For the phase condition $\phi = 0$, one hears alternating strong and weak beats. For the phase condition $\phi = \pi/2$ one hears a relatively smoother tone, presumably associated with the fact that the envelope never reaches zero. Further, at $\phi = \pi/2$ one can hear modulation in the pitch, at least for a target frequency of 250 Hz. It is a plausible conjecture that the above features, easily observed for large values of the target amplitude, mediate detection of the target at the detection level.

For both cases, $\phi = 0$ and $\phi = \pi/2$, the target results in modulation at a rate of ω_m , half the beating rate of $2\omega_m$. For $\phi = 0$, the effect of a finite target is to make alternating beats larger or smaller, thereby doubling the envelope periodicity. For $\phi = \pi/2$, the envelope periodicity is unchanged by the presence of the target, but the frequency modulation has a period of twice the envelope period. In the case of particular interest, where the maskers are separated by 5 Hz, the target is detected by detecting a variation with a frequency of 2.5 Hz, i.e., with a period of 400 ms. It would seem probable that detection experiments with stimulus durations of 100 or 200 ms do not give the subject adequate exposure to elicit optimum performance. It seemed a good idea to repeat the experiments of Phipps and Henning for 5 Hz separation using longer stimulus durations.

II. DETECTION EXPERIMENTS

A. Stimulus

For his experiments Green employed three oscillators. Presumably the phase relation between target and maskers varied somewhat from trial to trial because of small oscillator drifts. Phipps and Henning used a digital system in which maskers and targets were phase locked. The present experiments create the maskers by balanced modulation of a carrier

$$y_c = \sin(\omega_s t), \qquad (13)$$

and a multiplying signal

 $y_m = 2\cos(\omega_m t + \theta) . \tag{14}$

When the output of the balanced modulator is added to an attenuated (a) and phase shifted (ϕ) carrier, the result is the stimulus of Eq. (1). The balanced modulation technique produces stimuli with these characteristics:

(1) Masker amplitudes are identical.

(2) The target frequency is exactly midway between the masker frequencies.

(3) The target phase is fixed with respect to the maskers.

(4) The modulation factor, envelope and phase, is identical to that in the phase-locked version of Phipps and Henning.

(5) The signal rolls within the envelope at a rate which is the difference between ω_{\bullet} and an integral multiple of ω_{m} . (There is no rolling for the phase-locked stimulus.)

(6) There is no difference between advanced $\phi = \pi/2$ and retarded $\phi = -\pi/2$ conditions.

Stimuli were controlled by a microcomputer which also collected the response data. Each masker had a level of 60 dB SPL. Stimulus tones were 2 s long. When the target was present, it was gated on and off with the maskers. Tones did not start and stop at any particular value of the envelope; we do not expect this stimulus variability to be significant for 2-s stimuli. Subjects heard the stimuli diotically via Beyer DT-48 headphones in a soundproof room.

B. Subjects

Four subjects participated in the experiment. Subjects M and B (the author) were experienced in detection experiments. Subject J had only one year's experience. Subject E was a novice listener; unlike subjects B, M, and J, he received feedback on data runs.

C. Procedure

Each experimental run consisted of two halves. On one half the phase relationship was $\phi = 0$, on the other it was $\phi = \pi/2$. The order was randomly selected by the computer. Each half-run consisted of a 2IFC staircase experiment (Levitt 1970), in which the target intensity was initially equal to the masker intensity (a=1). After two correct responses the target level decreased by 2 dB. After one wrong response, the target level increased by 2 dB. The interstimulus interval was 0.5 s. Presentation of a stimulus pair began 0.5 s after the subject's response. Experimental intervals were indicated with lights on the response box.

For each experimental half a subject reversed the direction of the staircase 18 times. The first four turning points were ignored and the average level of the remaining 14 turning points was regarded as a measurement of the detection threshold. Data presented here are the average of two or more runs, performed after subjects had performed at least two practice runs.

III. RESULTS

The masked threshold levels with respect to 60 dB, the level of each of the maskers, for subjects B, E, J, and M are shown in Table I for two phase relations at each of three target frequencies. With few exceptions all subjects have equal thresholds and these are approximately independent of frequency. The average threshold for $\phi = 0$ is -15.3 dB (±2.0). The average threshold for $\phi = \pi/2$ is -14.8 dB (±3.8). Overall, therefore, the average threshold is about -15 dB, corresponding to $a \approx 0.2$, independent of phase condition.

Table I also shows data from the studies of Phipps and Henning and of Green. The comparison of these data with ours suggests that detection performance was lim-

TABLE I. The table shows masked threshold in dB with respect to the level of one of the two 60-dB maskers. The target has frequency f_s and the masker frequencies are above and below the target frequency by 2.5 Hz. Errors in parenthesis for the four subjects of the present experiments are one standard error. The results obtained by Phipps and Henning are shown in rows P and H. The data for Green's three subjects, read from graphs, are given in rows G1, G2, and G3.

	$f_{*} = 0.25 \text{ kHz}$		1 kHz		4 kHz	
Subject	$\phi = 0$	$\phi = \pi/2$	φ=0	$\phi = \pi/2$	φ ≖ 0	$\phi = \pi/2$
в	-16(1)	-14(1)	-16(2)	-24(2)	-15(1)	-11(1)
Е	-14(1)	-15(1)	-17(1)	-16(1)		
J	-16(1)	-12(0)	-15(0)	-18(0)	-12(2)	-13(1)
М	-14(1)	-14(1)	-20(0)	-17(1)	-13(1)	-9(1)
Р	-3	-14	3	-17	-16	-15
H	-4	-12	-3	-13	-13	-13
G1	-3		-5 ·		-6	
. G2	-6		-8		-6	
G3	-6		-5		-4	

ited by stimulus duration in the previous studies. The present thresholds, obtained with a stimulus duration of 2000 ms, are consistently lower than those found by Green (100 ms) and the present thresholds do not show the pronounced phase dependence found by Phipps and Henning (200 ms).

IV. DISCUSSION

Phipps and Henning found thresholds that were considerably higher than ours for target frequencies of 250 and 1000 Hz for the phase condition $\phi = 0$. We believe that the reason for the discrepancy is that Phipps and Henning listened to a segment of the stimulus which obscured the alternating envelope peaks for $\phi = 0$. Their stimulus was gated as shown by the dashed lines in Fig. 1. The alternating peaks occur just as the stimulus is switched on and off.¹ By contrast the valley region and the large frequency excursion for the $\phi = \pi/2$ condition were not obscured. The discrepancy between their data and ours supports the view that envelope variations mediate detection for $\phi = 0$. This explanation, however, does not account for the drop in threshold which Phipps and Henning found when they moved to 4000 Hz.

As noted in Sec. II, the two different phase conditions lead to very different stimuli, but, as noted in Sec. III, they result in approximately equal thresholds. There is no obvious reason why this should be so. Below we make a quantitative study of the envelope changes caused by the target to try to understand this result.

A. Envelope

The detection of differences between different envelopes is similar to standard difference limen tasks, such as intensity discrimination and amplitude modulation detection. Initially we expected that the results of these difference limen experiments could be used to predict performance in the two-masker task. This turns out not to be possible, as may be seen by examining the $\phi = 0$ condition. There the subject needs to distinguish the more intense beats from the less intense beats. The more intense beats are also longer in duration. For the time scale of this experiment the best comparison with difference limen experiments is obtained by integrating the power in the individual beats. At threshold (a=0.2) we find $\int \Delta I / \int I_{\min} = 0.875$. This can be compared with intensity jnds found by Jesteadt et al. (1977). Interpolating between their values for I=40 and I = 80 dB SPL gives $\Delta I/I = 0.16$, which is considerably smaller than 0.875. Alternatively consider that the threshold ratio of successive envelope peaks for $\phi = 0$ is 1.22. This can be compared to the threshold envelope peak to valley ratio in Zwicker's (1952) AM detection experiments, a value of 1.04, which is considerably smaller than 1.22.

It is not surprising that the difference limen experiments do not agree with the two-masker experiment. Whereas the former require subjects to detect a variation in an otherwise constant level, the latter requires subjects to detect a difference between two time-varying sensations. Evidently the latter is a more difficult task. Therefore we abandon attempts to relate the twomasker experiment to intensity difference limen experiments.

We next consider a model internal representation for the envelope. Our goal is to find some way in which the internal representations for the two phase conditions can be compared. Following standard energy detection theory we assume that the internal excitation is given by the square of the envelope passed through an integrator. The integrator is taken to be a one-pole low-pass filter (Green, 1965). This model predicts an internal excitation Ω , given by

$$\Omega(t) = k \int_{-\infty}^{t} e^{-k(t-t')} E^2(t') dt', \qquad (15)$$

where k is the inverse of the filter time constant. Because E is periodic (with period T) the convolution can be written in a form which involves integration over only a single period. The answer is

$$\Omega(t) = \frac{k}{e^{kT} - 1} \int_0^T dt' \, e^{kt'} E^2(t+t') \,, \tag{16}$$

a form which is convenient for numerical computation.

The excitation $\Omega(t)$ looks rather like the envelopes shown in Fig. 1. The most obvious effect of the integrator is to eliminate the zeros for the case of no target and for the case of $\phi = 0$. All excitations acquire valley regions like those in Fig. 1(b).

Our procedure is to suppose that there is some threshold value of $\Delta\Omega/\Omega$, above which the target is detected, and that this threshold applies to both $\phi = 0$ and $\phi = \pi/2$ conditions. We further suppose that for each phase condition, detection is mediated by that aspect of the excitation for which the presence of the target makes the biggest difference. Therefore, for the $\phi = 0$ condition, detection occurs when the ratio of the larger beat to the smaller beat exceeds the threshold criterion. For the $\phi = \pi/2$ condition, detection occurs when the valley excitation in the interval with the target exceeds the valley excitation in the interval with no target by the threshold ratio.

From numerical calculations we find that if $\Delta\Omega/\Omega$ =0.5 and if the time constant of the integrator is 10 ms then detection threshold occurs for a = 0.2 for both phase conditions, the result obtained in our experiments. A time constant of 10 ms is somewhat larger than is expected in an experiment which involves minimum integration time such as this one. For example, a value of 4 ms was found in the modulation transfer function studies of Viemeister (1979). We conclude that with a large, but not unreasonable, time constant one could explain the equality of detection thresholds in the two phase conditions as the result of equal excitation variations in the two conditions. According to this interpretation the observed equality does not reflect any particular invariance within the auditory system, rather one would regard it as accidental.

B. Frequency modulation

The frequency modulation (FM) present for $\phi = \pi/2$ has a curious form. The frequency excursion increases for increasing ω_m , and it *increases* for *decreasing* amplitude of the target. Under favorable conditions these two effects can be heard, and they can be seen in the output of an electronic frequency to voltage converter.

Although pitch modulation can be heard for some values of the target, in the $\phi = \pi/2$ condition, we cannot assume that the FM necessarily plays a role in detection. We would like to decide whether the FM can be expected to play a role by comparison with other psychoacoustical results, notably with Zwicker's (1952) FM detection data. However, near the detection level the FM modulating waveform, given by Eq. (12), is not even approximately the sine waveform used by Zwicker. In the limit of vanishing target (a=0) the frequency excursion actually diverges. But the frequency excursion itself is not a good indicator of the variation produced by FM. Previous experiments in our lab (Hartmann, 1977; Klein, 1980) compared the sensations of suprathreshold pitch modulation created by numerous different FM waveforms. Two empirical rules provide a rather good summary of the data: (1) Two different FM waveforms produce equal pitch variation sensations when their rms frequency excursions are equal; and (2) two different FM waveforms produce equal pitch variation sensations when the magnitudes of the first Fourier coefficients of the modulating waveforms are equal. A third model, based upon the average absolute value of the frequency excursion, is also often successful.

When these models of FM perception are applied to the $\phi = \pi/2$ condition they all make a paradoxical prediction. For a target at the observed detection level the above models predict that the FM sensation is about four times greater than the FM detection level which can be inferred from Zwicker's data, in the case of 250- and 1000-Hz targets. Further, the models predict that the FM sensation becomes monotonically *larger* as the target amplitude decreases.

An escape from this impossible dilemma is offered by Feth's model (1974) in which FM detection is based upon the envelope-weighted frequency excursion. Because the extreme frequency excursions coincide with envelope minima, Feth's model predicts that the sensation caused by FM decreases with decreasing target amplitude, as required for a stable detection result. The quantitative prediction of Feth's model for the FM sensation is rather interesting. The model predicts that when the target is at the detection level for the ϕ $=\pi/2$ condition then the FM sensation is exactly equal to the detection level found in Zwicker's experiments for 250 and 1000 Hz, but that the FM sensation is more than a factor of 2 below Zwicker's detection level for 4000 Hz. Feth developed his model in connection with discrimination experiments involving two closely spaced sine tones of unequal amplitude. The model was partially successful in accounting for the data. Feth's stimulus is similar in many ways to the two-masker plus target stimulus considered here. It is possible to take the view that for $\phi = \pi/2$ and $f_s = 250$ or 1000 Hz detection of the target is mediated by the FM sensation.

According to our calculations Feth's model would provide a good description of how this is done. In any event, it is the only one of the models considered here that could possibly apply to our case.

V. CONCLUSION

We have repeated the Phipps and Henning two-masker experiments for masker frequencies 2.5 Hz above and below the target frequency. We do not find the large dependence on phase angle found by Phipps and Henning, and we interpret the discrepancy as a result of the brief stimulus duration in the Phipps and Henning experiments. The discrepancy supports the view that detection is mediated by the changes that the target makes in the envelope of the stimulus.

Our experiments with long duration stimuli show thresholds which are approximately independent of phase condition. We have not found an entirely satisfactory way to account for the data. The thresholds do not correlate well with those which can be inferred from standard difference limen experiments. It is possible to construct a model, in which threshold is based upon percentage changes in internal excitation, that agrees with the phase independent thresholds. The model is plausible, but contains enough free parameters that no significance can be attached to the agreement with experiment, other than the fact that the final parameters are not unreasonable. This result, too, offers some support for the view that detection is mediated by changes in the envelope.

The role of frequency modulation for the $\phi = \pi/2$ condition is difficult to evaluate. Feth's model is consistent with the view that detection in this condition is mediated by detection of FM, for targets at 250 and 1000 Hz. However, it is not clear that this model provides a reasonable estimate when the FM is accompanied by large envelope variations. These variations can be expected to make the FM harder to detect. All the models, and subjective listening too, agree that the FM plays no role for 4000-Hz targets. In fact, our thresholds for $\phi = \pi/2$ and 4000 Hz are slightly higher than thresholds for other conditions. Possibly, in the general case, detection in the $\phi = \pi/2$ condition is mediated by some combination of envelope and FM detection.

Although the two-masker experiment may provide important information on critical band widths when the maskers are well separated in frequency, the case of small separation leads to a complicated stimulus. We have not even been able to decide whether envelope effects or frequency effects are paramount in detection. It seems to us, therefore, that the two-masker experiment for small masker separations is a difficult route to improved understanding of signal detection in hearing.

ACKNOWLEDGMENTS

This paper has benefited from helpful comments by Dr. G.B. Henning and M.A. Klein. The work was supported by the National Science Foundation, grant BNS 79-14155.

¹The time interval selected by Phipps and Henning for 5-Hz masker separation was particularly unfavorable for detecting the target in the $\phi = 0$ phase condition. They found a threshold of -3 dB. At this target level the change in the level of the smaller peak in Fig. 1(a) caused by the target is 4 dB. Therefore it is probable that had Phipps and Henning chosen a time interval centered on the smaller peak they would have obtained a lower threshold. But although the particular choice of time interval may have been partly responsible for the gap between $\phi = 0$ and $\phi = \pi/2$ levels, this choice was probably not responsible for all of the gap. There are two pieces of evidence to support this view. Phipps and Henning performed the experiment at other masker separations. In particular, for a masker separation of 10 Hz the time window, relative to the inverse frequency scale in Fig. 1, was twice as long as for 5 Hz and therefore included the smaller peak. The gap between the two phase conditions was found to be reduced from its 5-Hz value, but it was not eliminated altogether. Further we have performed informal experiments using 5-Hz masker separation and stimuli durations of 200 ms. Because our envelope was not correlated with the time interval, envelope features caused by the target were randomized from trial to trial. We found that the gap between $\phi = 0$ and $\phi = \pi/2$ phase conditions persisted in this case.

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