Phase Coherence of Conduction Electrons Below the Kondo Temperature

Gassem M. Alzoubi and Norman O. Birge*

Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824-2320, USA

(Received 6 July 2006; published 30 November 2006)

We have measured the phase decoherence rate \( \tau_\phi^{-1} \) of conduction electrons in disordered Ag wires implanted with 2 and 10 ppm Fe impurities, by means of the weak-localization magnetoresistance. The Kondo temperature of Fe in Ag, \( T_K \approx 4 \text{ K} \), is in the ideal temperature range to study the progressive screening of the Fe spins as the temperature \( T \) falls below \( T_K \). The contribution to \( \tau_\phi^{-1} \) from the Fe impurities is clearly visible over the temperature range \( 40 \text{ mK} - 10 \text{ K} \). Below \( T_K, \tau_\phi^{-1} \) falls rapidly until \( T/T_K \approx 0.1 \), in agreement with recent theoretical calculations. At lower \( T \), \( \tau_\phi^{-1} \) deviates from theory with a flatter \( T \)-dependence. Understanding this anomalous dephasing for \( T/T_K < 0.1 \) may require theoretical models with larger spin and number of channels.

DOI: 10.1103/PhysRevLett.97.226803

PACS numbers: 73.23.-b, 72.15.Qm, 73.20.Fz

The Kondo problem is a paradigm many-body problem in condensed matter physics. In recent years, Kondo physics has been observed in semiconducting quantum dots [1] and carbon nanotubes, while Kondo lattices play a major role in some strongly-correlated materials [2]. The original problem that motivated Kondo’s 1964 paper [3] was the increase in resistivity at low-temperature of metals containing magnetic impurities. Although Kondo solved this puzzle, his perturbation theory diverged in the limit of zero-temperature. Wilson [4] showed that, at temperatures far below the Kondo temperature \( T_K \), the magnetic impurity forms a spin-singlet with the surrounding conduction electrons and behaves as a nonmagnetic scatterer with cross section given by the unitarity limit.

The brief history given above might leave the impression that the behavior of dilute magnetic impurities in metals is completely understood. That is, however, not the case. An important aspect of the Kondo problem concerns the distinction between elastic and inelastic scattering. This distinction is extremely important in the context of quantum transport and mesoscopic physics, where it has been known since 1979 that elastic scattering from static disorder preserves quantum phase coherence, whereas inelastic scattering destroys it [5]. The magnetic impurity contribution to the conduction electron phase coherence rate, \( \tau_\phi^{-1} \), was first measured explicitly by two groups in 1987 [6,7], and has received renewed attention recently [8,9] in the context of the debate over zero-temperature decoherence in disordered metals [10,11]. Until very recently [12,13], however, there was no theoretical expression for the temperature dependence of the inelastic scattering rate valid for \( T \) not too far below \( T_K \), and very little reliable data in that temperature range [14].

The most reliable estimates of \( \tau_\phi^{-1} \) come from analysis of low-field magnetoresistance data in the context of weak-localization theory. In disordered metals without magnetic impurities, \( \tau_\phi^{-1} \) is dominated by electron-phonon scattering at temperatures above about 1 K and by electron-electron scattering at lower \( T \) [11]. In the presence of magnetic impurities, \( \tau_\phi^{-1} \) contains the additional contribution \( \gamma_m \), which peaks at \( T \approx T_K \). In order to observe this peak in \( \gamma_m \) while keeping the magnetic impurity concentration low enough to avoid interactions between impurities, one must choose a system with \( T_K \) below about 10 K; otherwise, \( \tau_\phi^{-1} \) is dominated by electron-phonon scattering. In order to acquire data far below \( T_K \), however, it is important to keep \( T_K \) as high as possible. The optimal range for \( T_K \) is a few Kelvins, which is achieved with Fe impurities in Ag [15,16].

We fabricated Ag wires of dimensions \( L = 780 \text{ \mu m} \), \( w = 0.1-0.2 \text{ \mu m} \) and \( t = 47 \text{ nm} \) on oxidized Si substrates using electron-beam lithography, thermal evaporation from a 99.9999%-purity Ag source, followed by liftoff of the bilayer resist. All the wires studied were evaporated simultaneously to ensure similar microstructure and resistivity. The 15 samples were divided into four batches—one that was kept pure, and three that were implanted with 2, 6, and 10 ppm of Fe impurities, respectively. SRIM simulations of the implantation at 70 keV indicate that about 90% of the implanted ions stay in the Ag wires, with the rest going into the substrate. As none of the 6 ppm samples survived, we report data taken on a pure sample and samples with 2 and 10 ppm Fe impurities. Table I lists the sample parameters. All samples were measured immersed in the mixing chamber of a dilution refrigerator with filtered leads. Four-probe resistance measurements were made using a lock-in amplifier with a ratio transformer to improve sensitivity [11]. The voltage drop across the sample was limited to \( eV_i \leq k_B T \) to avoid heating the electrons.

Raw magnetoresistance (MR) data for the three samples, at \( T \approx 1.8 \text{ K} \), are shown in the inset to Fig. 1. MR data for each sample were fit using the following procedure. Because Ag has moderate spin-orbit scattering (\( \tau_{so} \approx 40 \text{ ps} \) for all of our samples), the MR is positive at low temperatures, when \( \tau_\phi \gg \tau_{so} \). At higher temperatures, when \( \tau_\phi < \tau_{so} \), the MR starts out positive but then turns around at a field scale \( B \approx 20 \text{ mT} \). Data at several temperatures in this higher temperature range (which was...
different for each sample) were first fit with three free parameters: \( L_\phi = (D \tau_\phi)^{1/2} \), where \( D \) is the diffusion constant, \( L_{so} = (D \tau_{so})^{1/2} \), and the sample width, \( w \). For each sample, these fits gave consistent values of \( L_{so} \) and \( w \) over a broad temperature range \([17]\). We then fixed these values of \( L_{so} \) and \( w \), and fit the MR curves for all temperatures with \( L_\phi \) as the only free parameter. Using the value of \( D \) obtained from the resistance and sample dimensions, we finally obtain \( \tau_\phi \) as a function of temperature for each sample.

Figure 1 shows the decoherence rate, \( \tau_\phi^{-1} \), obtained from the MR data and fitting procedure described above, for the three samples. The \( \tau_\phi^{-1} \) data for the pure sample follow theoretical expectations above 200 mK, with a modest amount of saturation at lower temperature \([18]\). Above 200 mK, the data are fit well (solid line) by a combination of electron-phonon scattering (\( \tau_\phi^{-1} \propto T^3 \)) and electron-electron scattering (\( \tau_\phi^{-1} \propto T^2/3 \)). The prefactor of the \( T^{2/3} \) term is equal to 0.36 ns\(^{-1}\) K\(^{-2/3}\), in good agreement with the theoretical value \([19]\). The additional contribution to \( \tau_\phi^{-1} \) from the Fe impurities, \( \gamma_m \), is clearly observable in the 2 ppm sample up to 4 K and in the 10 ppm sample up to 10 K. Above 10 K, extraction of \( \gamma_m \) from \( \tau_\phi^{-1} \) is not reliable, due to a apparent difference in the magnitude of the electron-phonon scattering rate in the 10 ppm and pure samples.

The suppression of the weak-localization magnetoresistance by magnetic impurities was first discussed in \([20]\) for the case of static impurity spins. Later, Fal’ko \([21]\) pointed out that the impurity spins have their own internal dynamics given by the Korringa rate, \( \gamma K \propto k_B T \). The ratio of the spin-flip scattering and Korringa rates is \( \gamma_m/\gamma_K = n_s/\nu_F k_B T \), where \( n_s \) is the density of magnetic impurities and \( \nu_F \) is the total density of states of conduction electrons at the Fermi level. Below \( T_K \), the Korringa rate saturates, so the ratio becomes \( \gamma_m/\gamma_K = n_s/\nu_F k_B T K \). If \( \gamma_K \gg \gamma_m \), then the impurity spin configuration is randomized during the time between spin-flip scattering events of the conduction electrons. The contribution of magnetic scattering to the total decoherence rate in the spin-singlet channel of the weak-localization magnetoresistance is given by \([21]\)

\[
\tau_\phi^{-1} = \tau_{in}^{-1} + 2\gamma_m \quad \text{for} \quad \gamma_m > \gamma_K \\
\tau_\phi^{-1} = \tau_{in}^{-1} + \gamma_m \quad \text{for} \quad \gamma_m < \gamma_K
\]

where \( \tau_{in}^{-1} \) is the dephasing rate due to electron-electron and electron-phonon scattering. For our Ag sample with 2 ppm Fe, assuming \( T_K \) in the range 2–4 K (see below), we find \( \gamma_m/\gamma_K = 0.02–0.04 \ll 1 \); hence, we should use Eq. (2) to extract \( \gamma_m \) from \( \tau_\phi^{-1} \). Figure 2 shows \( \gamma_m \) for the 2 ppm and 10 ppm samples obtained from the \( \tau_\phi^{-1} \) data in Fig. 1 using Eq. (2), with \( \tau_{in}^{-1} \) obtained from the data on the nominally pure sample. The data sets from the 2 ppm and 10 ppm samples are consistent with each other when we use Eq. (2) to analyze both of them.

The criterion \( \gamma_m/\gamma_K \ll 1 \) is a necessary condition for the theoretical approach of Micklitz et al. \([13]\) in their numerical renormalization group (NRG) calculations of \( \gamma_m \). Those authors note that Eq. (2) is not strictly correct because the decoherence induced by electron-electron interactions \([19]\) is not an exponential process. Eq. (12) in \([13]\) shows how to add the decoherence rate due to e-e interactions with the rate due to all other processes including \( \gamma_m \). We have compared the results of analyzing our data with Eq. (2) and with Eq. (12) in \([13]\), and we find that the difference is at most 15% for the 2 ppm sample at the

![Figure 1](https://example.com/figure1.png)

**FIG. 1** (color online). Total decoherence rate vs temperature for a pure Ag sample (△), and for samples with implanted Fe concentrations of 2 ppm (●) and 10 ppm (■). Inset: Raw magnetoresistance data at \( T = 1.8 \) K for the three samples, and fits to weak-localization theory.
On the other hand, previous measurements of \( T \) = 0.4 K only. Fitting to the 2 ppm (10 ppm) data alone gives to reproducing the temperature dependence of \( \gamma_m \) to several striking conclusions. First, as noted recently the data and fits shown in Fig. 2 lead immediately to striking conclusions. First, as noted recently the data and fits shown in Fig. 2 lead immediately to striking conclusions. First, as noted recently the data and fits shown in Fig. 2 lead immediately to striking conclusions. First, as noted recently the data and fits shown in Fig. 2 lead immediately to striking conclusions. First, as noted recently

The solid line is the theoretical calculation of Micklitz et al. [13] fit to the data for \( T > 0.4 \) K. The dotted line is the Suhl-Nagaoka approximation for \( s = 1/2 \). Inset: Same plot with a linear vertical scale. Uncertainties are larger for the 10 ppm sample due to smaller signal-to-noise in the measurements.

FIG. 2 (color online). Inelastic scattering rate due to magnetic impurities for the 2 ppm (●) and 10 ppm (■) samples. Data for the 2 ppm sample are multiplied by 5. The solid line is the theoretical calculation of Micklitz et al. [13] fit to the data for \( T > 0.4 \) K. The dotted line is the Suhl-Nagaoka approximation for \( s = 1/2 \). Inset: Same plot with a linear vertical scale. Uncertainties are larger for the 10 ppm sample due to smaller signal-to-noise in the measurements.

The lowest temperature, which is not observable on the plots in Fig. 2. The solid line in Fig. 2 shows a fit of the NRG calculation from [13] to our data at temperatures above 0.4 K only. Fitting to the 2 ppm (10 ppm) data alone gives \( T_K = 4.8 \) K (5.4 K). For comparison, the Suhl-Nagaoka approximation with \( T_K = 4.8 \) K is shown as the dotted line. The data and fits shown in Fig. 2 lead immediately to several striking conclusions. First, as noted recently the Suhl-Nagaoka approximation does not come close to reproducing the temperature dependence of \( \gamma_m \) for \( T < T_K \). Second, the NRG theory of Micklitz et al. [13] fits the data reasonably well over the temperature range \( T/T_K = 0.1-2 \). And third, the NRG theory deviates strongly from the data when \( T/T_K < 0.1 \).

According to [13], the maximum value of \( \gamma_m \), occurring at \( T = T_K \), is 0.23 \( \times 4\pi\gamma/(\pi\hbar\nu_F) \). For 10 ppm of magnetic impurities in Ag, the theoretical estimate is \( \gamma_{\phi}^{\max} = 16 \) ns\(^{-1} \), whereas the data in Fig. 2 show a maximum value about twice as large. This significant discrepancy may point to the inadequacy of the spin-1/2 theory of [13] to account for the large spin (\( s = 2 \)) of Fe impurities in Ag. On the other hand, previous measurements of \( \gamma_m \) in Ag samples implanted with Mn impurities [11], with \( s = 5/2 \) and \( T_K = 40 \) mK, were consistent with the theoretical estimate. (Those data were analyzed with the Suhl-Nagaoka approximation, which is in close agreement with the theory of [13] for \( T > T_K \).) It thus appears that the inelastic scattering cross section of Fe in Ag is roughly twice that of Mn in Ag.

To further test the theory, we seek an independent estimate of \( T_K \) to compare with the value obtained from the fit to \( \gamma_m \) shown in Fig. 2. The traditional method of determining \( T_K \) from the resistivity vs temperature, \( \rho(T) \), is not reliable for our samples because the electron-phonon scattering contribution to \( \rho(T) \) starts to grow significantly when \( T > 8 \) K, before the Kondo contribution has completely died out. Instead, we analyze the high-field magnetoresistance, \( \rho(B) \), shown at several temperatures in Fig. 3. We fit the \( \rho(B) \) data using NRG calculations from Ref. [22]. For \( T \ll T_K \), these are nearly identical to the \( T = 0 \) analytical solution from [23]. The two-parameter fit to the 83 mK data set gives the values \( \Delta\rho_K = \rho(B = 0, T = 0) - \rho(B \gg B_K, T = 0) = 0.57 \) nΩ cm/ppm and \( B_K = 1.2 \) T, where \( g\mu_B B_K = k_B T_K \), with \( \mu_B \) the Bohr magneton [24]. Using \( g = 2 \) [16], that in turn gives \( T_K = 1.6 \) K. Similar fits to the \( \rho(B) \) data sets at higher temperatures, keeping \( \Delta\rho_K \) fixed, show that they can not be fit consistently with such a low value of \( T_K \).

To circumvent this problem, we have performed a global fit to all the \( \rho(B) \) data sets with only three parameters: \( \Delta\rho_K, B_K \), and \( T_K \), where \( B_K \) determines the magnetic field scale over which \( \rho(B) \) decreases, while \( T_K \) determines the temperature scale over which \( \rho(B = 0) \) decreases. The results of this global fit are shown by the solid lines in Fig. 3, with the values \( B_K = 1.36 \) T, \( T_K = 2.96 \) K, and \( \Delta\rho_K = 0.58 \) nΩ cm/ppm. The value of \( B_K \) corresponds to a Kondo temperature of 1.8 K. The discrepancy between the values of \( T_K \) extracted from the field scale and from the

FIG. 2 (color online). Inelastic scattering rate due to magnetic impurities for the 2 ppm (●) and 10 ppm (■) samples. Data for the 2 ppm sample are multiplied by 5. The solid line is the theoretical calculation of Micklitz et al. [13] fit to the data for \( T > 0.4 \) K. The dotted line is the Suhl-Nagaoka approximation for \( s = 1/2 \). Inset: Same plot with a linear vertical scale. Uncertainties are larger for the 10 ppm sample due to smaller signal-to-noise in the measurements.

FIG. 3 (color online). Change in resistivity vs magnetic field for the 10 ppm sample at temperatures of 83, 188, 550, 860, and 1880 mK, from top to bottom. The zero for the 83 mK data is shown as the dotted line. Similar fits to the 83 mK data set gives the values \( \Delta\rho_K = 0.57 \) nΩ cm/ppm and \( B_K = 1.2 \) T, where \( g\mu_B B_K = k_B T_K \), with \( \mu_B \) the Bohr magneton [24]. Using \( g = 2 \) [16], that in turn gives \( T_K = 1.6 \) K. Similar fits to the \( \rho(B) \) data sets at higher temperatures, keeping \( \Delta\rho_K \) fixed, show that they can not be fit consistently with such a low value of \( T_K \).
temperature scale highlights the fact that we are using a spin-1/2 theory. For free spins in a metal without Kondo effect, the magnetoresistance is proportional to $-(s_r)^2$, where $s_r$ is the average spin polarization given by the appropriate Brillouin function. A comparison of the functions $-(s_r)^2$ for $s=1/2$ and $s=2$ shows that the full width at half maximum of the $s=1/2$ function is 1.85 times larger than for the $s=2$ function. If a similar relationship holds for the $\rho(B)$ curves in the Kondo regime, then the value $B_K = 1.2$ T we found from the $s=1/2$ fit to the 83 mK data would become $B_K = 2.2$ T for $s=2$, which corresponds to a Kondo temperature $T_K = 3.0$ K. That is consistent with the value of $T_K$ obtained from the temperature dependence of $\rho(B=0)$, but still lower than the value $T_K = 5$ K obtained from the fit to $\gamma_m$.

The experimental value $\Delta \rho_K = 0.58 \text{n}\Omega \text{cm/ppm}$ should be compared with the unitarity limit for $s$-wave scattering, $\Delta \rho_K = 4\pi n_0 \sigma_F / n e^2 k_F = 0.43 \text{n}\Omega \text{cm/ppm}$ for Ag, where $k_F$ is the Fermi wave vector. (The value for $d$-wave scattering is 5 times higher.) Our measured value of $\Delta \rho_K$ is larger than the theoretical $s$-wave value, but smaller than the measured values in other Fe-noble metal Kondo systems: 1.3 nΩ cm/ppm in Cu:Fe and 1.0 nΩ cm/ppm in Au:Fe [25].

The most serious discrepancy between theory and experiment is the flattening of $\gamma_m(T)$ for $T/T_K < 0.1$, shown in Fig. 2. Although the behavior of the data is reminiscent of Eq. (3b) in [26] (which describes inelastic scattering by an underscreened Kondo impurity when $T \ll T_K$), a recent calculation of the inelastic scattering cross section for the underscreened single-channel model with $s = 1$ shows that such a model cannot fit our data for $0.1 < T/T_K < 1$ [27]. The issue of screening for Fe in Ag is unclear. At first glance, the five channels corresponding to the $d$-orbitals should overscreen the $s = 2$ Fe spin. In reality, however, the different channels have different coupling constants, and the impurity spin and orbital degeneracies are broken by crystal fields and spin-orbit scattering. Sacramento and Schlottmann showed that a multichannel Kondo model with $s = 2$ and $n = 4$ (i.e., neither under nor overscreened) gave a good fit to data for several equilibrium quantities in the Ag:Fe and Cu:Fe systems [16]. A calculation of the phase decoherence rate in that model might resolve the discrepancy between theory and experiment shown in Fig. 2.

This work was supported by the NSF under Grant No. DMR-0405238, and by the Keck Microfabrication Facility under NSF No. DMR-9809688. We are grateful to A. Rosch and T. Costi for sending us the results of numerical calculations from [13,22], respectively. We also thank C. Bäuerle, L. Glazman, S.D. Mahanti, R. Ramazashvili, M. Vavilov, and G. Zarand for helpful discussions.

Note added.—We recently became aware of a related work with similar results [28]. Those authors conclude that neither underscreened nor overscreened models can fit the low-temperature inelastic scattering rate.