Collisional and Radiative Energy Loss of a Heavy Quark in a QPG Plasma

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- Collisional energy loss
- Dead cone for radiation from heavy quarks?
- Radiative energy lass
- Results for RHIC

Weak points of the existing calculations of coll energy loss

R_{AA} or energy loss is determined by the elementary elastic scattering cross sections.
q channel:

$$\frac{d\sigma_F}{dt} = \frac{g^4}{\pi(s-M^2)^2}\left[\frac{(s-M^2)^2}{(t-\kappa m_D^2)^2} + \frac{s}{t-\kappa m_D^2} + \frac{1}{2}\right]$$

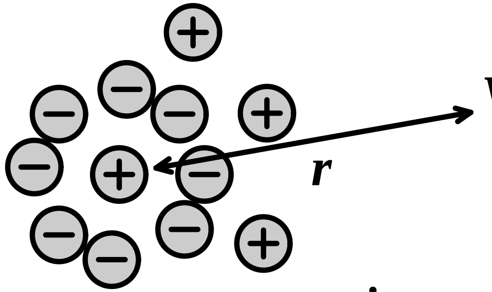
Neither $g2= 4\pi\ \alpha(t)$ nor $\kappa m_D{}^2$= are well determined

$\alpha(t)$ =is taken as constant [0.2 < α < 0.6] or $\alpha(2\pi T)$
$m_{Dself}{}^2\ (T) = (1+n_f/6)\ 4\pi a_s(\ m_{Dself}{}^2)\ xT^2$ (Peshier hep-ph/0607275)

But which κ is appropriate?
κ =1 and α =.3: large K-factors are necessary to describe data

Is there a way to get a handle on α and κ ?

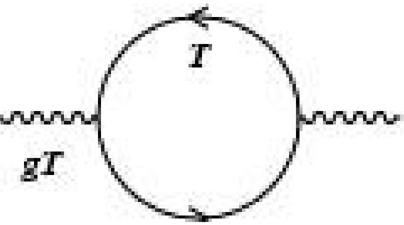
A) Debye mass



$V(r) \sim \frac{\exp(-m_D\, r)}{r}$

m_D regulates the long range behaviour of the interaction

PRC78 014904, 0901.0946



Loops are formed

If t is small (<<T) : Born has to be replaced by a hard thermal loop (HTL) approach like in QED:
(Braaten and Thoma PRD44 (91) 1298,2625)

For t>T Born approximation is ok

QED: the energy loss (ω = E-E')

$$-\frac{dE}{dx} = \frac{1}{2Ev}\int \frac{d^3k}{(2\pi)^3 2k}\int \frac{d^3k'}{(2\pi)^3 2k'}\int \frac{d^3p'}{(2\pi)^3 2E'} \times (2\pi)^4 \delta^{(4)}(p+k-p'-k') \sum \frac{n_i(k)}{d_i} |\mathcal{M}_i|^2\, \omega\,.$$

Energy loss indep. of the artificial scale t^* which separates the 2 regimes

.

Extension to QCD

HTL in QCD cross sections is too complicated for simulations

Idea: - Use HTL ($t<t^*$) and Born ($t>t^*$) amplitude to calculate dE/dx
make sure that result does not depend on t^*

- use the cross section in Born approximation

$$\frac{d\sigma_F}{dt} = \frac{g^4}{\pi(s-M^2)^2}\left[\frac{(s-M^2)^2}{(t-\mu)^2} + \frac{s}{t-\mu} + \frac{1}{2}\right]$$

and determine μ in that way that HTL calculation and Born approximation give the same energy loss

In reality a bit more complicated: with Born matching region of t^* outside the range of validity of HTL (<T) -> add to Born a constant μ'

Constant coupling constant -> Analytical formula -> Phys.Rev.C78:014904
Running -> numerically

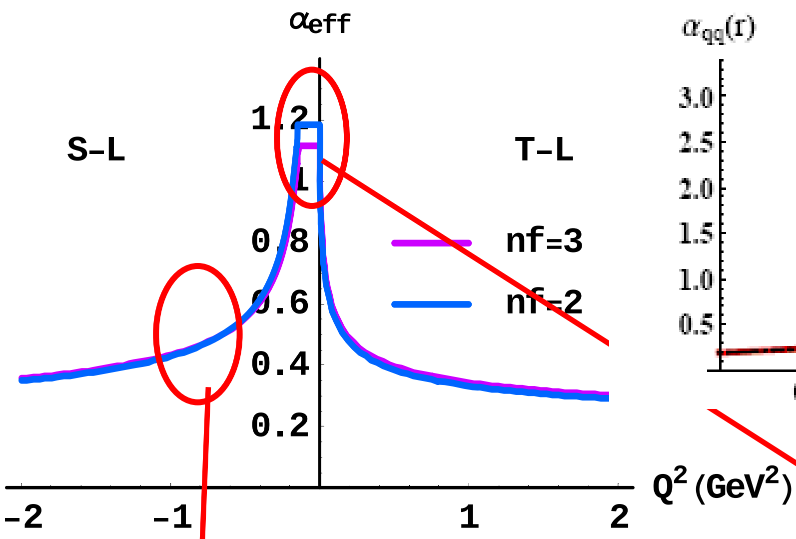
B) Running coupling constant

- Effective $\alpha_s(Q^2)$ (Dokshitzer 95, Brodsky 02)

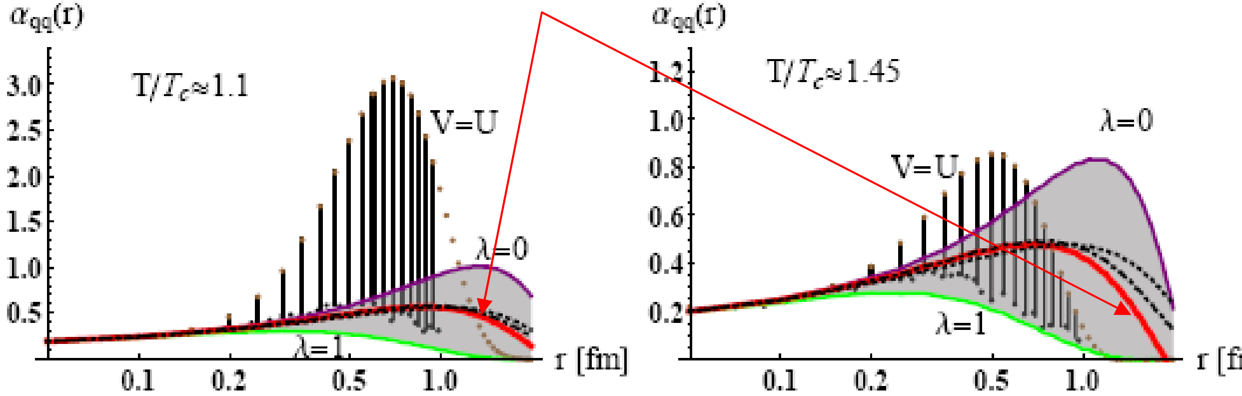
Observable = effective coupling * Process dependent fct

"Universality constrain" (Dokshitzer 02) helps reducing uncertainties:

$$\frac{1}{Q_u}\int_{|Q^2|\le Q_u^2} dQ\,\alpha_s(Q^2) \approx 0.5$$



Our choice

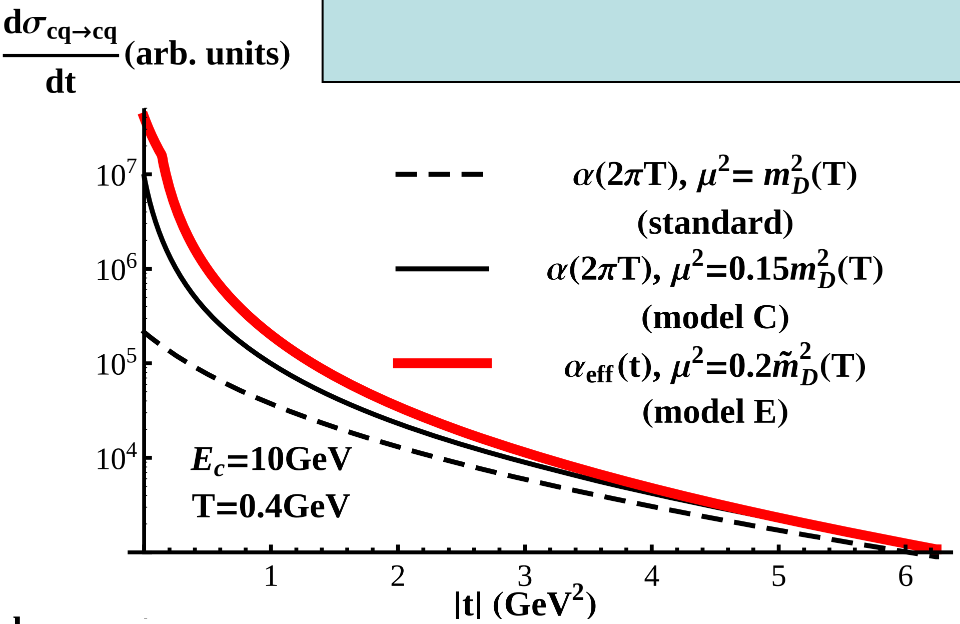


Comp w lattice results PRD71,114510

Large values for intermediate momentum-transfer

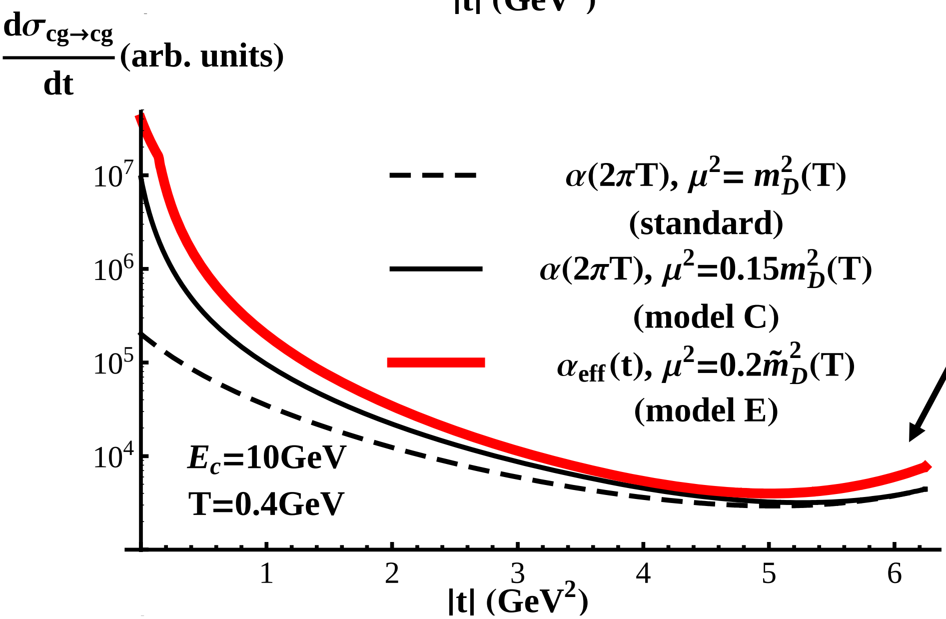
IR safe. The detailed form very close to $Q^2 = 0$ is not important does not contribute to the energy loss

The matching gives $\mu \approx 0.2\ m_D$ for running α_s for the Debye mass and $\mu \approx 0.15\ m_D$ not running!



Large enhancement of
cross sections at small t

Little change at large t



Largest energy transfor
from u-channel gluons

Au + Au @ 200 AGeV

. c,b-quark transverse-space distribution according to Glauber

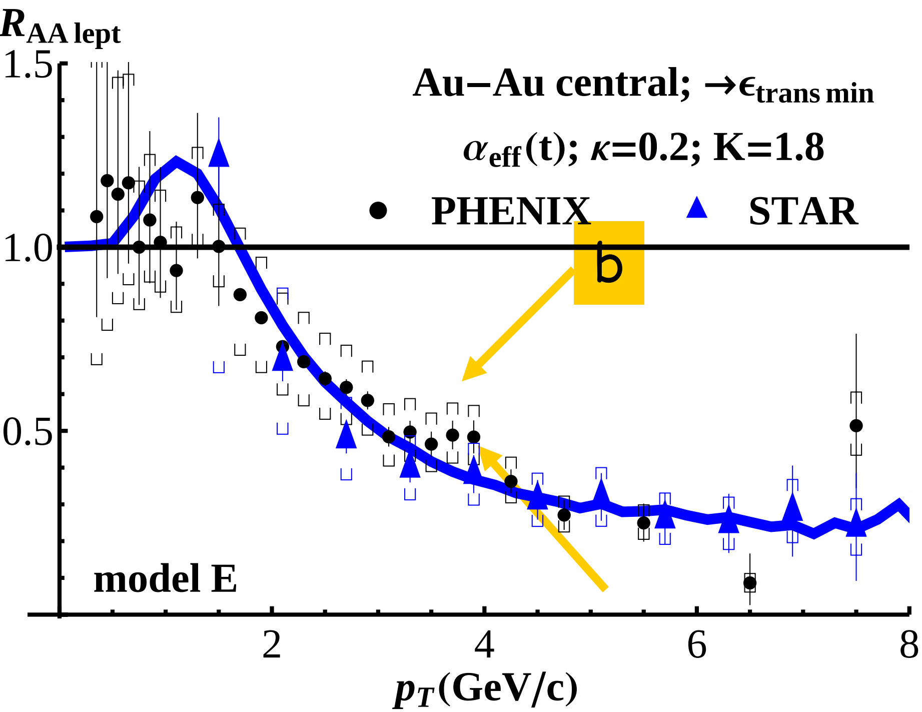
• c,b-quark transverse momentum distribution as in d-Au (STAR)... seems very similar to p-p (FONLL) ⇒ Cronin effect included.

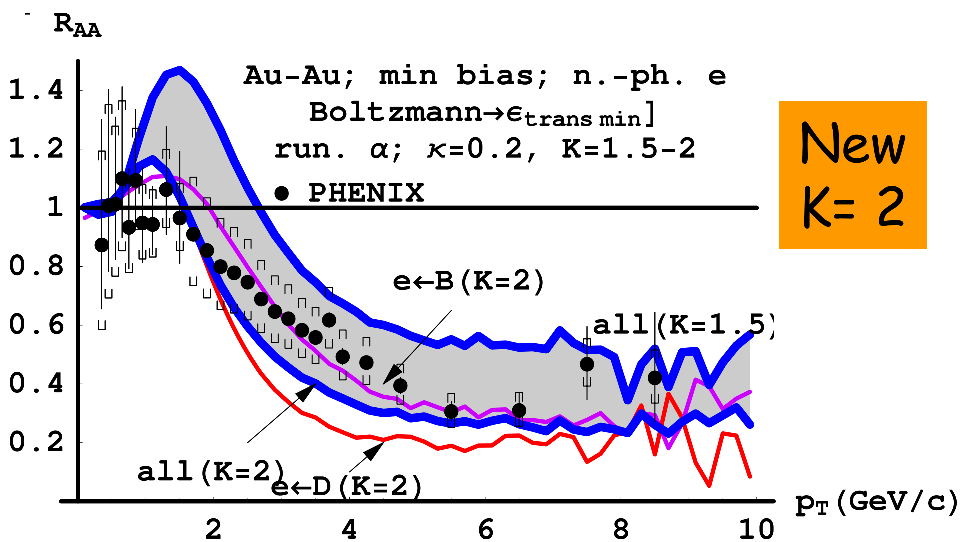
• c,b-quark rapidity distribution according to R.Vogt (Int.J.Mod.Phys. E12 (2003) 211-270).

• QGP evolution: 4D / Need local quantities such as T(x,t) ⇒ taken from hydro dynamical evolution (Heinz & Kolb)

•D meson produced via coalescence mechanism. (at the transition temperature we pick a u/d quark with the a thermal distribution) but other scenarios possible.

RAA=dn/dp$_t$ (AA)/(dn/dp$_t$(pp)* N$_{bin}$)





New
K= 2

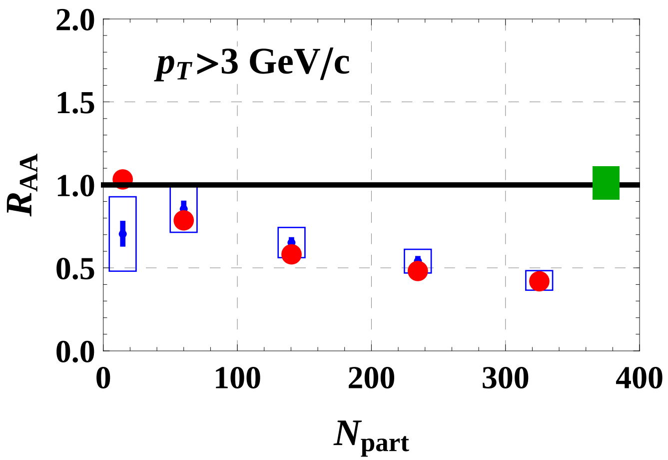
Central and minimum bias events described by the same parameters.
The new approach reduces the K- factor

K=12 -> K=2

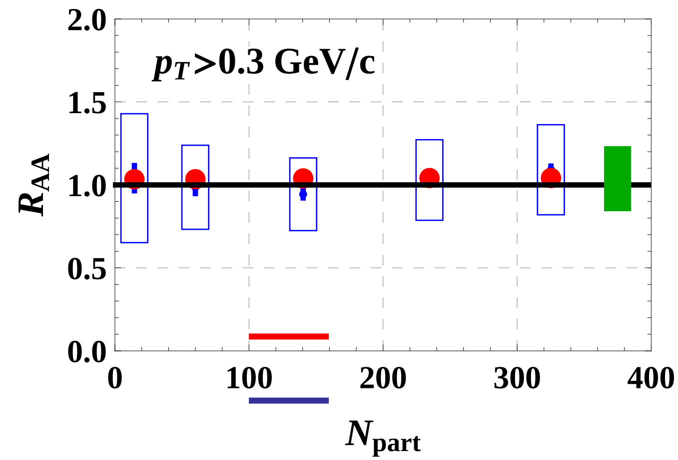
p_T > 2 bottom dominated!!
more difficult to stop,
compatible with experiment

Difference between b and c becomes smaller
in minimum bias events

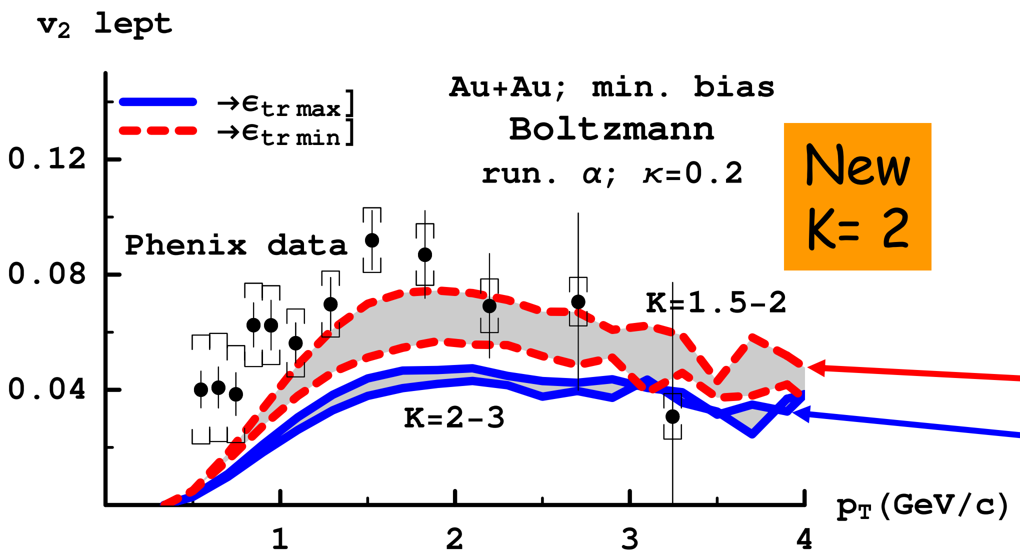
Centrality dependence of integrated yield



New K= 2



minimum bias out of plane distribution v_2 =<cos2φ>



New K= 2

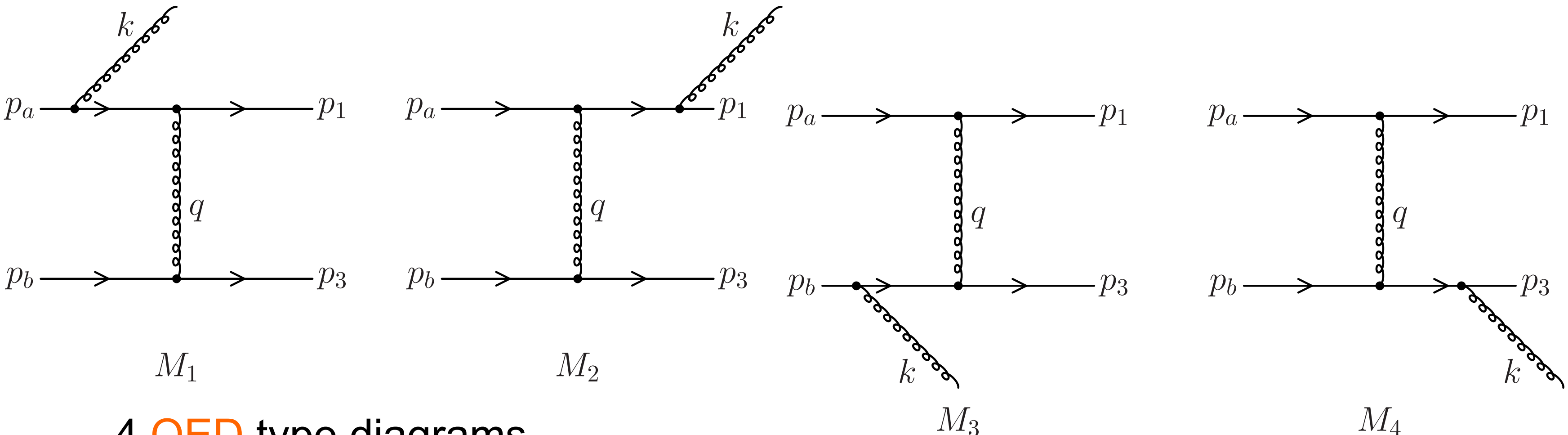
v_2 of heavy mesons depends on where fragmentation/ coalescence takes place

end of mixed phase

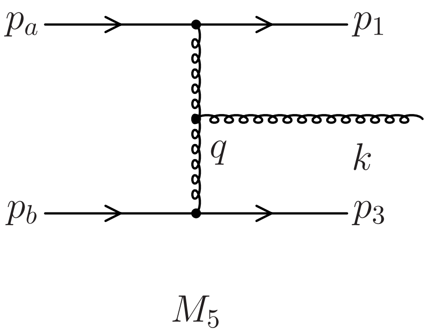
beginning of mixed phase

Deadcone and Radiative Energy Loss

Low mass quarks : radiation is the dominant energy loss process
Charm and bottom: radiation should be of the same order as collisional



4 QED type diagrams



1 QCD type diagram

Thanks to the commutator rel of the Color SU(3) operators $T^b T^a = T^a T^b - i f_{abc} T^c$

we can regroup M1-M5 into 3 gauge invariant subgroups

$$M^1_{QED} = T^a T^b (M_1 + M_2) \quad M^2_{QED} = T^a T^b (M_3 + M_4)$$

$$M_{QCD} = i f_{abc} T^c (M_1 + M_3 + M_5)$$

M_{QCD} dominates the radiation

Approximations

straight forward calculation is of little use: 5 dim numerical integration
result not transparent

To come to a transparent form:

- interest: energy loss of large p_t quarks -> leading order in $\sqrt{s}$

- spinor QCD and scalar QCD calculations are in leading order rather similar: corrections of the order

$$M_{QCD} = M_{SQCD}(1 - \frac{(\omega/E)^2}{1-\omega/E})$$

because radiation is dominated by

$$\omega/E << 1$$

we can calculate M_{QCD} in scalar QCD (SQCD)

- light cone gauge (only 2 diagrams contribute)

- proper choice of the coordinate system

M_{SQCD} in light cone gauge

In the limit $\sqrt{s} \to \infty$ the radiation matrix elements factorizes in

$$M_{tot} = M_{elast} \cdot P_{radiation}$$

$$|M_{SQCD}|^2 = D^{QCD} g^2 4(1-x)^2 |M_{elast}|^2 \left(\frac{\vec{k}_t}{k_t^2+(\omega/E)^2 m^2} - \frac{\vec{k}_t-\vec{q}_t}{(\vec{q}_t-\vec{k}_t)^2+(\omega/E)^2 m^2}\right)^2$$

k_t , ω = transv mom/ energy of gluon
E = energy of the heavy quark
D^{QCD} = color factor

Emission from heavy q

Emission from g

no emission from light q

m=0 -> Gunion Bertsch

M_{QED}:

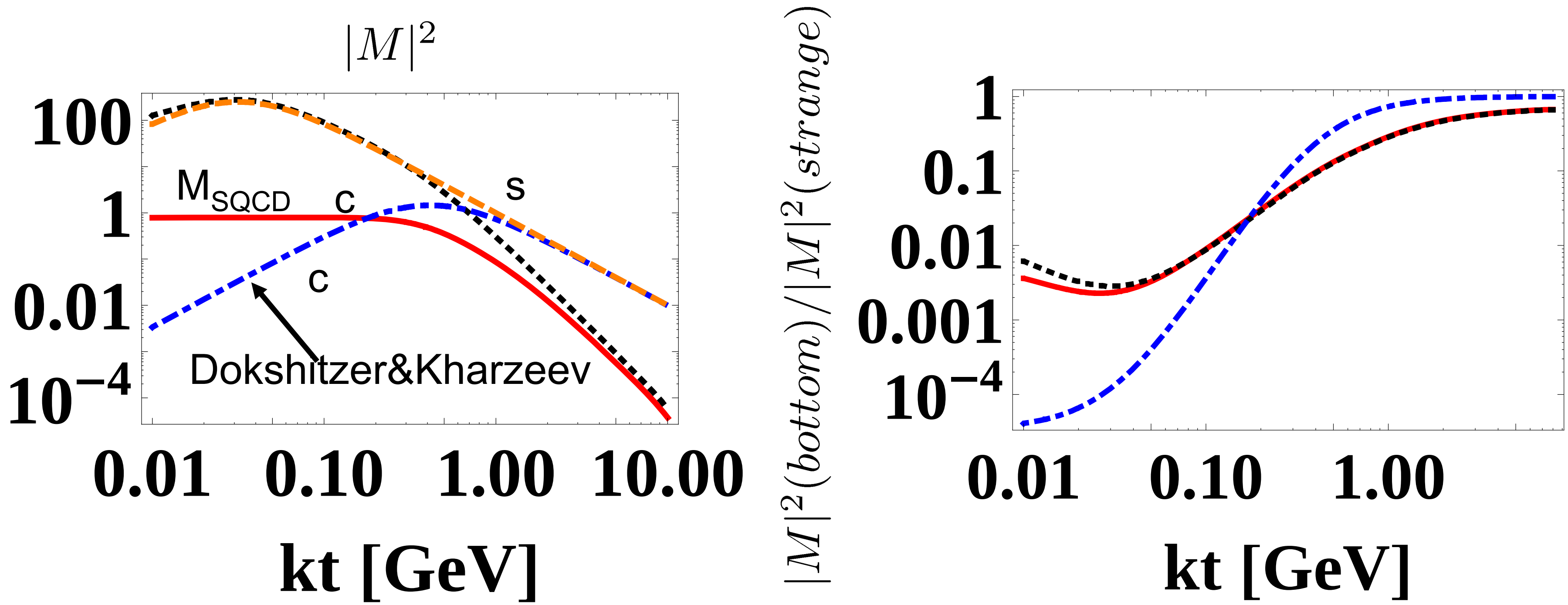
$$q_t \to \omega/E \cdot q_t$$

low k_t emission : Dead cone ?

The gauge invariant QCD matrix elements does not suppress completely radiation off heavy quarks at small k_t:

$$|M_{SQCD}|^2 \underbrace{=}_{k_t \to 0} \frac{q_t^2}{q_t^2 + m^2 x^2}.$$

Less suppression -> radiative energy loss more important



No real dead cone but strong suppression (as compared to light q)

radiative energy loss

Phase space limitation not serious k_t integration up to $k_t = \infty$

$$\int_0^\infty \frac{d\sigma^{qQ\to qQg}}{d^2k_t dx d^2q_t} d^2k_t = 6\pi \left(\frac{\left(q_t{}^2+2m^2x^2\right)\log\left(\frac{q_t{}^4+4m^2x^2q_t{}^2+\sqrt{q_t{}^4+4m^2x^2q_t{}^2}q_t{}^2}{m^2x^2\left(\sqrt{q_t{}^4+4m^2x^2q_t{}^2}-q_t{}^2\right)}+1\right)}{2\sqrt{q_t{}^4+4m^2x^2q_t{}^2}} - 1 \right).$$

After q_t integration: energy loss per collision:

$$x\frac{d\sigma}{dx} = (\omega/E)\frac{d\sigma}{d(\omega/E)} \approx \frac{4C_F\alpha_S^3}{3}\frac{log(\frac{3x^2m^2}{m_d^2})}{x^2m^2}$$

Describes in between 15% the numerical results

radiative energy loss

Energy loss per unit length

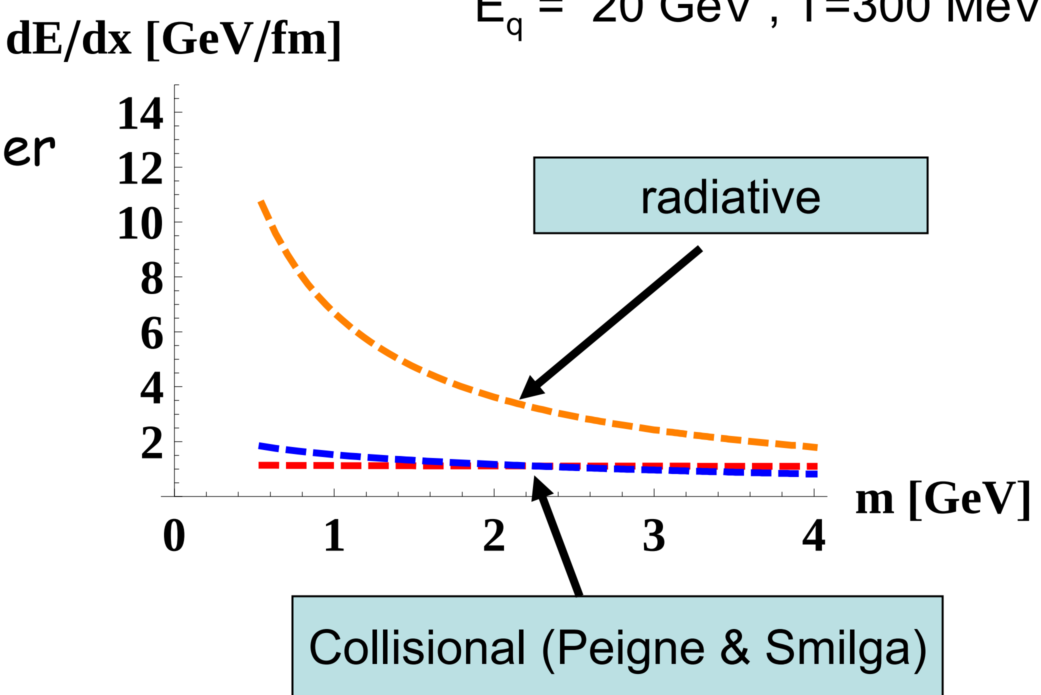
$$\frac{dE}{dz} = \int d^3k \;\rho_k \int \Delta E \frac{d\sigma}{dx}\; dx = \int d^3k \;\rho_k E \int x \frac{d\sigma}{dx}$$

For large q masses:
Collisional and radiative energy loss of the same order

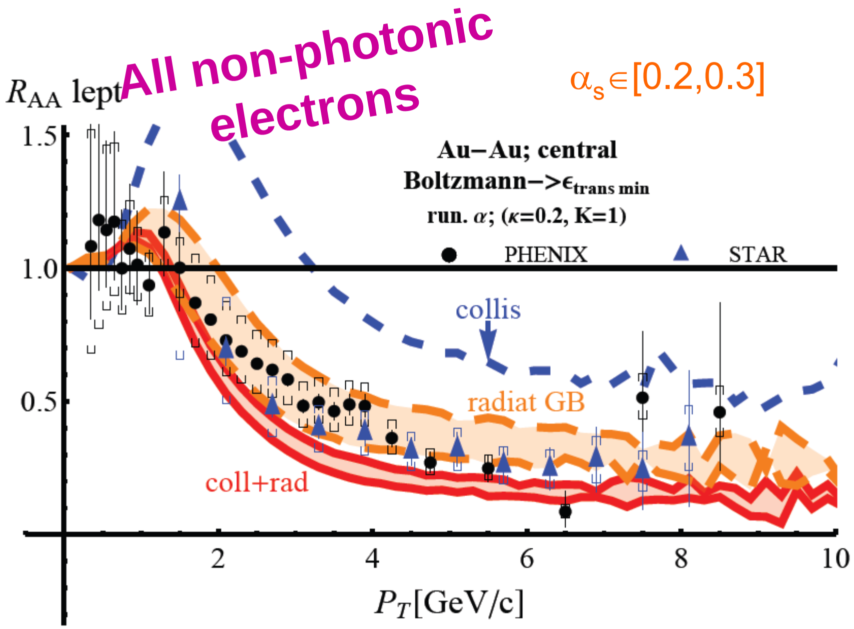
Small q masses:
radiative dominante

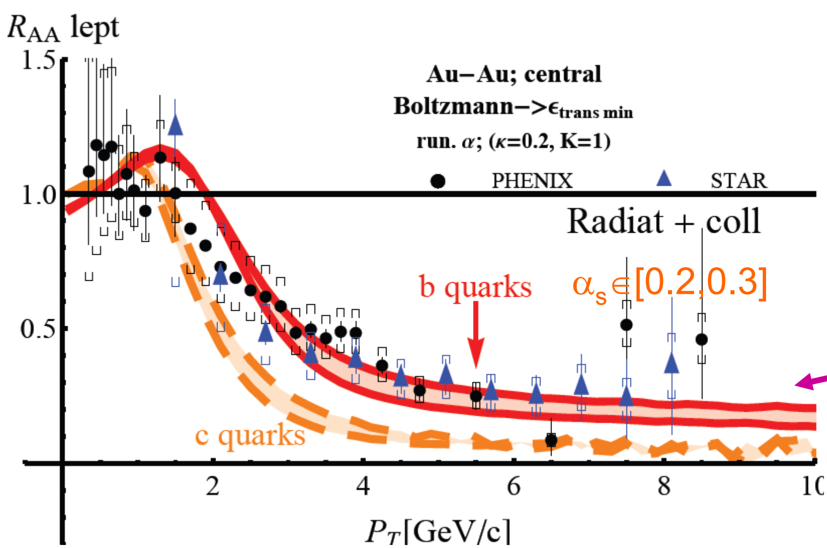
Rad: $\Delta E \propto E$
Coll: $\Delta E \propto \ln E$

E_q = 20 GeV , T=300 MeV



Results I

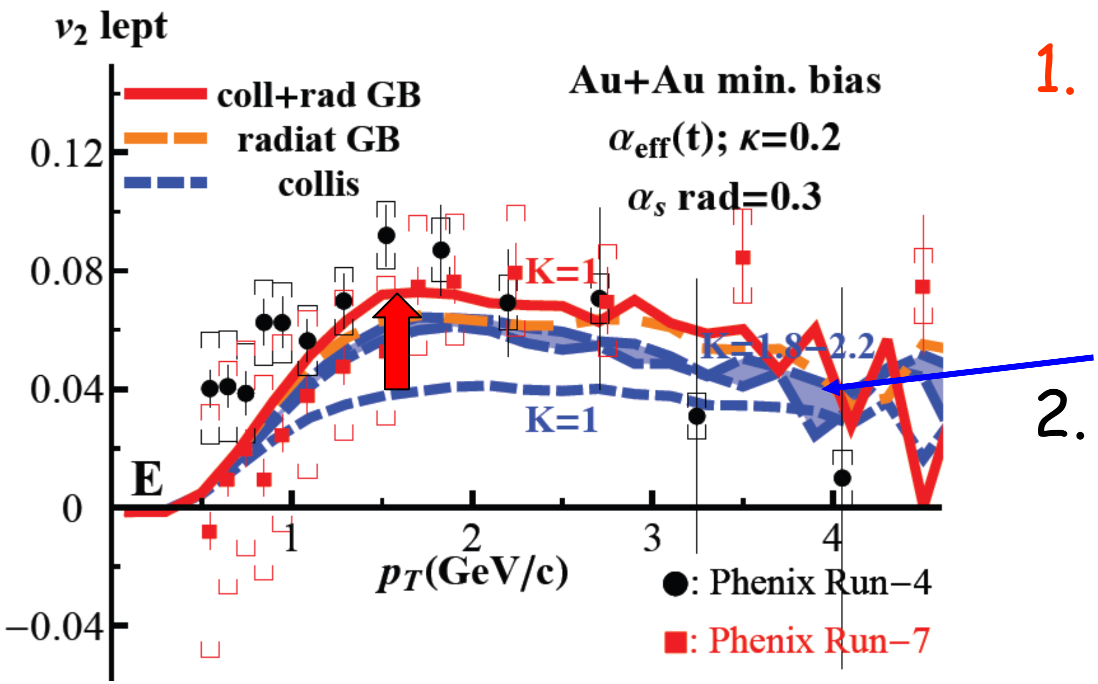




1. Too large quenching (but very sensitive to freeze out)
2. Radiative Eloss indeed dominates the collisional one
3. Flat experimental shape is well reproduced

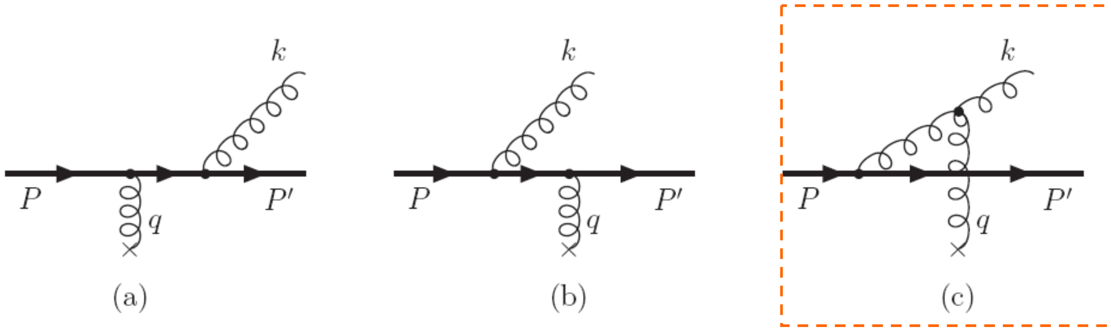
separated contributions $e \leftarrow D$ and $e \leftarrow B$.

Results II



1. Collisionnal + radiative energy loss + dynamical medium : *compatible* with data
2. Shape for radiative E loss and *rescaled* collisional E loss are pretty similar
3. To our knowledge, one of the first model using radiative Eloss that reproduces v_2

Formation time of the gluons



$$t_f \approx \frac{2(1-x)\omega}{(\vec{k}_\perp - \vec{q}_\perp)^2 + x^2 M^2 + (1-x)m_g^2}$$

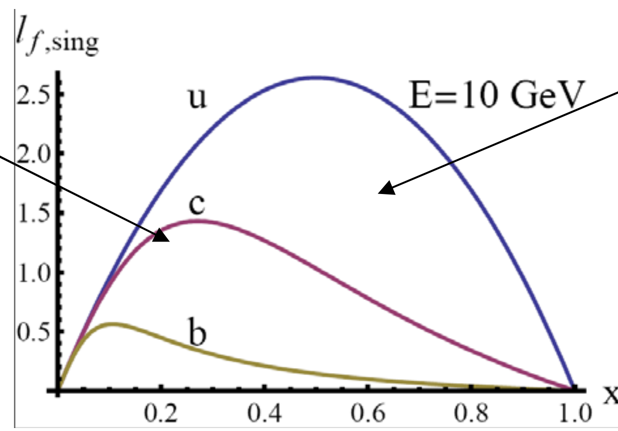
At 0 deflection

$\vec{q}_t = \vec{k}_t$:

$$l_{f,\text{sing}} \approx \frac{2x(1-x)E}{m_g^2 + x^2 M^2}$$

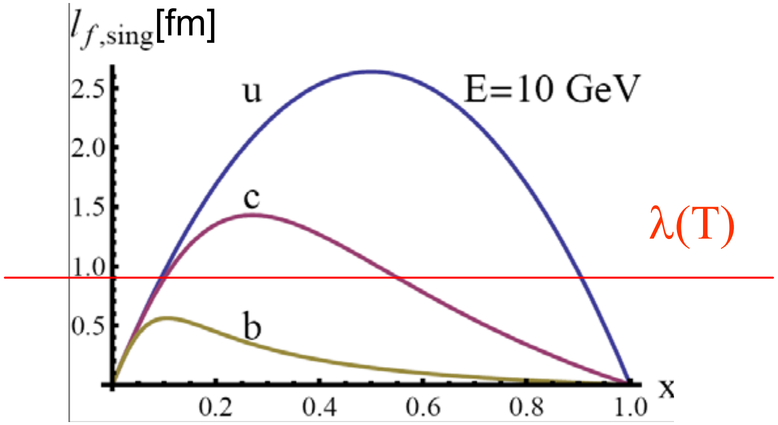
[fm]

For $x<x_{cr}=m_g/M$, basically no mass effect in gluon radiation



For $x>x_{cr}=m_g/M$, gluons radiated from heavy quarks have a smaller formtion time then those from light quarks and gluon => radiation process less affected by coherence effects in multiple scattering

Formation time of the gluons



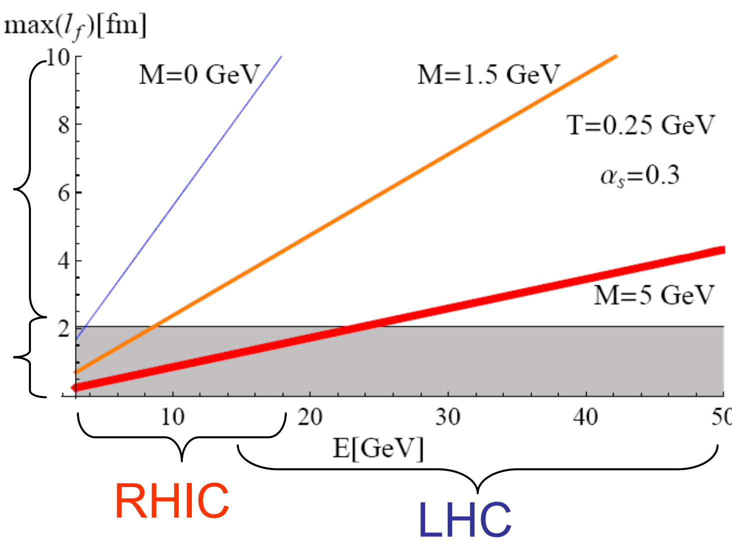
Comparing the formation time with the mean free path:

Coherence effect for HQ gluon radiation : $\Leftrightarrow \quad \frac{E}{M} \gtrsim m_g \lambda_Q \sim \frac{1}{g_s}$

Mostly coherent

Mostly incoherent

(of course depends on the physics behind λ_Q)



coherence effects may be not that important for HQ.

(will provide at least a maximal value for the quenching)

Conclusions

All experimental data are compatible with the assumption that QCD describes

energy loss and elliptic flow v_2

observed in heavy ion collisions.
Specials features : running coupling constant
adjusted Debye mass

Description of the expansion of the medium (freeze out, initial cond) influences the results enormously (->studies in progress)

Refinements still necessary
LPM
Running coupling constant for gluon emission vertex
Treatment of frequent collisions with low momentum transfer

Monte Carlo Implementation

I) For each collision with a given $q_\perp$, we define the conditional probability of radiation:

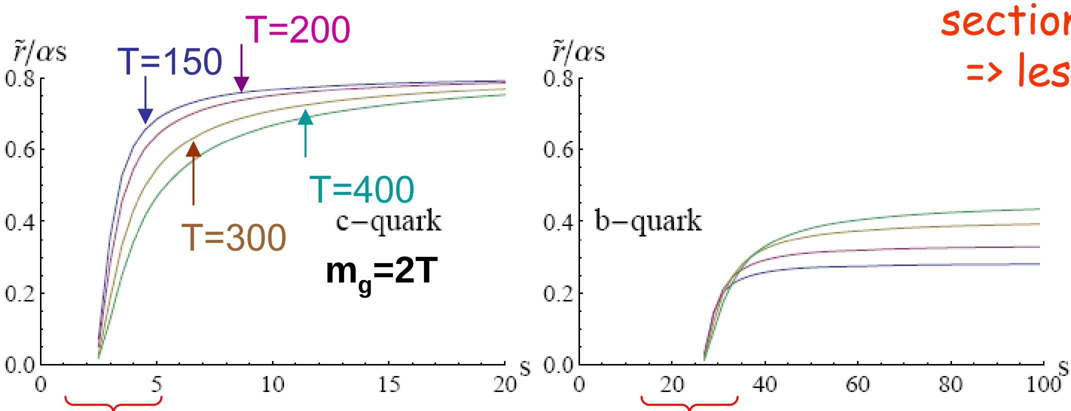
$$r(q_\perp) := \frac{\int_0^{+\infty} \frac{d^2\sigma_{\rm rad}}{d\omega dq_\perp^2} d\omega}{\frac{d\sigma_{\rm el}^{Qq}}{dq_\perp^2}}$$

In practice, w_{min}=5% E to avoid IR catastrophy

II) For each collision with a given invariant mass squared s, we define the conditional *total* probability of radiation:

$$\tilde{r}(s) = \frac{\sigma_{\rm rad}}{\sigma_{\rm el}} \approx \frac{\int_{-|t|_{\rm max}}^{0} r(\sqrt{-t}) \frac{d\sigma_{\rm el}^{Qq}(t)}{dt} dt}{\int_{-|t|_{\rm max}}^{0} \frac{d\sigma_{\rm el}^{Qq}(t)}{dt} dt}$$

Probes the elastic cross section at larger values of t => less sensitive to a_{eff} at small t-values



Threshold for radiation

Monte Carlo Implementation

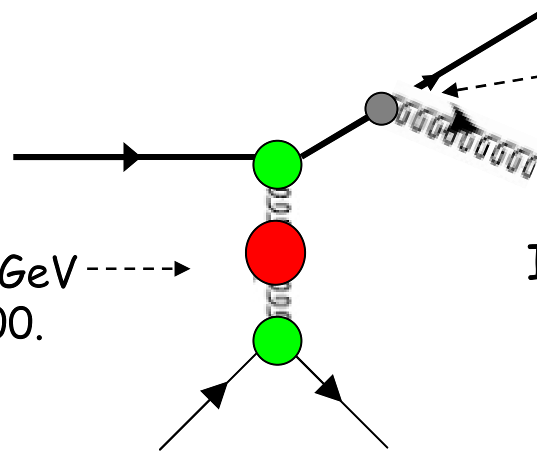
III) For a given HQ energy E, we sample the entrance channel according to the thermal distribution of light quarks and gluons and $s_{el}(s)$ and accept according to $\tilde{r}(s)$ the conditional probability

IV) We sample "downwards" $q_\perp$, ω and then $k_\perp$

Hard shocks with |t|>25% s are rejected (not treated properly in our formalism)

V) $P^+ \rightarrow (1-x)\, P^+$ and transverse kick of $\mathbf{q}_\perp - \mathbf{k}_\perp$.

Fixed α_s



$\langle q_\perp \rangle$ from 0.6 GeV (col) → 1.1 GeV (rad) for E=15GeV and T=400.

Approximation:

In "reality", several collisions at intermediate t-values accumulate