Relativistic Shock Waves in Viscous Gluon Matter

Ioannis Bouras

with Andrej El, Oliver Fochler, Francesco Lauciello, Etele Molnar, Harri Niemi, Zhe Xu, Carsten Greiner and Dirk H. Rischke

I. Bouras, E. Molnar, H. Niemi, Z. Xu, A. El, O. Fochler, C. Greiner and D. H. Rischke, Phys. Rev. Lett. 103:032301 (2009)

I. Bouras, H. Niemi, E. Molnar et al., in preparation

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

26th Winter Workshop on Nuclear Dynamics

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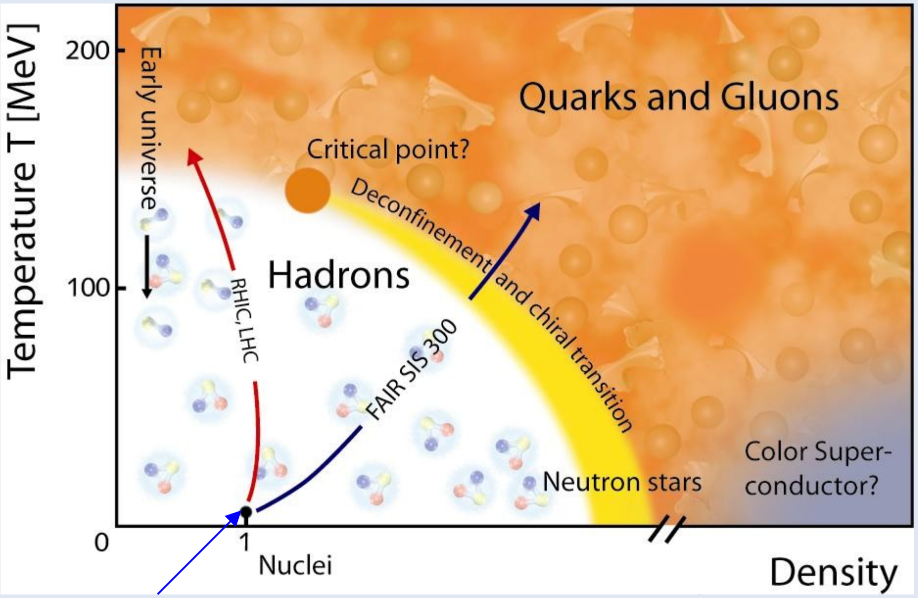
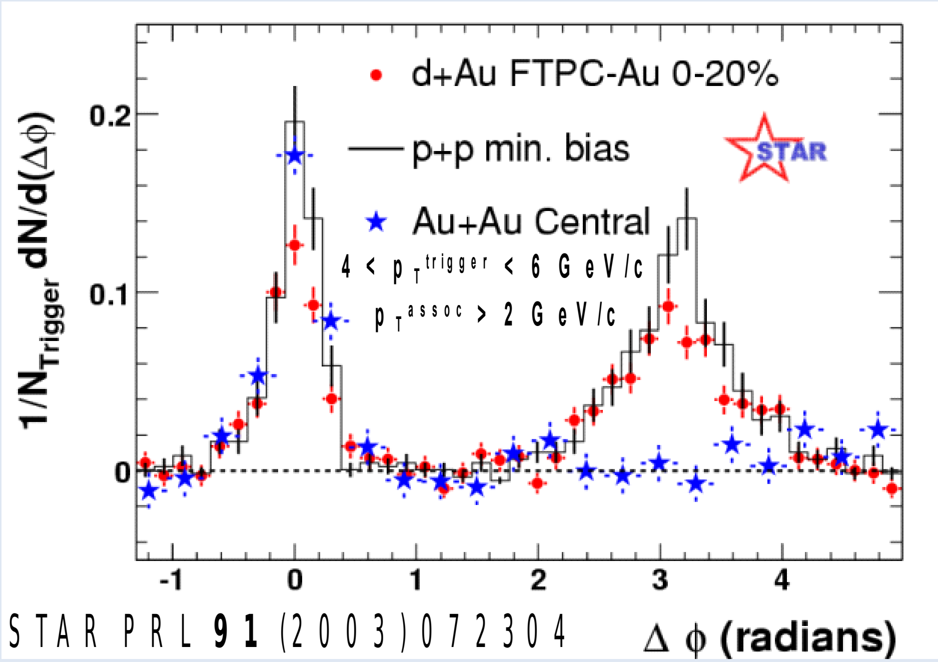
January 2nd - 9th, 2010

Motivation

- RHIC data indicates jet-suppression in heavy-ion collisions → signal for a new phase of matter, namely ” QGP ”
- Flow data show that the matter is behaving like a nearly perfect fluid



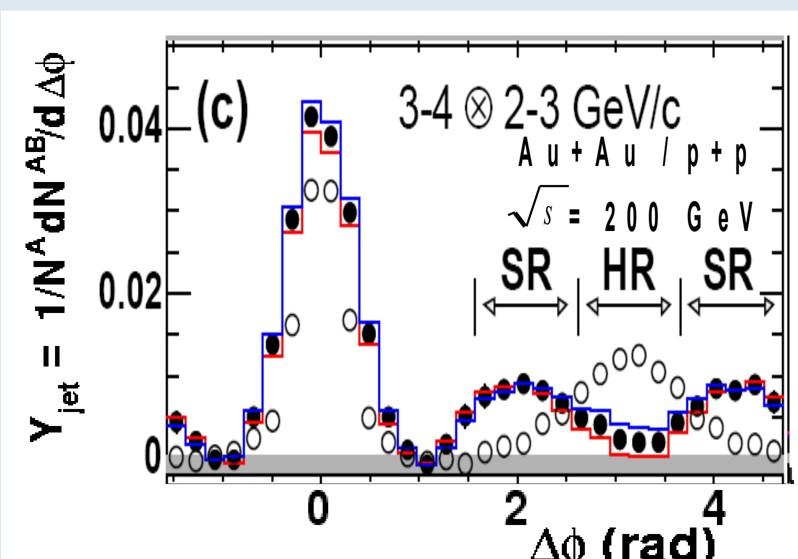
ordinary world



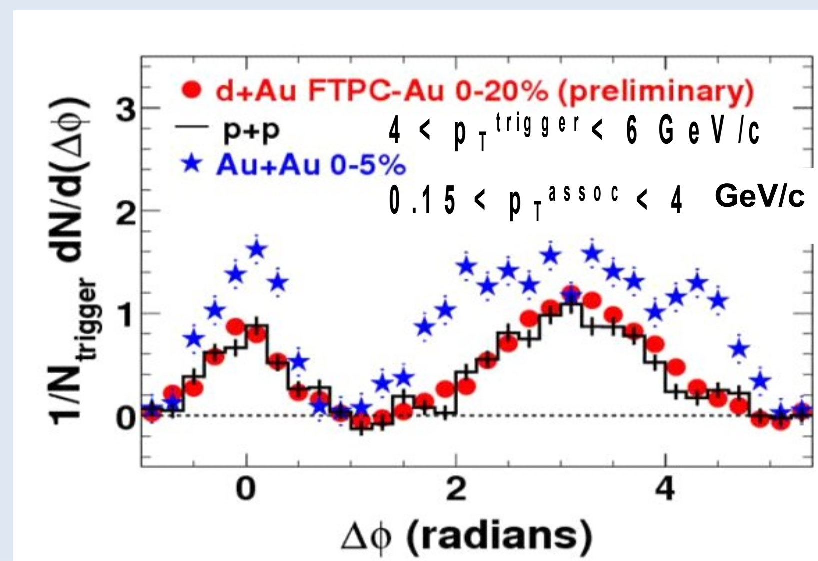
Motivation

- **Double-peak structure is observable for lower momenta**
  → One possible consequence is the formation of Mach cones

H. Stöcker, *Nucl. Phys. A* ***750****, 121 (2005)*
J. Ruppert and B. Müller, *Phys. Lett. B* ***618,*** *123 (2005)*
J. Casalderrey-Solana, E.V. Shuryak and D. Teaney, *J. Phys. Conf. Ser.* ***27****, 22 (2005)*
V. Koch, A. Majumder and X.N. Wang, *Phys. Rev. Lett.* ***96****, 172302 (2006)*
B. Betz*, PRC* ***79****:034902, (2009)*



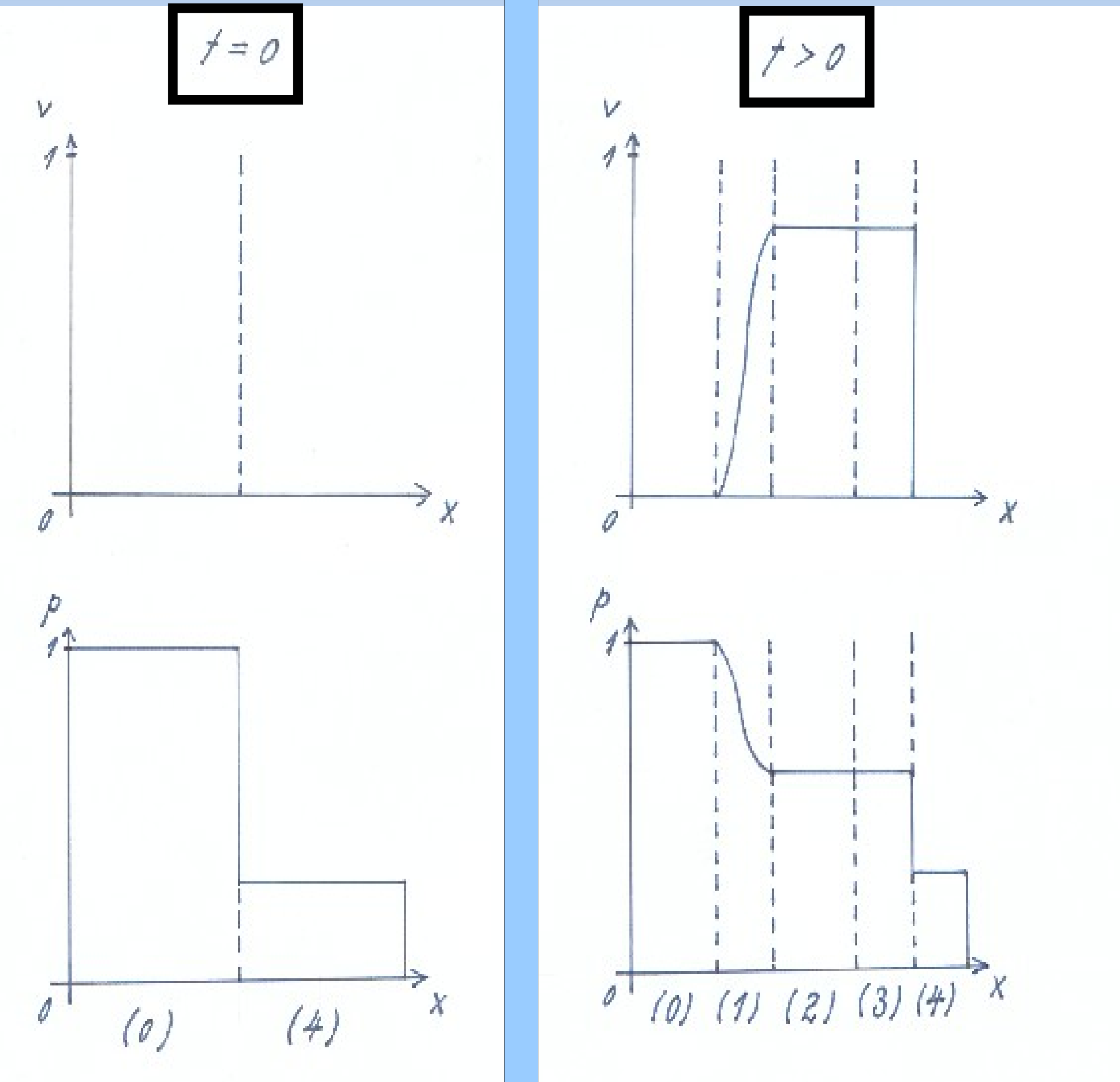
PHENIX arXiv:0705.3238 [nucl-ex]



STAR, Nucl. Phys. **A774**, 129 (2006)

The Relativistic Riemann problem

Initial conditions



What happens if you remove the membran?

A shock wave travels to the right with a speed higher than the speed of sound and a rarefaction wave travels to the left with the speed of sound

Numerical methods:
The Parton Cascade BAMPS

Boltzmann
Approach for
Multi-
Parton
Scatterings

- **Transport algorithm solving the Boltzmann equation using Monte Carlo techniques**

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22} + C_{23} + \ldots$$

- **Stochastic interpretation of collision rates**

$$P_{2i} = v_{rel} \frac{\sigma_{2i}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

- **pQCD interactions, 2<->3 processes**

See also the talks:
Jan Uphoff
Oliver Fochler
Andrej El

Numerical methods: The Parton Cascade BAMPS

For this setup :

- **Boltzmann gas, isotropic cross sections, elastic processes only**
- **Implementing a constant η/s , we locally get the cross section σ_{22}:**

$$\eta = \frac{4}{15}\frac{\epsilon}{R^{tr}}$$

Transport collision rate R^{tr}

For isotropic elastic collisions:

$$R_{22}^{tr} = n\frac{2}{3}\sigma_{22}$$

$$\epsilon = 3nT$$

$$s = 4n - n\ln(\lambda_{fug})$$

$$\lambda_{fug} = \frac{n}{n_{eq}} \qquad n_{eq} = \frac{g}{\pi^2}T^3$$

g = 16 for gluons

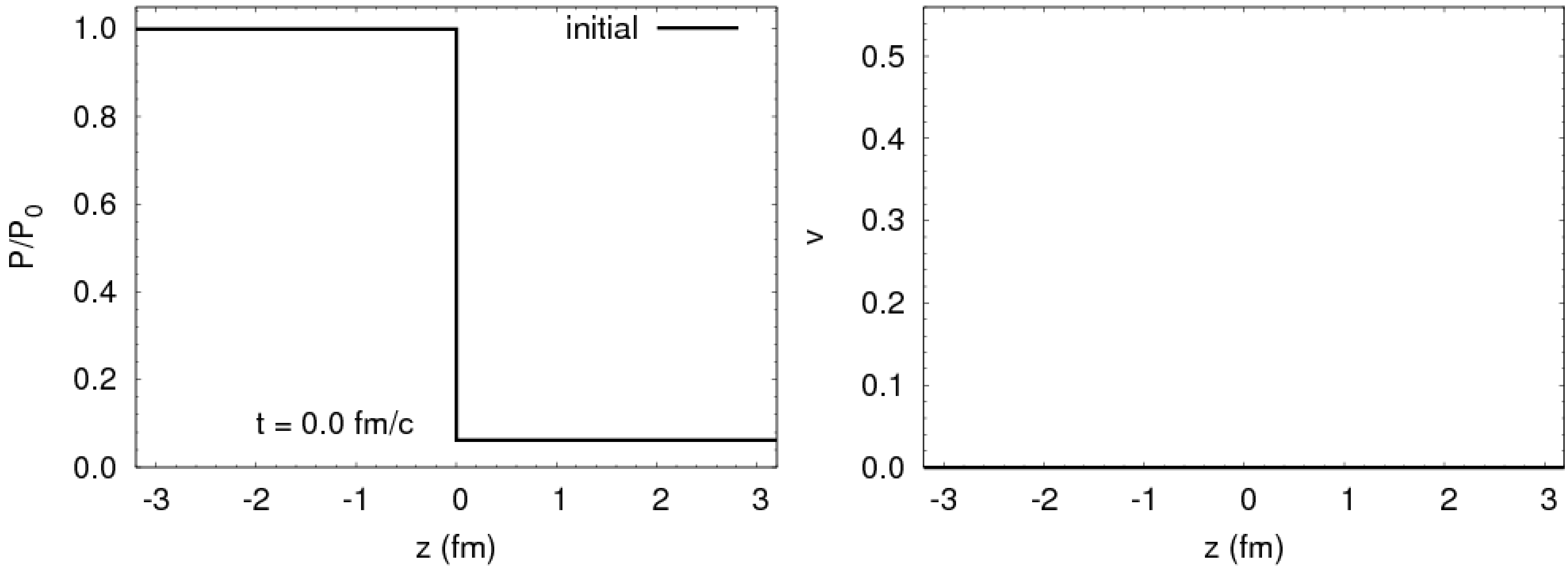
Z. Xu & C. Greiner,
Phys.Rev.Lett.100:172301,2008

$$\Rightarrow \quad \sigma_{22} = \frac{6}{5}\frac{T}{s}\left(\frac{\eta}{s}\right)^{-1}$$

Numerical Results:
The Relativistic Riemann Problem

Initial conditions

$T_L = 400\ MeV$
$T_R = 200\ MeV$
$t = 0\ fm/c$



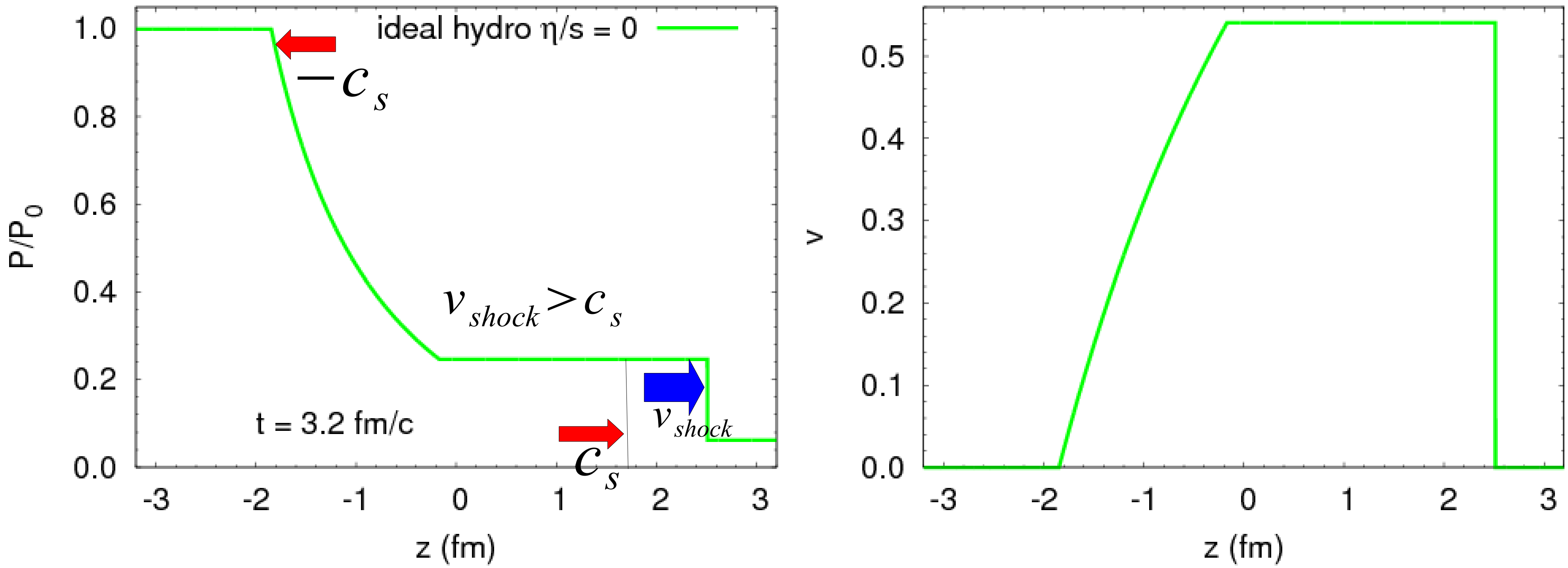
- **Two pressure regions seperated by a membran**
- **The velocities on both sides are zero**

What happens if you remove the membran?

Numerical Results: The Relativistic Riemann Problem

$T_L = 400\ MeV$
$T_R = 200\ MeV$
$t = 3.2\ fm/c$

Analytical Solution for a perfect fluid

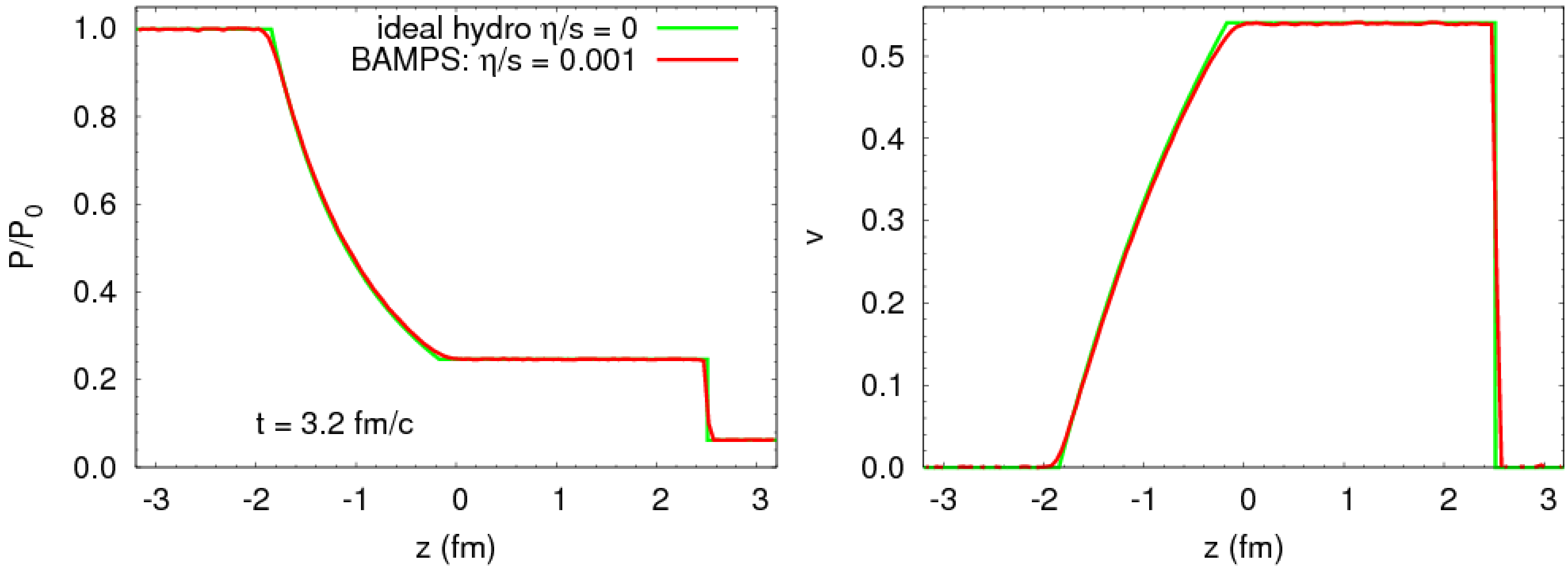


- **A shock wave travels to the right with a speed <u>higher</u> than the speed of sound**
- **A rarefaction wave travels to the left with the speed of sound**

Numerical Results:
The Relativistic Riemann Problem

$T_L = 400\ MeV$
$T_R = 200\ MeV$
$t = 3.2\ fm/c$

Boltzmann solution



The IDEAL HYDRO LIMIT is reproduced by using a very high cross section, i.e. a very small mean free path !!!

Numerical Results:
The Relativistic Riemann Problem

Boltzmann solution

$T_L = 400\ MeV$
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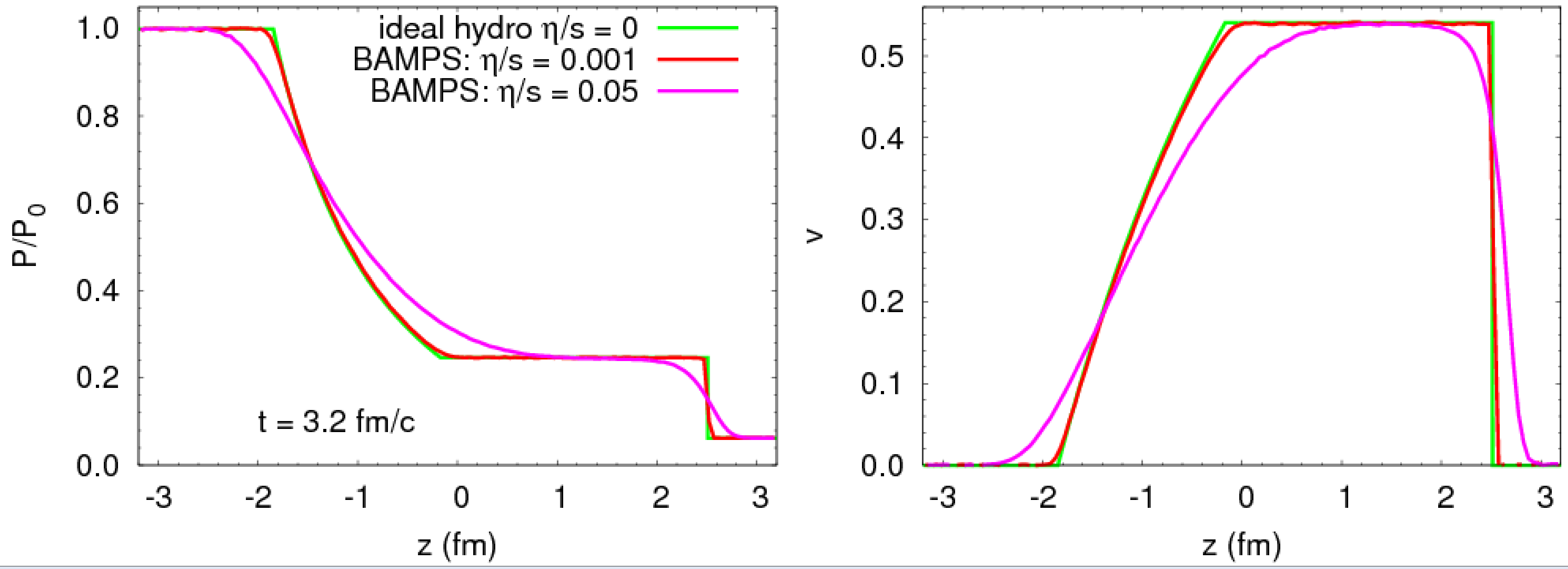
$t = 3.2\ fm/c$



Numerical Results: The Relativistic Riemann Problem

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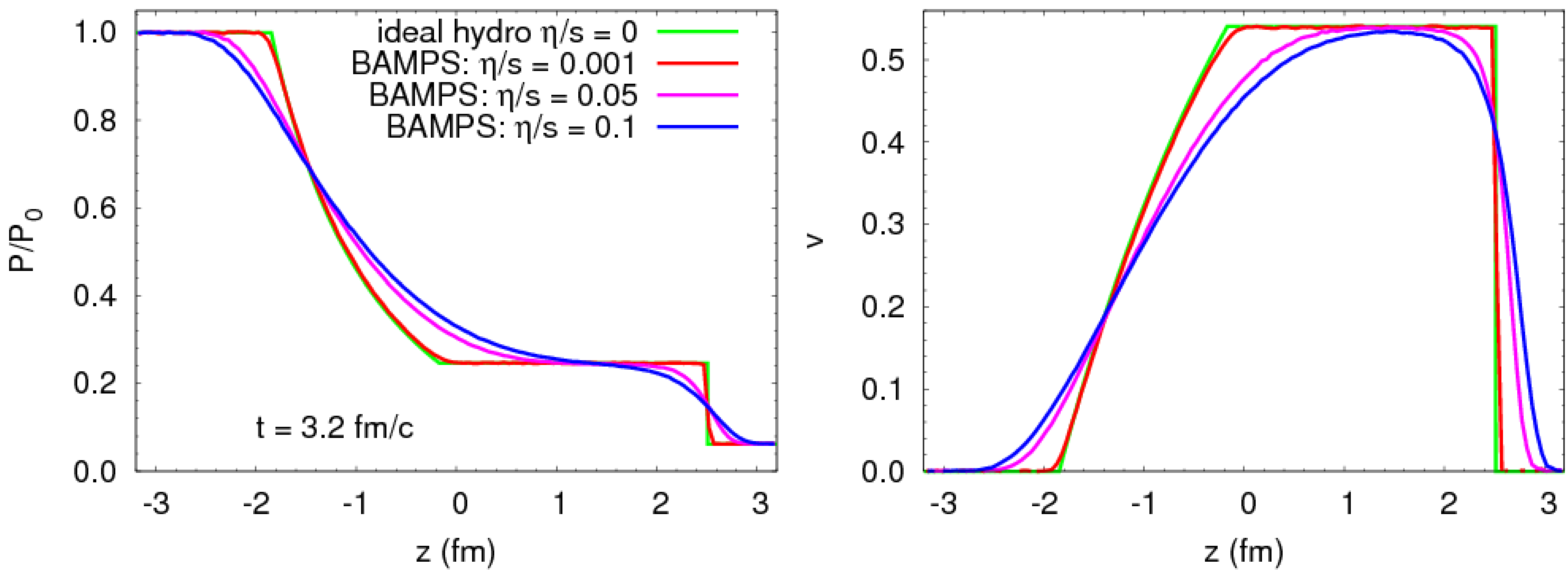

Numerical Results:
The Relativistic Riemann Problem

Boltzmann solution

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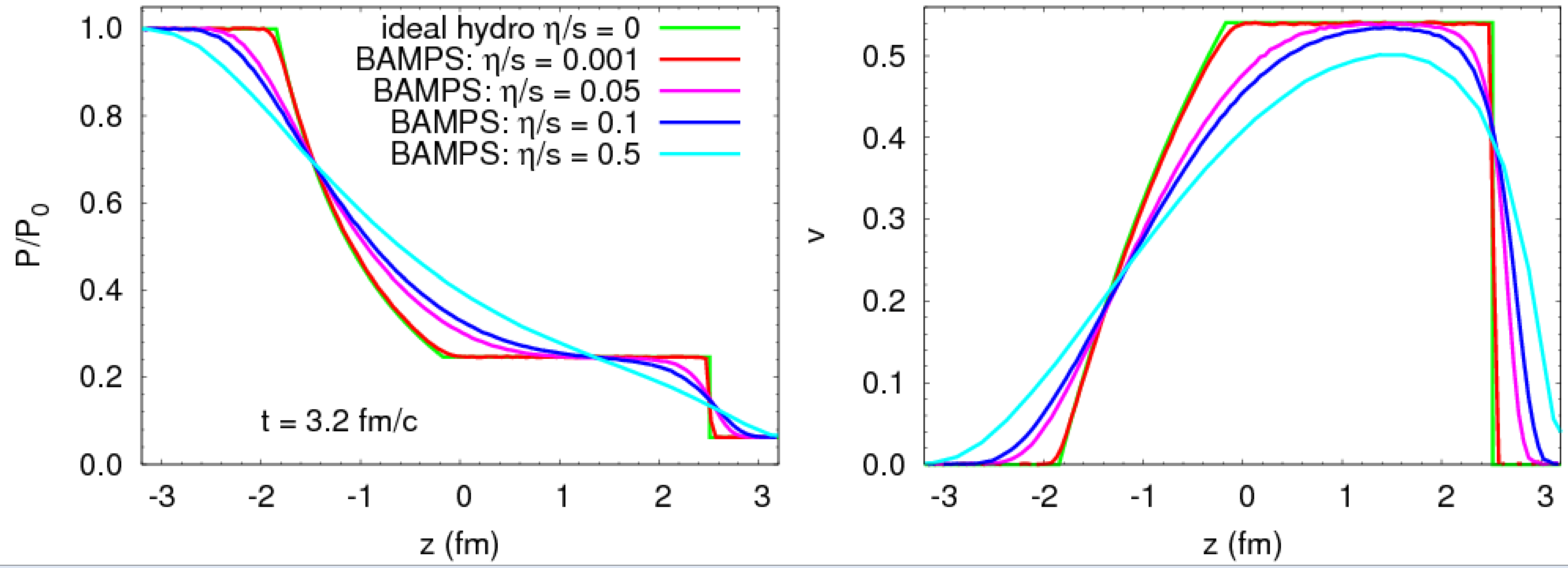

Numerical Results:
The Relativistic Riemann Problem

$T_L = 400\ MeV$
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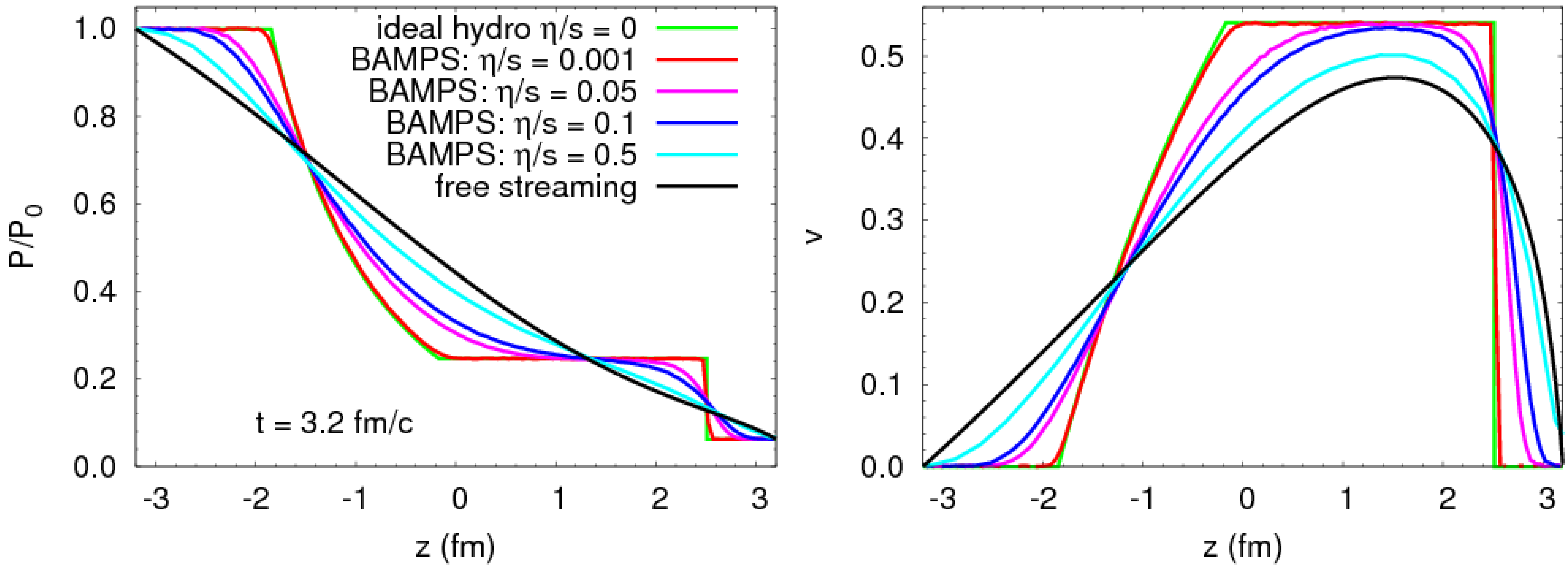
$t = 3.2\ fm/c$

Boltzmann solution



<u>Transition from ideal hydro to free streaming</u>

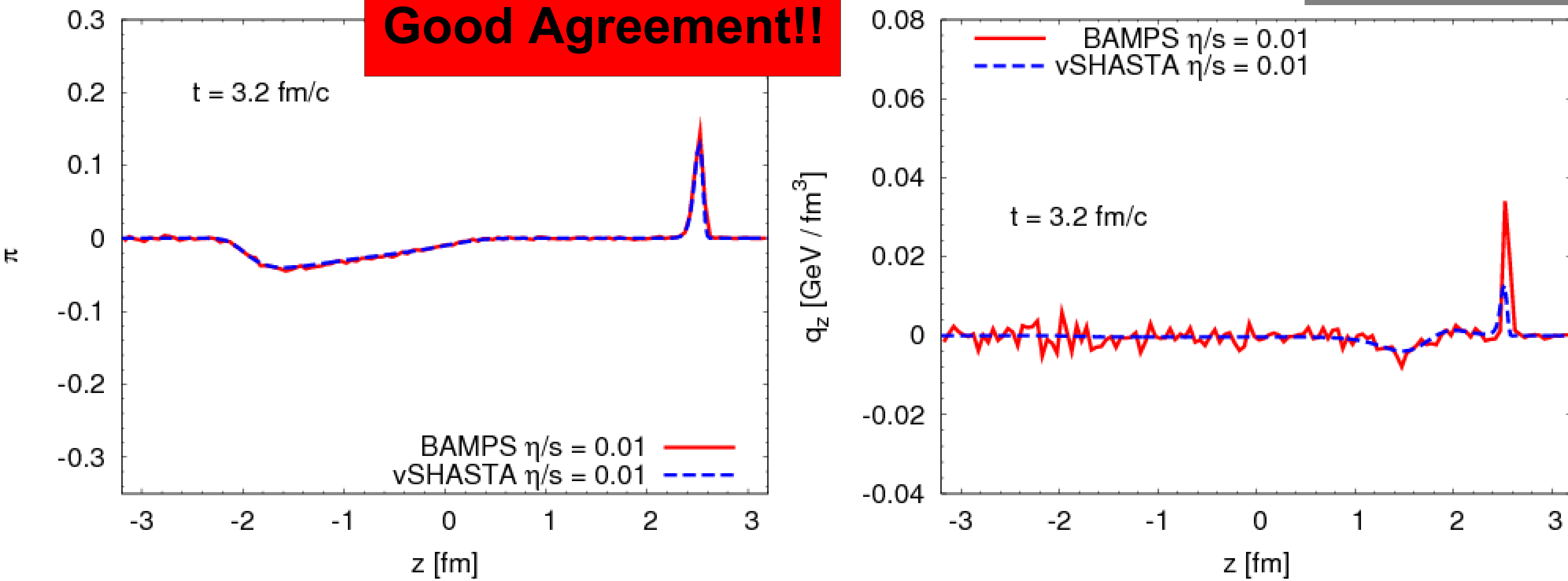
- **Shock plateau shrinks and vanish with strong viscous effects**
- **Shock front gets finite width and rarefaction wave moves faster than the speed of sound**

Numerical Results:
The Relativistic Riemann Problem

Comparison between BAMPS and vSHASTA

$$T_L = 400\,MeV$$
$$T_R = 200\,MeV$$
$$t = 3.2\,fm/c$$

Good Agreement!!



vSHASTA

1 + 1 dimensional viscous hydro model using the Israel-Stewart equations

E. Molnar, H. Niemi and D. Rischke
Eur.Phys.J.C60:413-429,2009
arXiv:0907.2583

η/s = 0.01 →

$$K_{local} = \lambda_{mfp} \partial_\mu u^\mu$$

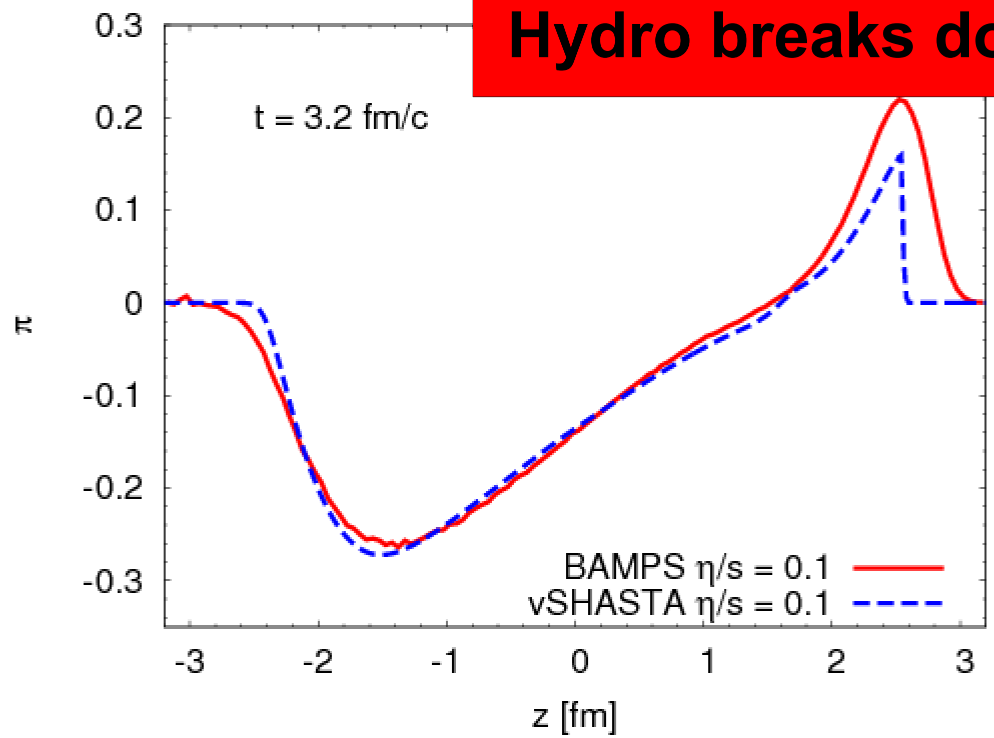
*is **SMALL** in the region of the shock front*

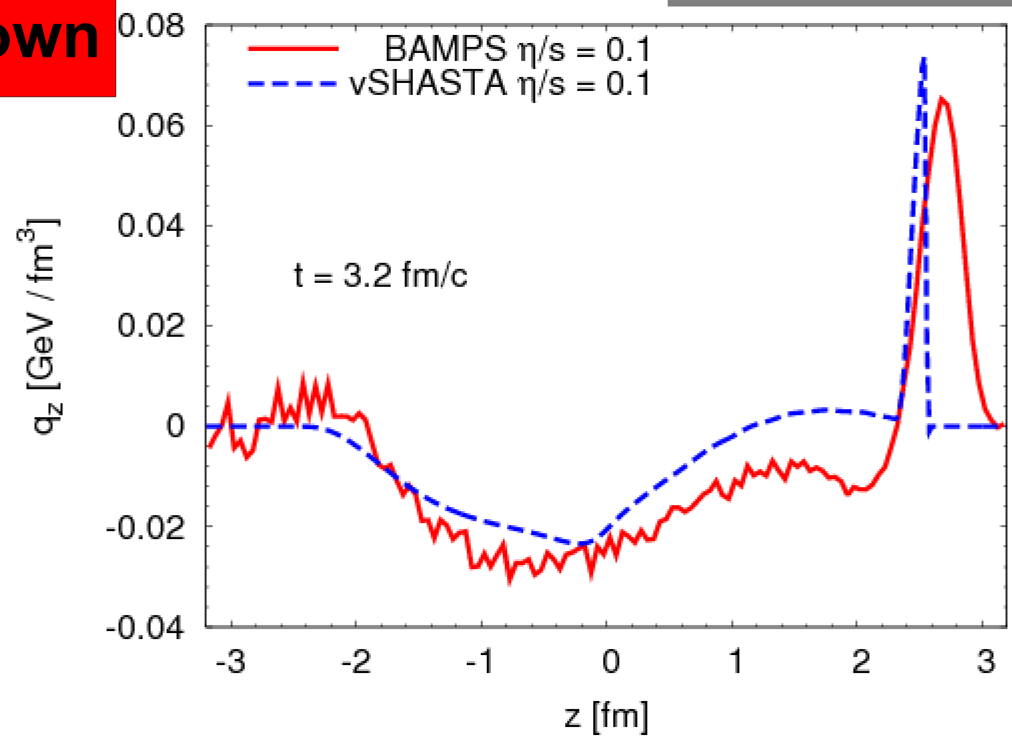
Numerical Results:
The Relativistic Riemann Problem

Comparison between BAMPS and vSHASTA

$T_L = 400\,MeV$
$T_R = 200\,MeV$
$t = 3.2\,fm/c$

Hydro breaks down





<u>vSHASTA</u>
1 + 1 dimensional viscous hydro model using the Israel-Stewart equations

E. Molnar, H. Niemi and D. Rischke
Eur.Phys.J.C60:413-429,2009
arXiv:0907.2583

η/s = 0.1

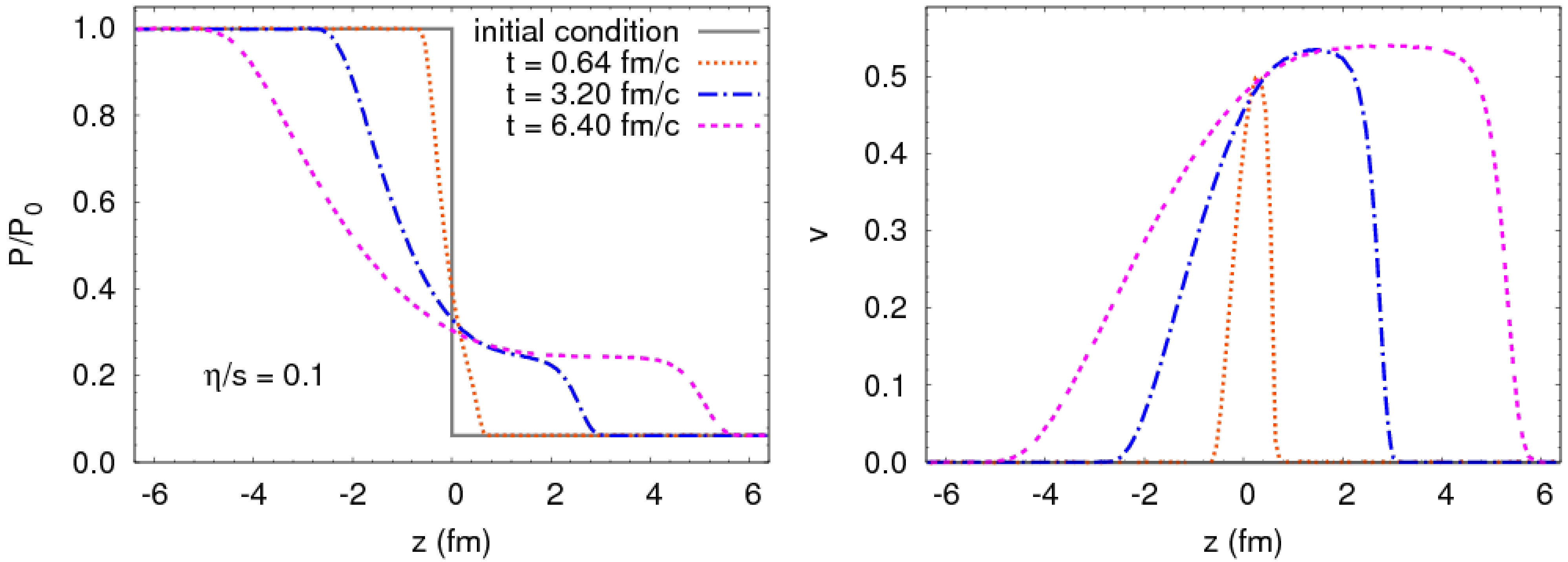
$K_{local} = \lambda_{mfp} \partial_\mu u^\mu$

*is **LARGE** in the region of the shock front*

Shock Evolution

Evolution from free streaming to a shock

$T_L = 400\ MeV$
$T_R = 200\ MeV$



- **A shock needs time to develop**
- **For early times it behaves like free streaming, for later times it behaves like an ideal fluid**
- **Definition of a shock: Existence of a shock plateau**

Scaling behaviour and Global Knudsen Number

Knudsen number defined as:

$$K = \frac{\lambda_{mfp}}{L}$$

We define the characteristic length *L*

$$L = t \cdot (v_{shock} + c_s)$$

and use from kinetic theory

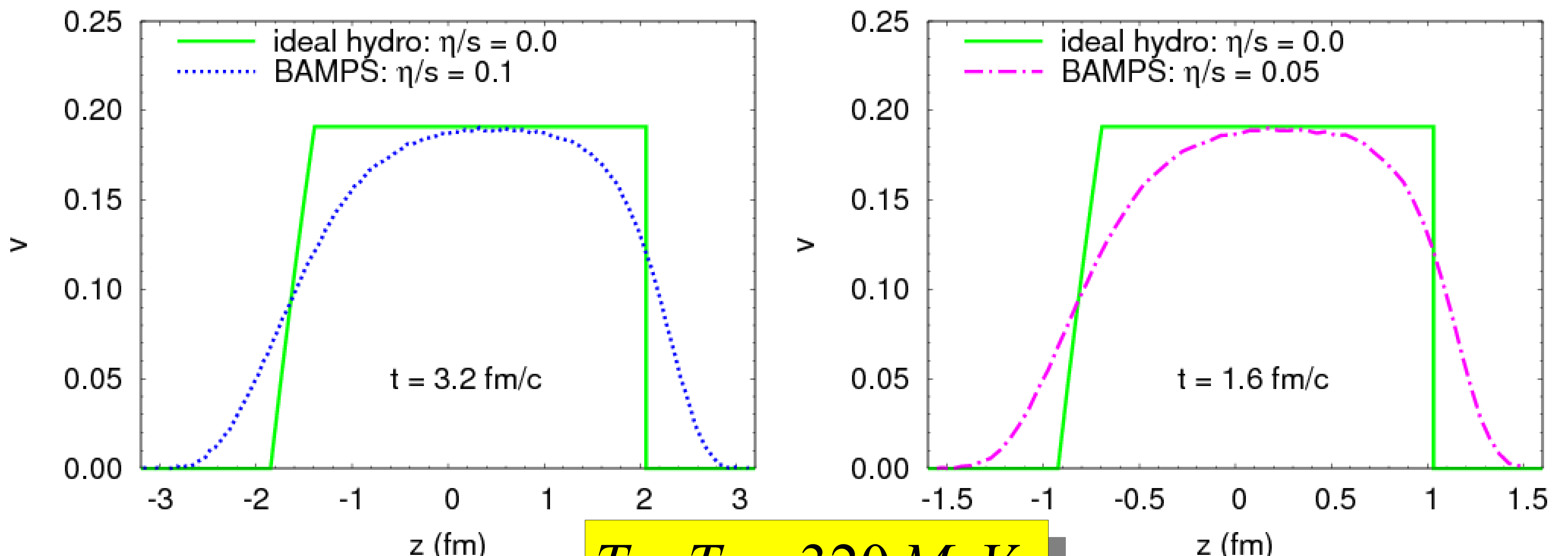
$$\lambda_{mfp} = \frac{10}{3T} \cdot \left(\frac{\eta}{s}\right)$$

$$K = \frac{10}{3} \frac{1}{t \cdot (v_{shock} + c_s) \cdot T} \cdot \left(\frac{\eta}{s}\right)$$

T is the lower temperature of the medium

2 systems behave the same, if they have the same Knudsen number

Scaling behaviour and Global Knudsen Number



$T = T_4 = 320\, MeV$

$\eta / s = 0.1$

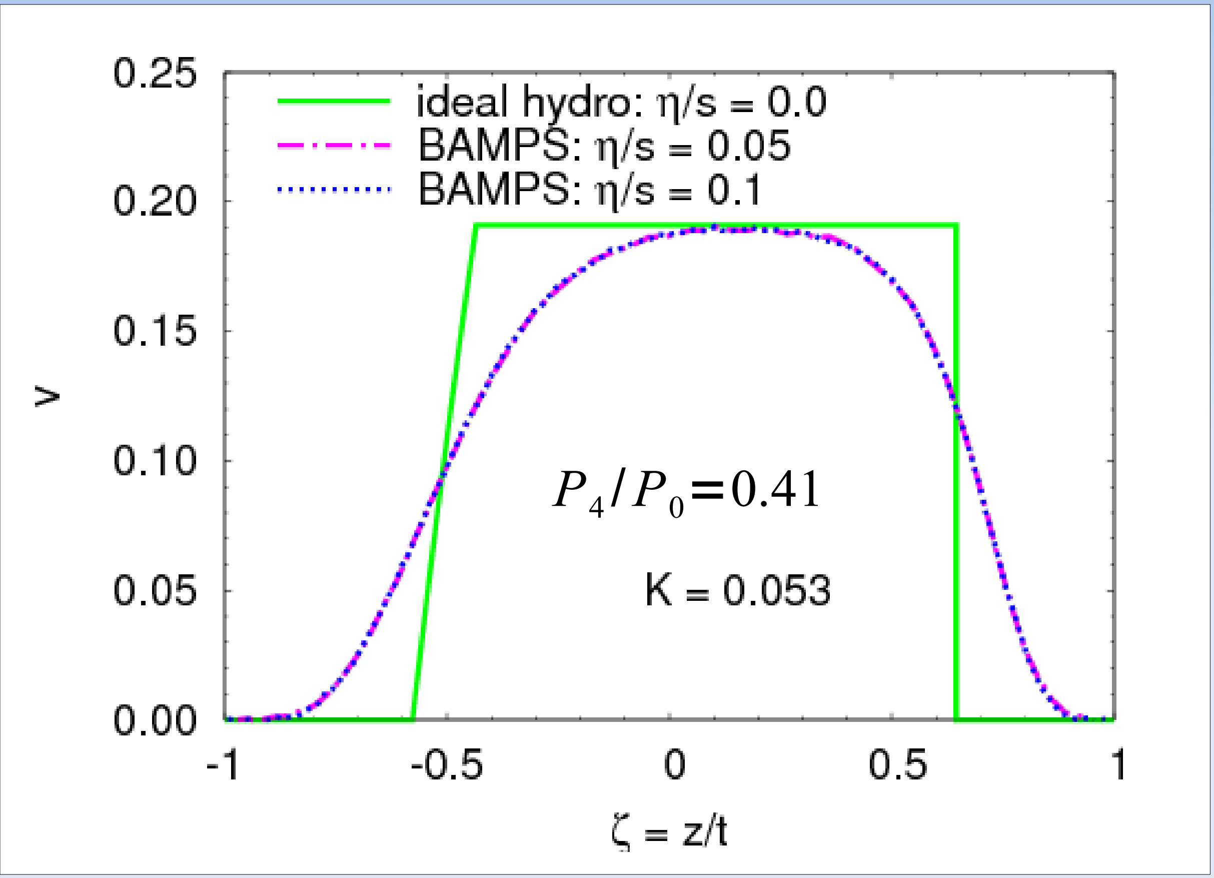
$t = 3.2\ fm/c$

$K = 0.053$

$\eta / s = 0.05$

$t = 1.6\ fm/c$

Scaling behaviour and Global Knudsen Number



The velocity profile is only a function of

$\zeta = z/t$ **and** K **,**

$$v(z,t,\eta/s) = F(\zeta, K)$$

and universal for a given ratio P_4/P_0 **.**

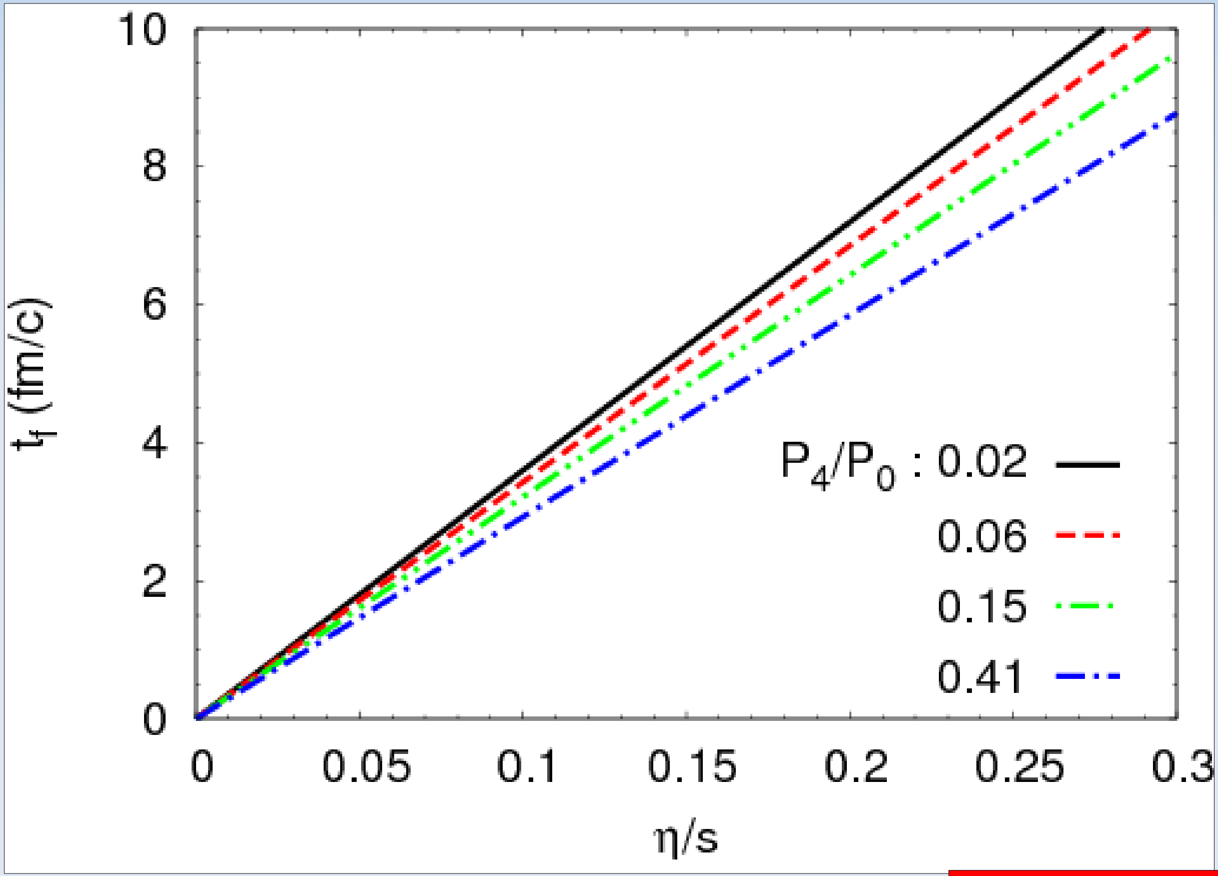
$$K_f = \frac{10}{3}\frac{1}{t_f\cdot(v_{shock}+c_s)\cdot T}\cdot\left(\frac{\eta}{s}\right) = 0.053$$

$$P_4/P_0 = 0.41$$

We define a shock when a shockplateau exist !!!

Scaling behaviour and Global Knudsen Number

Is the formation of shocks (Mach cones) possible in gluonic matter?



$$t_f = \frac{10}{3} \frac{1}{K_f \cdot (v_{shock} + c_s) \cdot T} \cdot \left(\frac{\eta}{s} \right)$$

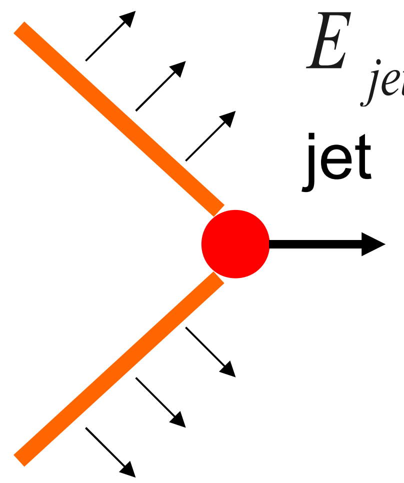
$$T = 350\,MeV$$

Lifetime QGP ~ 6 fm/c ➡ **The formation of Mach cones is in principle possible if *η/s* < 0.2**

Mach Cones in BAMPS

Setup

$$E_{jet} = 20 \text{ GeV}$$

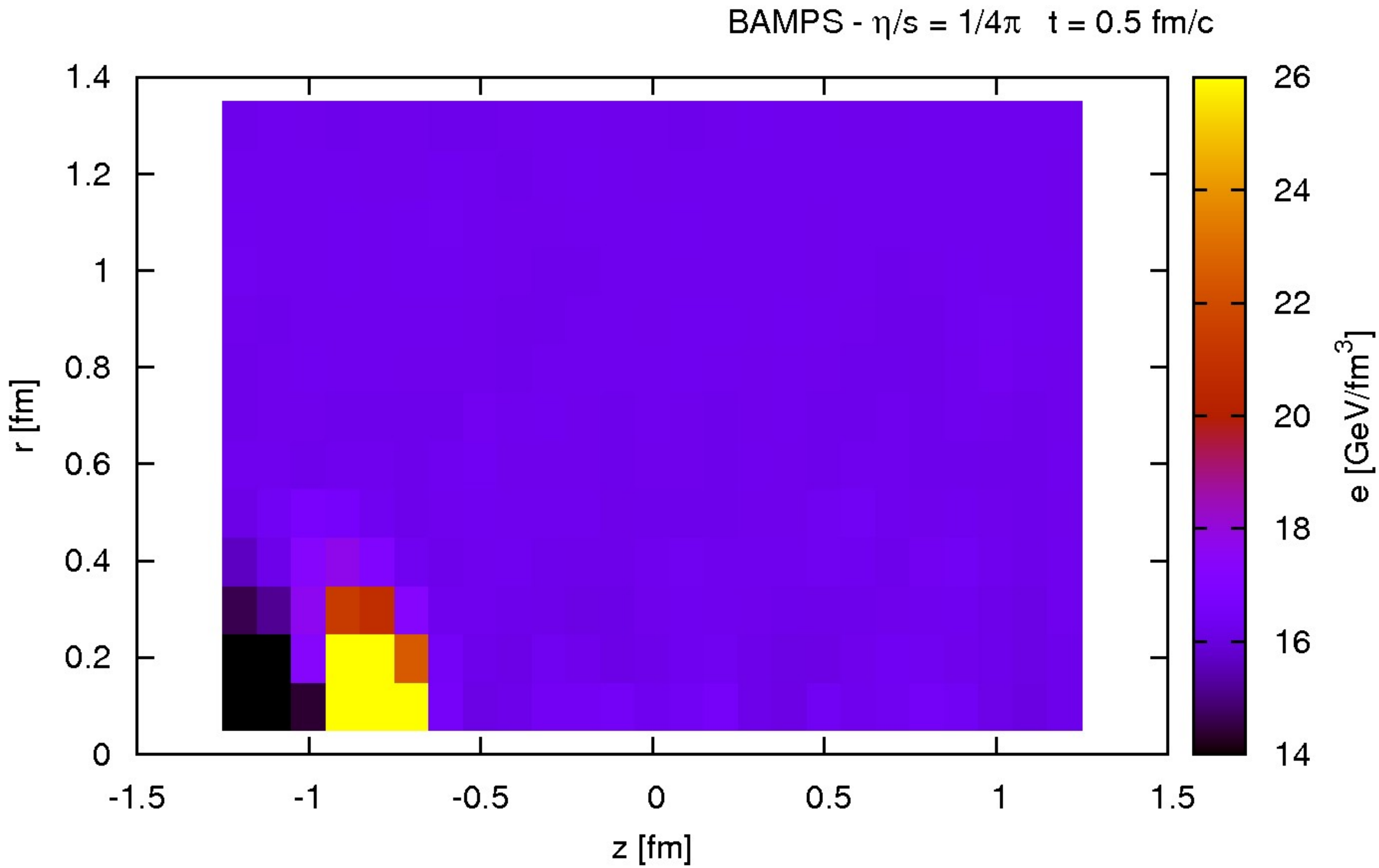


Medium $\frac{\eta}{s} = \frac{1}{4\pi} = 0.08 \quad T = 400 \text{ MeV}$

interactions: 2 -> 2 with isotropic distribution of the collision angle

Mach Cones in BAMPS: Evolution

$\eta / s = 1/4\pi$
$T = 400$ MeV
$E_{jet} = 20$ GeV
$t = 0.5$ fm/c



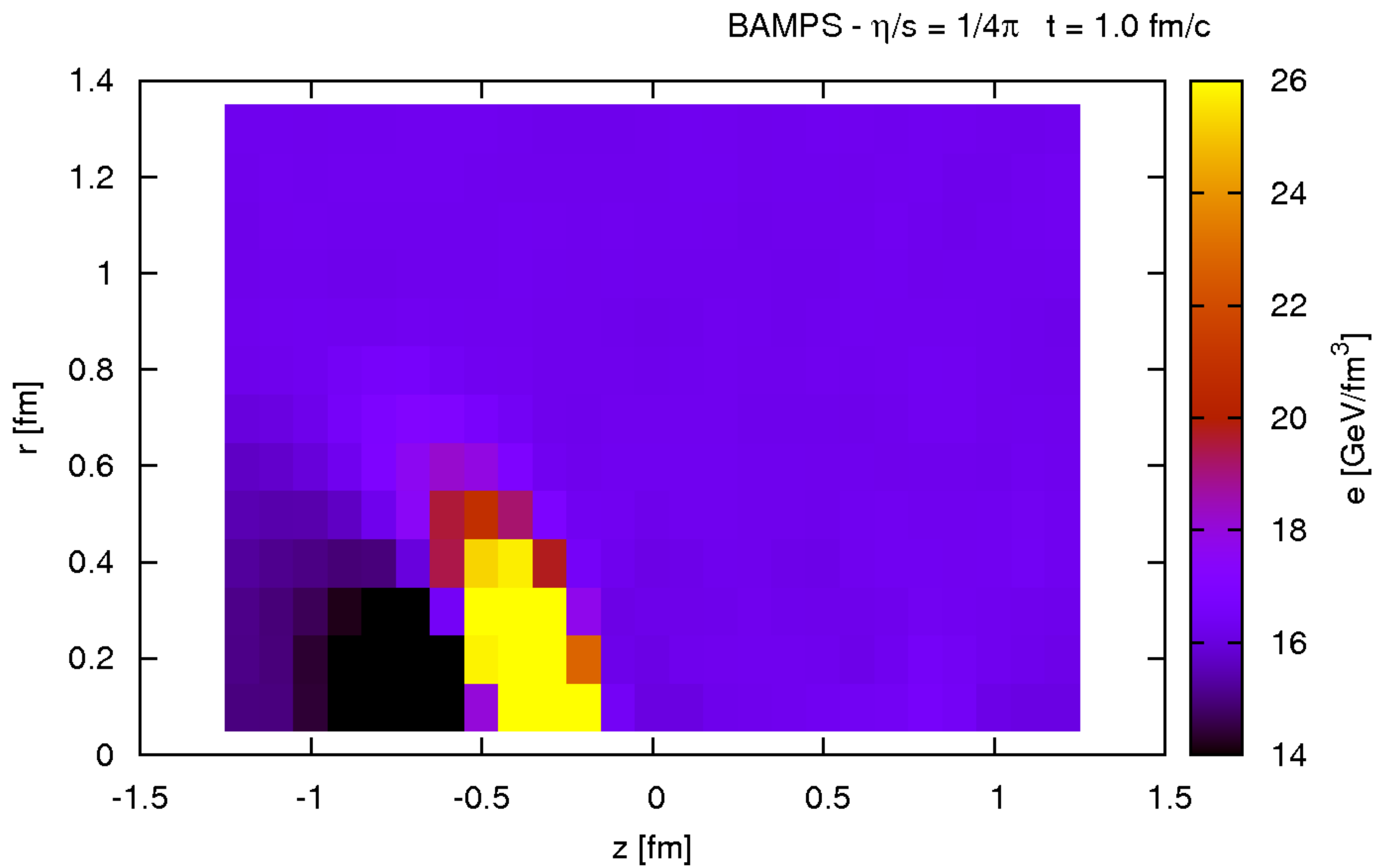
Mach Cones in BAMPS: Evolution

$\eta / s = 1/4\pi$

$T = 400$ MeV

$E_{jet} = 20$ GeV

$t = 1.0$ fm/c



Mach Cones in BAMPS: Evolution

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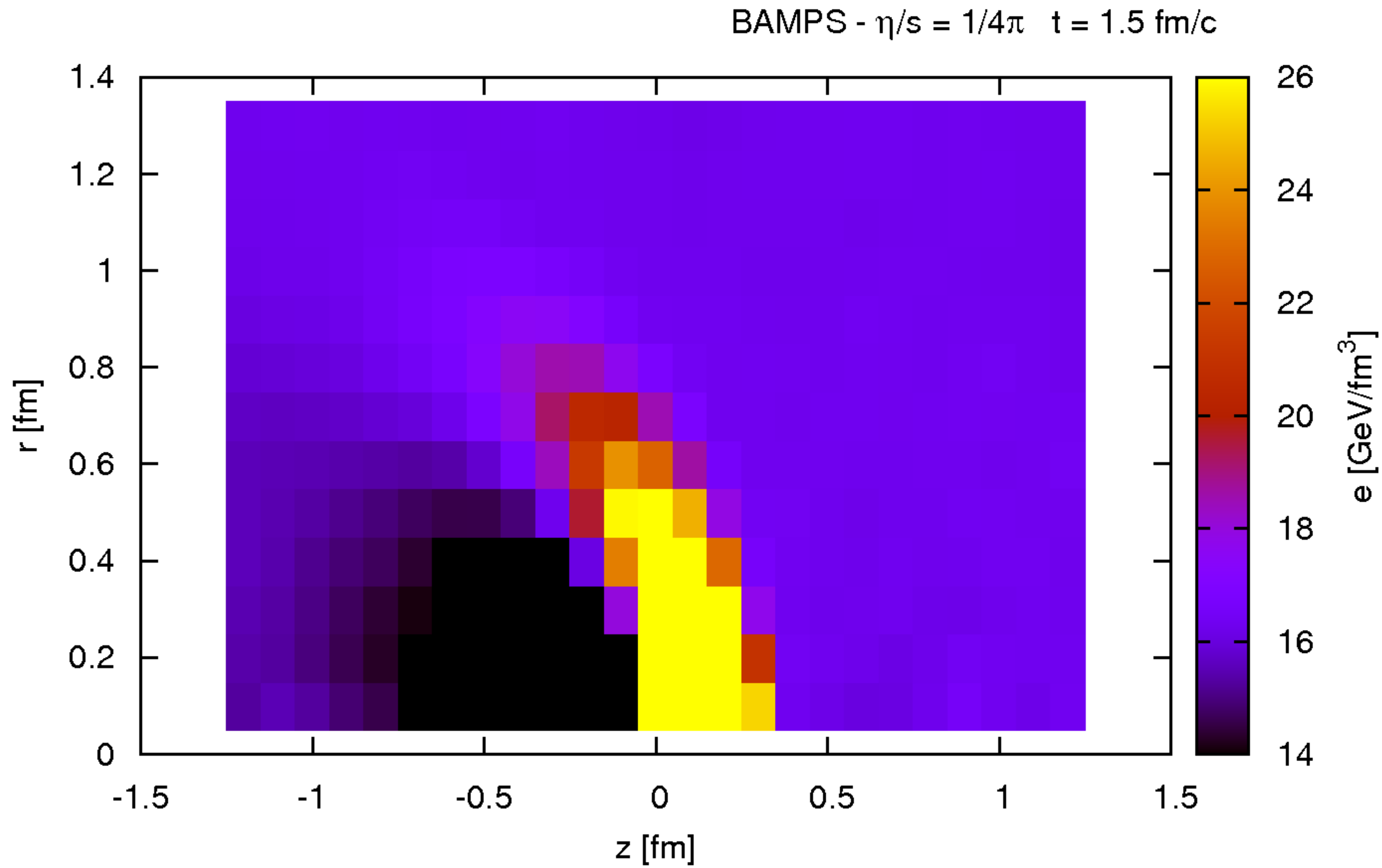


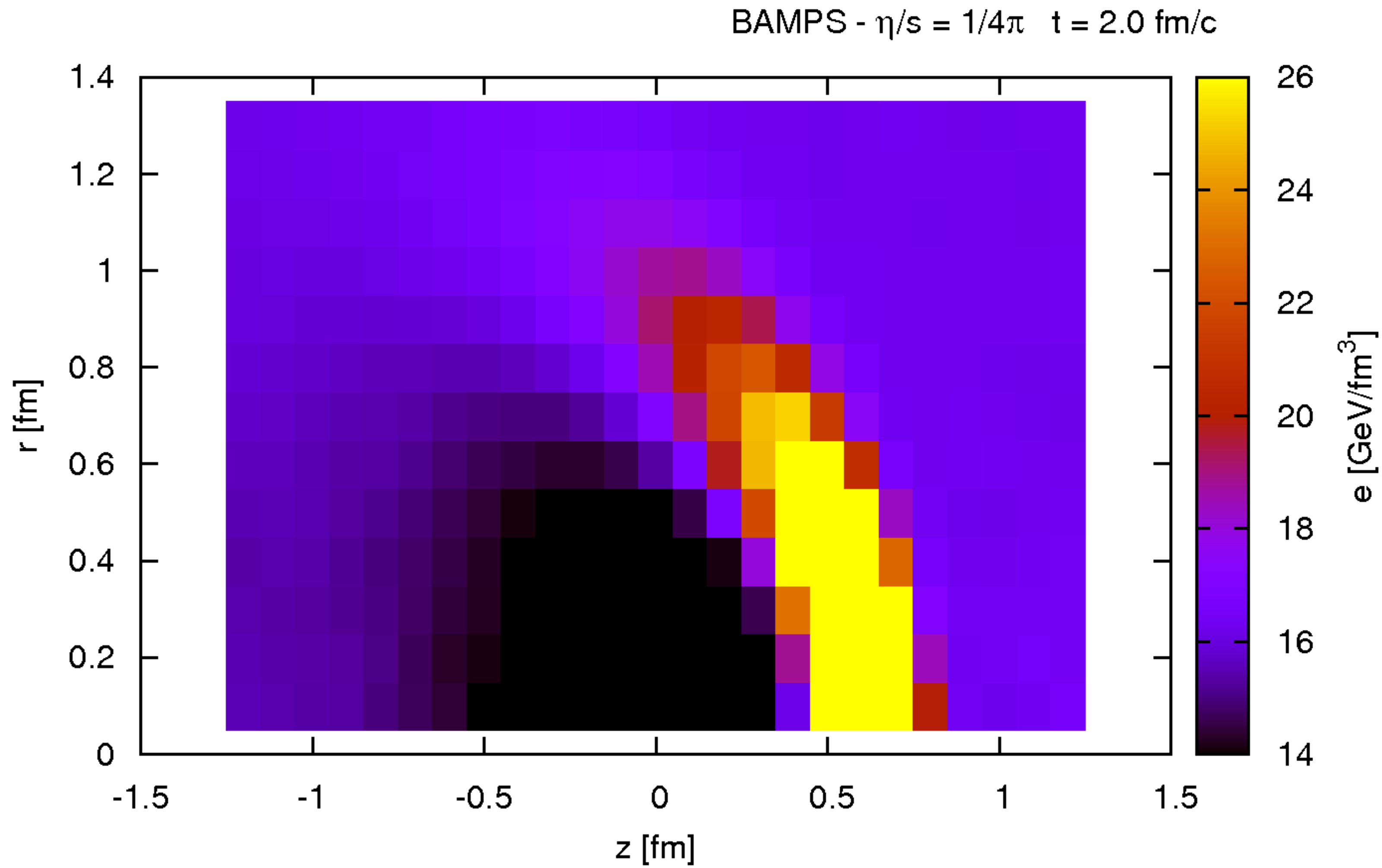
Mach Cones in BAMPS: Evolution

$\eta / s = 1/4\pi$

$T = 400$ MeV

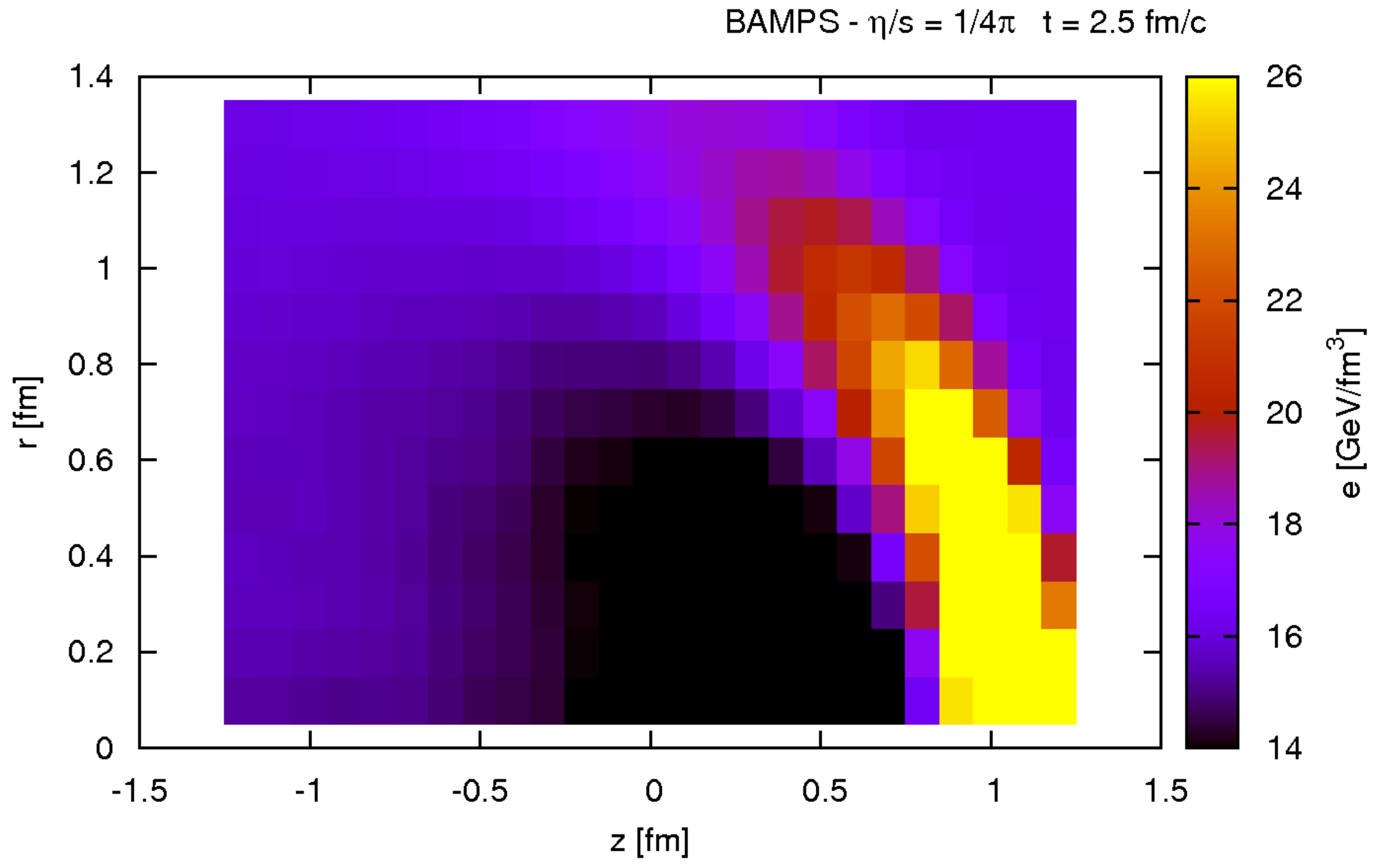
$E_{jet} = 20$ GeV

$t = 2.0$ fm/c



Mach Cones in BAMPS: Evolution

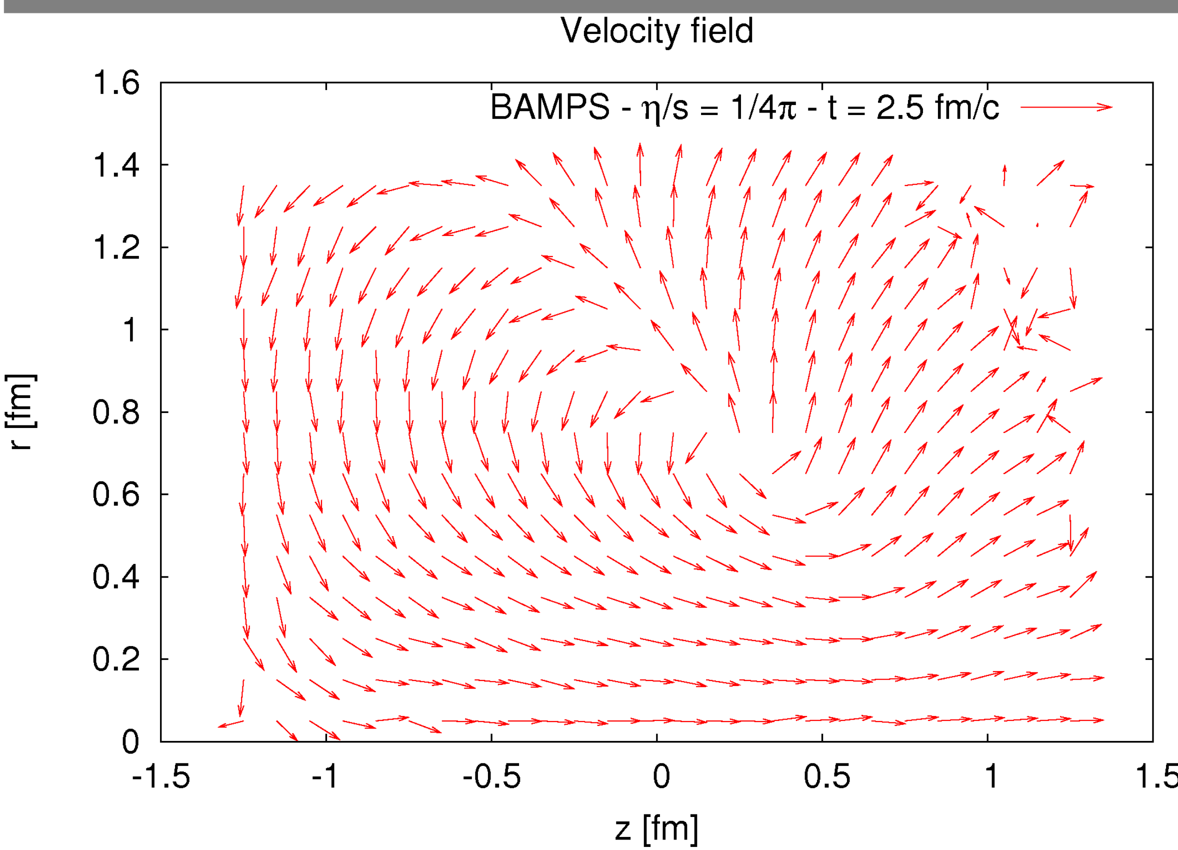
$\eta / s = 1/4\pi$
$T = 400\ \mathrm{MeV}$
$E_{jet} = 20\ \mathrm{GeV}$
$t = 2.5\ \mathrm{fm/c}$

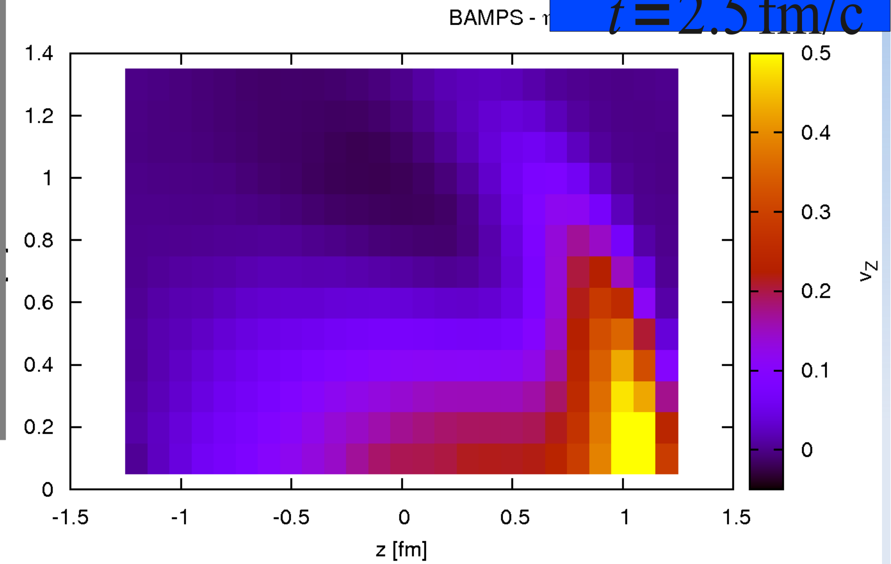


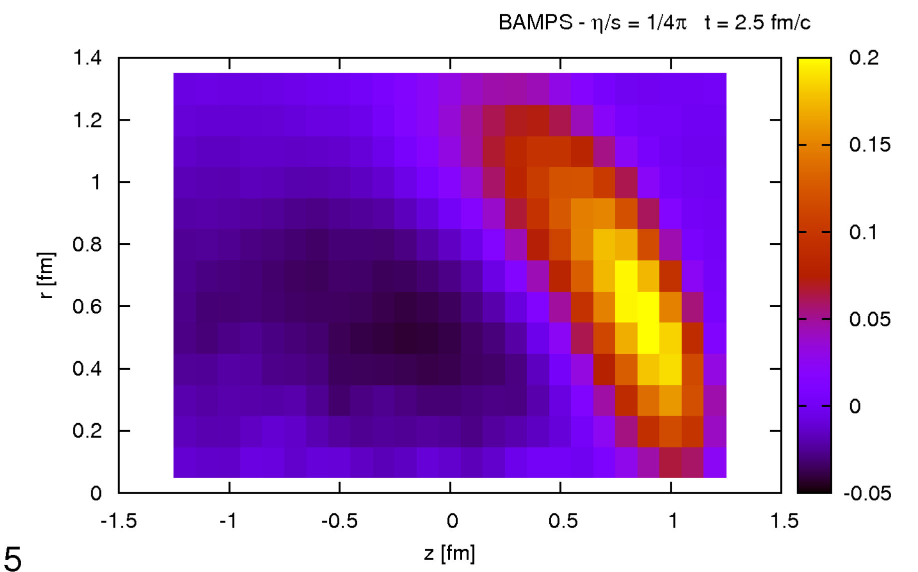
Mach Cones in BAMPS: Velocity profiles

$\eta/s = 1/4\pi$
$T = 400$ MeV
$E_{jet} = 20$ GeV
$t = 2.5$ fm/c

- **The results agree qualitatively with hydrodynamic and transport calculations**
  → *B. Betz, PRC 79:034902, 2009*
  → D. Molnar, arXiv:0908.0299v1
- **Strong collective behaviour is observed**
- **A diffusion wake is also visible, momentum flows in direction of the Jet**

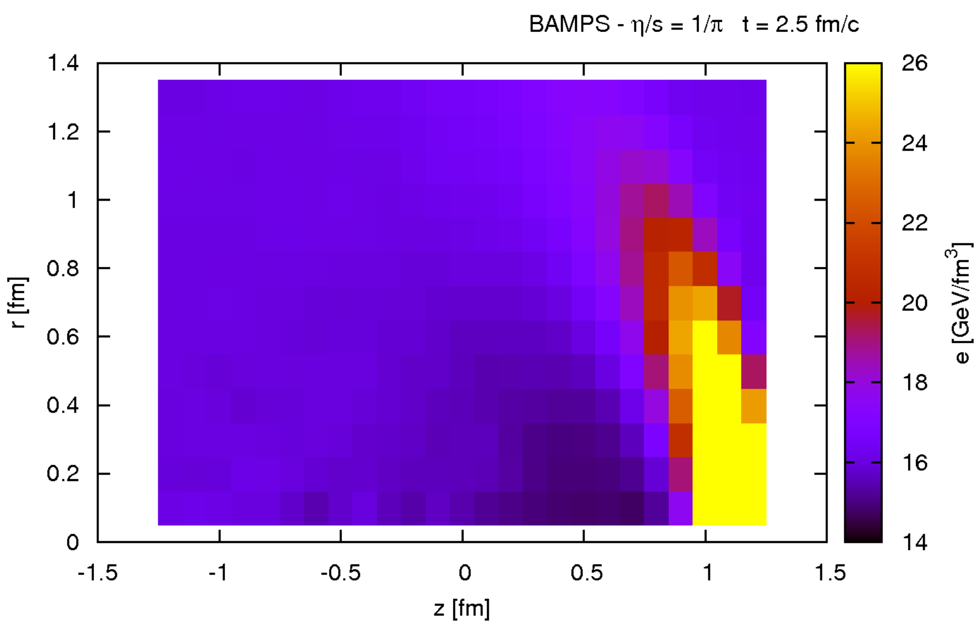




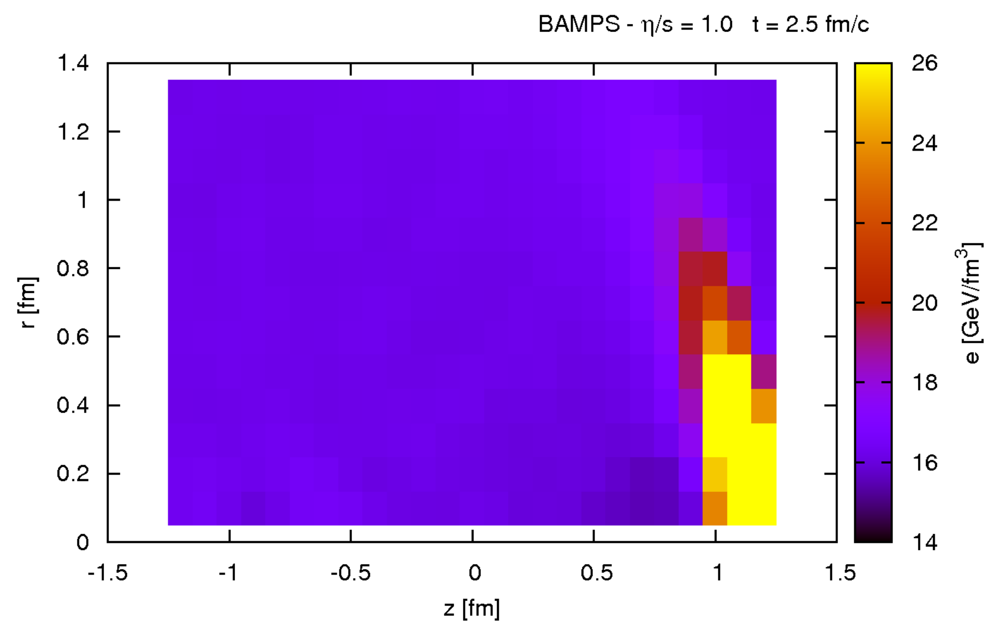


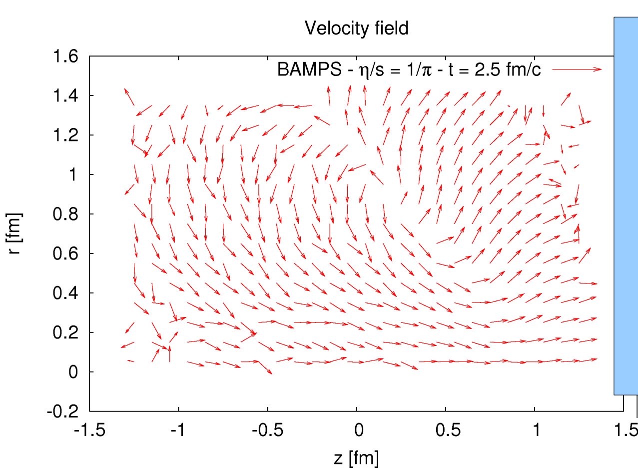
Mach Cones in BAMPS:
More dissipative medium?

$\eta/s = 1/\pi = 0.32$

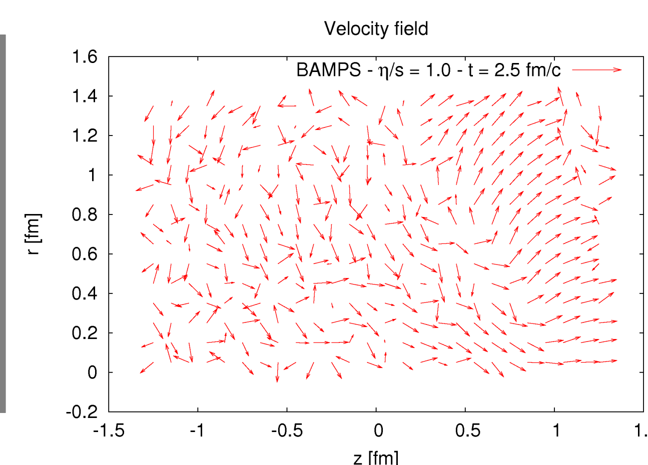


$\eta/s = 1.0$

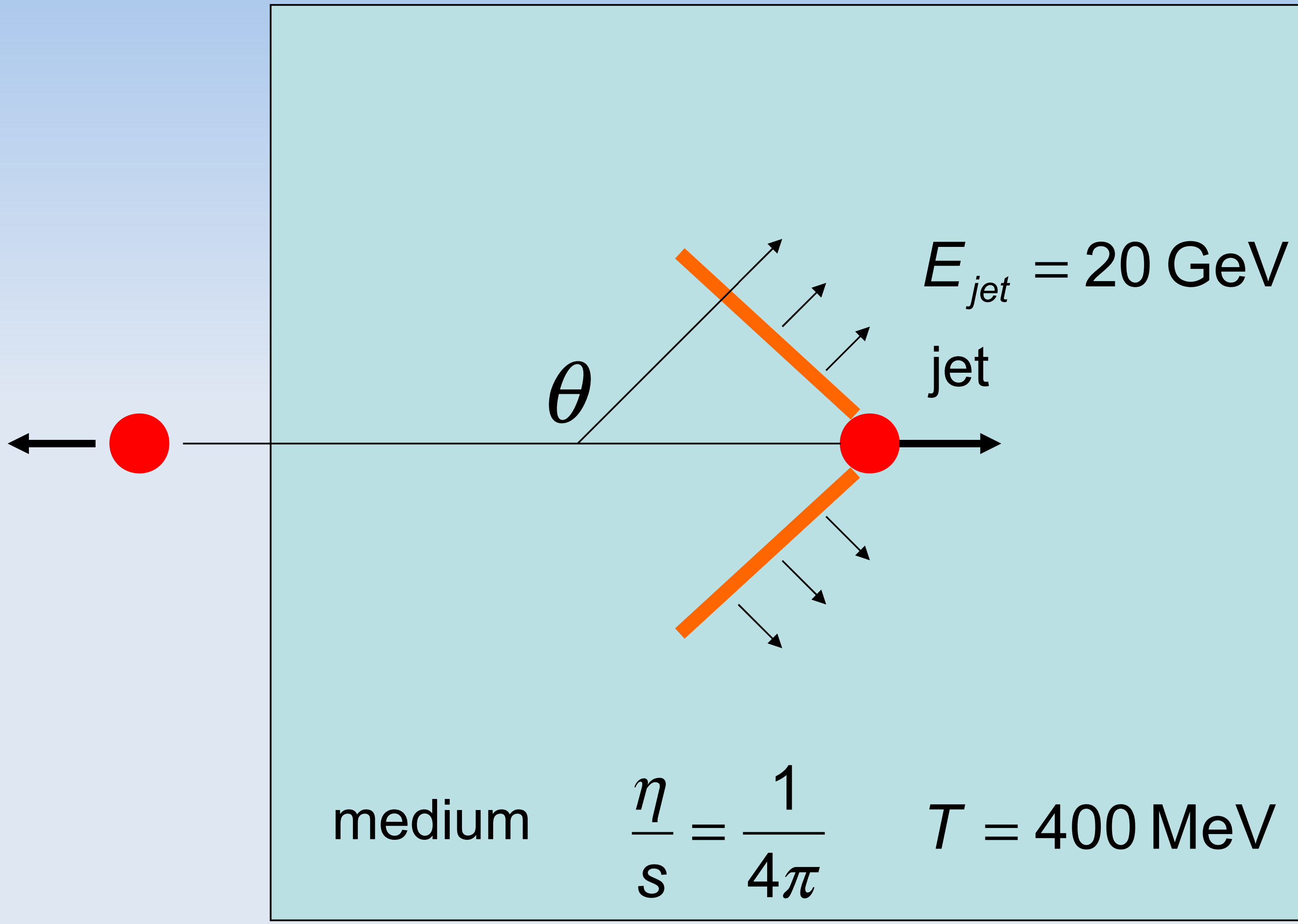




- Mach Cone structure vanish with more dissipation
- Collective behaviour also vanish
- Mach Cone angle changes, see next slides

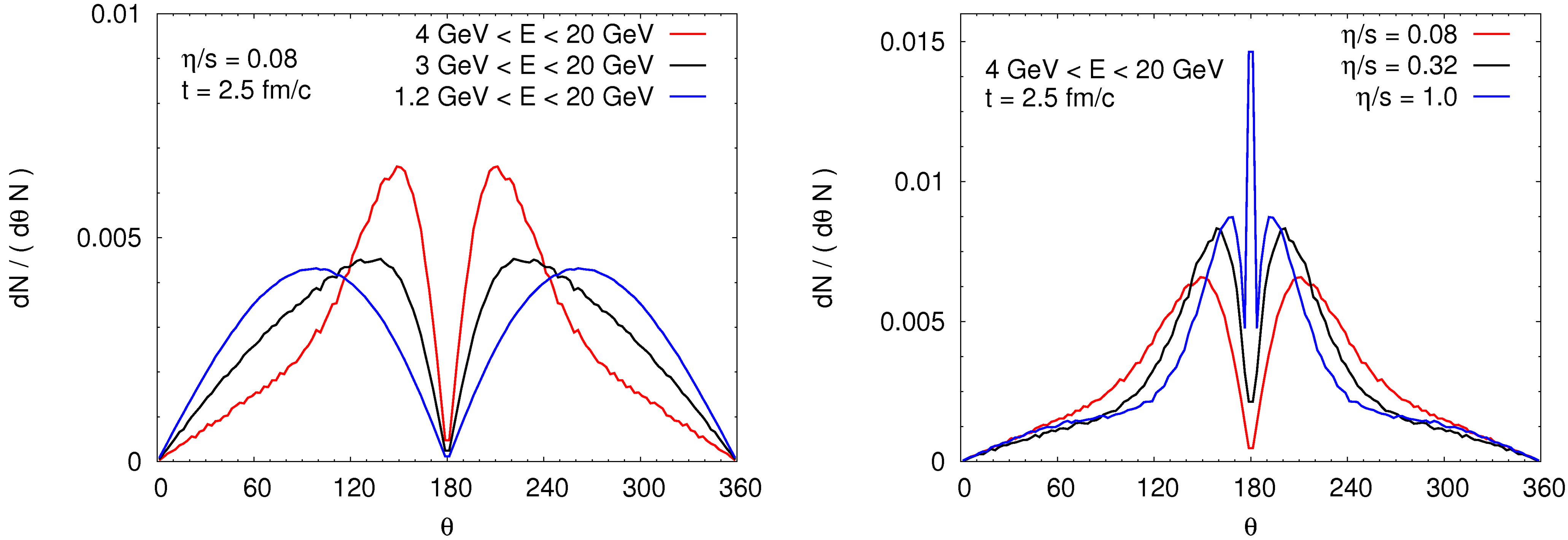


Mach Cones in BAMPS: 2-Particle Correlations



Mach Cones in BAMPS: 2-Particle Correlations

- Double-Peak structure is observable
- Absolute value of the "Mach Cone angle" is not clear, depends on the background cut
- As stronger the collective behaviour of the medium, as larger the emission angle of the Mach Cone



Conclusion and Outlook

- We solve the relativistic Riemann problem using BAMPS from ideal hydro to free streaming
- We compared BAMPS and vSHASTA → BAMPS is good for the comparison to every viscous hydro model !!!
- We investigated the evolution of the shock wave
- Shock waves are in principle possible at RHIC or LHC

- Full 3-dimensional simulations of Mach Cones were done → Strong collective behaviour is observed
- Mach Cones vanish when medium is strong dissipative
- 2-particle correaltions are observed – double peak structure exist

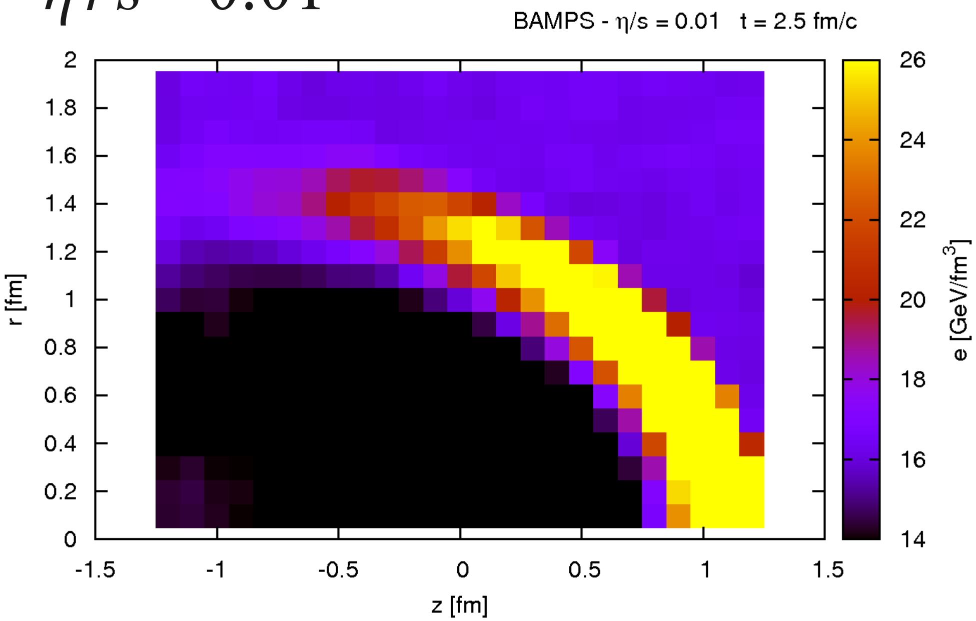
<u>Future Taks:</u>

- Use BAMPS as comparison model for other viscous hydro models
- Investigate Mach Cones in more detail and in more realistic scenarios → expanding box, phase transition, 2-> 3 processes

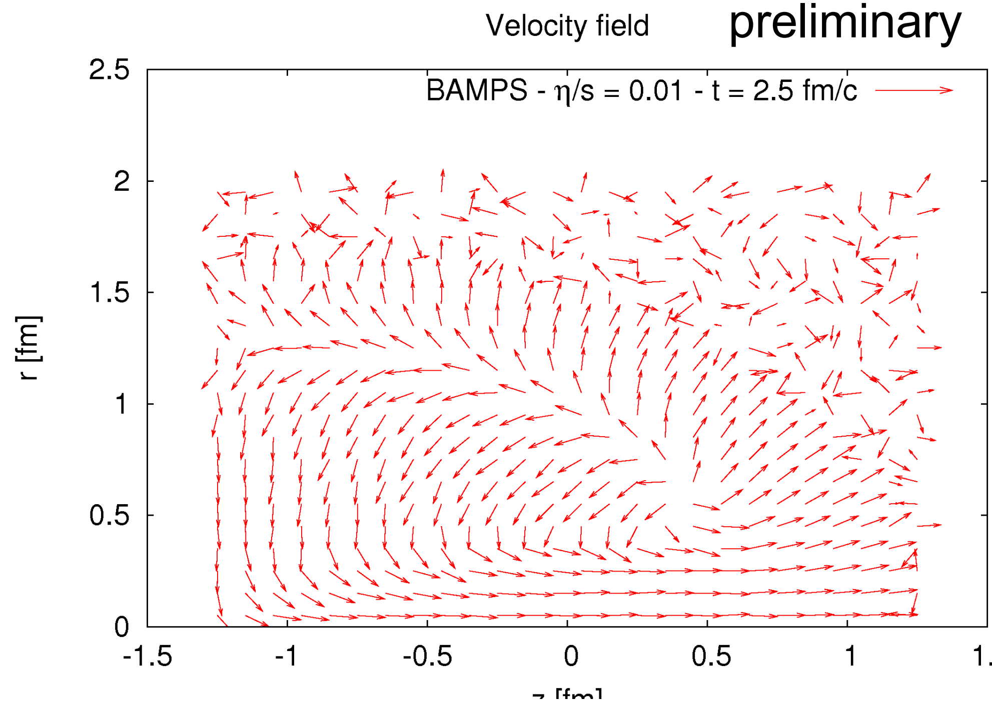
Thank you for your attention

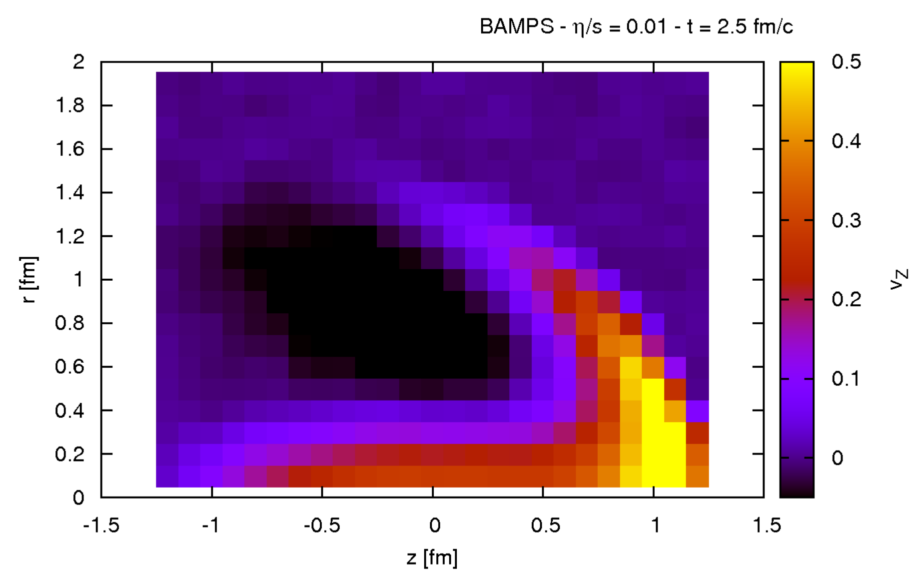
Mach Cones in BAMPS (2D)

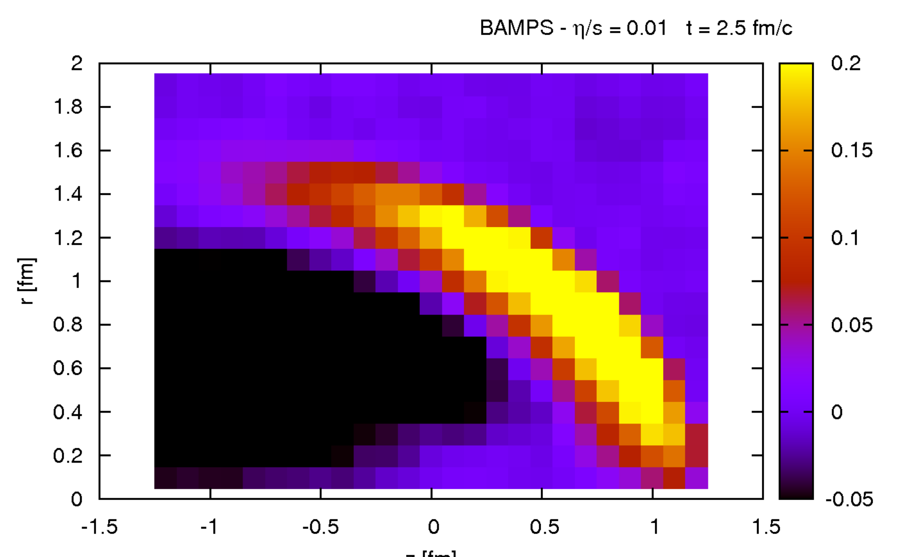
$\eta / s = 0.01$



preliminary







The relativistic Riemann problem

The relativistic hydrodynamic equations

- The local conservation of charge, energy and momentum

$$\partial_\mu N^\mu = 0$$
$$\partial_\mu T^{\mu\nu} = 0$$

with

$$T^{\mu\nu} = (\epsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$
$$N^\mu = n u^\mu$$
$$g^{\mu\nu} = diag(1, -1, -1, -1)$$
$$u^\mu = (\gamma, \gamma v) \quad with \quad u^\mu u_\mu = 1$$
$$\gamma = 1/\sqrt{1 - v^2}$$

- The equations of relativistic hydrodynamics of an ideal fluid in one dimension

$$\partial_t N^0 + \partial_z (v_z N^0) = 0$$
$$\partial_t T^{0z} + \partial_z (v_z T^{0z}) = -\partial_z (p)$$
$$\partial_t T^{00} + \partial_z (v_z T^{00}) = -\partial_z (v_z p)$$

$$N^0 = \gamma n$$
$$T^{00} = \gamma^2 (\epsilon + p) - p$$
$$T^{0z} = \gamma^2 (\epsilon + p) v$$

Equation of state $p = p(\epsilon, n)$

The relativistic Riemann problem

Shock discontinuities

- **Shock waves represent discontinuous solutions of ideal hydrodynamics. The partial derivatives of the charge density and the energy momentum are not right defined at that location**
- **Therefore using the Rankine-Hugeniot-Taub relations**

$$n_3 \gamma_3 v_3 = n_4 \gamma_4 v_4$$
$$(\epsilon_3 + p_3)\gamma_3^2 v_3 = (\epsilon_4 + p_4)\gamma_4^2 v_3$$
$$(\epsilon_3 + p_3)\gamma_3^2 v_3^2 + p_3 = (\epsilon_4 + p_4)\gamma_4^2 v_4^2 + p_4$$

We get

$$v_{shock} = \sqrt{\frac{(p_4 - p_3)(\epsilon_3 + p_4)}{(\epsilon_4 - \epsilon_3)(\epsilon_4 + p_3)}}$$

- **The quantities defined in the local rest frame of the shock front**

Numerical Results: Hydro Limits

$T_L = 400\ MeV$
$t = 1\ fm/c$

Expansion into the medium

$T_R = 350\ MeV$

$T_R = 100\ MeV$

Expansion into the vacuum

$T_R = 0\ MeV$