Lattice QCD + Hydro/Cascade Model of Heavy Ion Collisions

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Outline

• Calculation of $T_c$ via lattice QCD – domain wall fermion method
• Parameterization of LQCD EoS
• Model of heavy ion collision including:
  – Initial, non-equilibrium flow (Pratt)
  – 2D viscous hydrodynamics (Romatschke’s vh2)
  – Parton cascade (URQMD)
Lattice QCD with Domain Wall Fermions
Many recent high-precision calculations are performed with some variant of staggered fermion discretization (stout-link, asqtad, p4, HISQ)

Single quark flavor for staggered fermions correspond to 4 flavors of continuum quark flavor.

Spontaneous breakdown of SU(4) chiral symmetry -> 15 Goldstone bosons.

However, lattice effects explicitly break SU(4) chiral symmetry -> U(1). Only one GB. Other pions have non-zero mass of $O(a^2)$

To recover a one flavor theory on the lattice, take $\frac{1}{4}$ root.
Domain Wall Fermions

• Domain Wall Fermions (DWF) faithfully preserve SU($N_f$) chiral symmetry to arbitrary accuracy even at finite lattice spacing.
• Therefore, meson spectrum, e.g. 3 light pions, is more correctly reproduced by DWF method.
• Penalty: QCD with DWF is recovered as a 4-d space-time slice of a 5-d theory.
Staggered v. DWF

• Primary reason to use staggered fermions: cost.
• Size of fifth dimension in DWF calculations: 8-32.
• Staggered fermions approach smaller lattice spacing at high precision faster than DWF.
• Since $T = 1/(N_t a)$, lattice calculations are done at fixed $N_t$ and varying lattice spacing.
• Until recently, only large lattice spacings feasible for exploration of finite T QCD ($N_t=4, 6$). In this regime, DWF formulation does not work so well.
DWF at $N_t = 8$

- Well-known disagreement for $T_c$ among staggered fermion calculations. Cannot agree on whether chiral, deconfining transitions are distinct. $T_c = 150-200$ MeV
- Calculations at $N_t=8$ for DWF are feasible. Useful check on the staggered calculations.
- Work done in collaboration with RBC Collaboration (arXiv:0911.3450)
- Vary lattice coupling ($\beta=6/g^2$) to change temperature.
- Calculate chiral, deconfinement observables.
• $\chi_l/T^2$ -> Chiral susceptibility. Peaks in transition region
• $\Delta_{l,s}/T^3$ -> Chiral condensate. Non-zero at low temperature, zero at high temperature.
• Deconfinement observables: isospin and charge susceptibilities.
• Inflection point determined by fitting data to ansatz.
• Consistent with peak in chiral susceptibility.
• However, SB limit already saturated at low temperature, as expected as DWF formulation is unimproved at high temperature.
Caveats

- Limitations in this calculation:
  - Small volume (Finite volume effects not controlled)
  - Lacks precision of staggered studies.
  - Quark mass not held constant in this calculation -> \( m_\pi \approx 300 \text{ MeV} \) at \( T = 170 \text{ MeV} \), but larger at low temperature, smaller at high temperature.
  - Single lattice spacing – cannot make continuum extrapolation (4-7% error suggested by other calculations)
  - Single set of masses – guess at extrapolation to physical quark masses.
Comparison with $L_s = 96$ calculation

$T_c = 171(17)(10)$ MeV
Hydro/Cascade Model
Description of Model

- Hybrid model includes:
  - Pre-thermalization flow (Pratt arXiv:0810.4325)
  - 2D viscous hydrodynamic evolution (Romatshke’s vh2)
  - Hadron cascade, after Cooper-Frye freezeout (URQMD)

- Examine the effect of varying:
  - Equation of state (LQCD EoS vs. 1st order transition)
  - Viscosity
  - Pre-thermalization flow.
  - Initial conditions/freezeout temperature

- Collaborators:
  - Ron Soltz, Andrew Glenn, Jason Newby (LLNL and ORNL)
  - Scott Pratt

- Talk by R. Soltz at CATHIE/TECHQM
Parameterizing LQCD EoS

- Already saw a more detailed study in talk by Petreczky, but also many others.
- Let $f(T)$ be parameterization of EoS.
- Suggestion by K. Rajagopal:
  - $1/f(T) = 1/g(T) + 1/h(T)$
  - $g(T)$ -> low temperature
  - $h(T)$ -> high temperature
  - $h(T) = d_2/T^2 + d_4/T^4$
  - $g(T) = (a + (T/T_0)^b) \times \text{HRG}(T)$
- Fix low $T$ to HRG by setting $a = 1.0$
Re-parameterized EoS
Speed of Sound

Hydro EOS Comparison

\[ C_s^2 \]

- HRG fit (50\%) to p4
- HRG fix (a=1.0) to p4
- vh2 default
- First Order Tc=180

T (GeV)
Description of existing runs

- **Initial Flow**
  - From Glauber profile.
  - $b = 3.4, 5.5$ fm., $T_{\text{initial}} = 250-350$ MeV

- **Vh2 2-D hydro:**
  - $\eta/s = 0.08 - 0.40$
  - EoS = Romatchske EoS, LQCD, LQCD+HRG

- **Cooper-Frye freezeout**
  - $T_{\text{freezeout}} = 120 - 170$ MeV

- **URQMD for hadronic cascade**

- **Match spectra to tune parameters**
Spectra, \( T_f = 120-170 \) MeV
V2 – with/without initial flow
$V_2$, $b = 3.4$, 5.5 fm.
$V_2, \eta/s=0.08-0.40$
Conclusions

• Calculation of crossover temperature with DWF to compare with staggered-type calculations.

• $T_c \sim 170$ MeV, but with large error because of statistics and several systematic uncertainties.

• No splitting evident for deconfinement, chiral observables

• Not really in disagreement with either of conflicting staggered calculations.

• Exploratory calculation – need to do a calculation that corrects many of the flaws of current calculation.

• One is underway, thinking about other methods, but still computationally too expensive…
Conclusions (cont.)

- Hybrid model including pre-thermalization flow + 2D viscous hydrodynamics + URQMD (almost) working.
- Still work in progress.
- Goals:
  - Study collective flow, femtoscopy.
  - Effects of varying $\eta/s$, initial conditions, $T_{\text{freezeout}}$
  - Does pre-thermal flow help explain HBT puzzle?
  - Quantify effects of varying EoS
  - Systematic comparison to experimental data.
Backup
Residual Mass at $L_s = 32$
HBT Radii, varying viscosity

- $R_{\text{out}}$ vs $k_T$ (MeV)
- $R_{\text{side}}$ vs $k_T$ (MeV)
- $R_{\text{long}}$ vs $k_T$ (MeV)
- $R_{\text{out}}/R_{\text{side}}$ vs $k_T$ (MeV)

Legend:
- STAR 0-30%
- PHENIX 0-30%
- STAR 30-50%
- $\eta/s=0.080000$
- $\eta/s=0.160000$
- $\eta/s=0.240000$
- $\eta/s=0.320000$
- $\eta/s=0.400000$