Viscous Hydrodynamics

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Contents

1. Radiative energy loss and v_2 spectra for viscous hydrodynamics

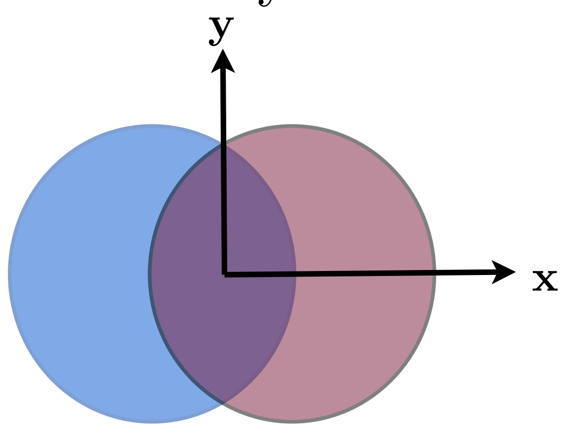
   K.D., Derek Teaney, Guy Moore arXiv:0909.0754

2. Photons and Dileptons for viscous hydrodynamics

   Photons: K.D. arXiv:0903.1764
   Dileptons: K.D., Shu Lin NPA 809:246-258, 2008.

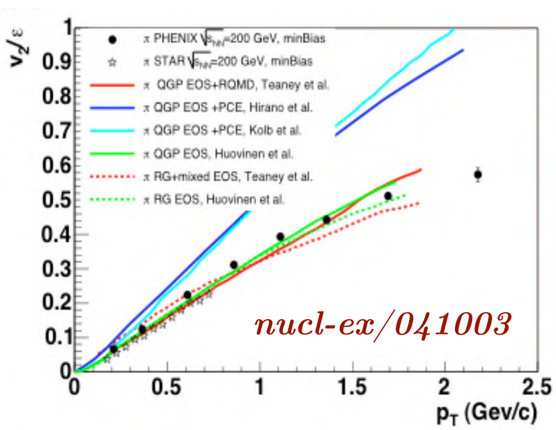
Introduction

- BNL Press Release 05: *RHIC Scientists serve up perfect Liquid*
- Conclusion reached by a detailed study of "flow" measurements



$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \approx 20\%$$

- Fact that ideal hydrodynamics "worked" was surprising to many



Must quantify these findings using viscous hydrodynamic simulations

How does viscosity manifest itself in spectra?

1. Viscous correction to equation of motion

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{where} \qquad T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p g^{\mu\nu} - \eta\langle\partial^\mu u^\nu\rangle$$

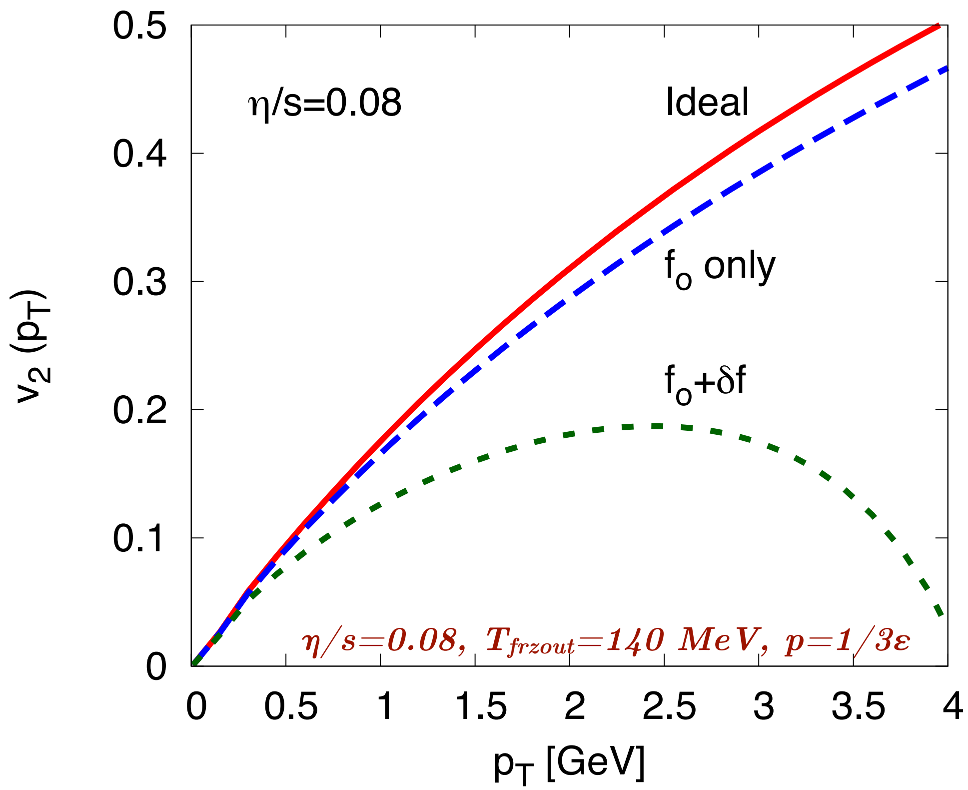
2. Viscous correction to spectra

$$E\frac{d^3N}{d^3p} = \frac{\nu}{(2\pi)^3}\int_\sigma f_o + \delta f \; p^\mu d\sigma_\mu$$

$$\delta f = -\frac{\eta}{sT^3} \times f_0(p) p^i p^j \langle\partial_i u_j\rangle$$

All simulations to date have used quadratic ansatz.

How does viscosity manifest itself in spectra?



We need to have a quantitative understanding of δf and quadratic ansatz.

δf in relaxation time approximation

- Start with Boltzmann equation

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\frac{f(p) - f_0(p)}{\tau_R(E_p)}$$

- Substitute $f(p) = f_o(p) + \delta f(p)$ and find

$$\delta f \propto \frac{\tau_R(E_p)}{E_p} f_0(p) p^i p^j \langle \partial_i u_j \rangle$$

- So we get back quadratic ansatz when $\tau_R \propto E_p$
but what about $\tau_R \propto (E_p)^\beta$?

Generalize quadratic ansatz

- Most general form of off equilibrium correction is

$$\delta f = -\chi(\tilde{p}) \times f_0 \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle$$

where $\tilde{p} \equiv \frac{p}{T}$ and $\hat{p}^i \equiv \frac{p^i}{|\mathbf{p}|}$

- Now we take the ansatz $\chi(\tilde{p}) \propto \tilde{p}^{2-\alpha}$

Two Extreme Limits

- Quadratic: Relaxation time growing with energy

$$\tau_R \propto E_p \qquad \frac{dp}{dt} \propto \text{const.} \qquad \chi(p) \propto p^2$$

- Linear: Relaxation time independent of Parton energy

$$\tau_R \propto \text{const.} \qquad \frac{dp}{dt} \propto p \qquad \chi(p) \propto p$$

- As we will show reality is somewhere in between

Connection between δf and viscosity

$$T^{ij} \equiv p\delta^{ij} - \eta\langle\partial^i u^j\rangle = \int_{\mathbf{p}} \frac{p^i p^j}{E_p}\, f_o + \delta f(p)$$

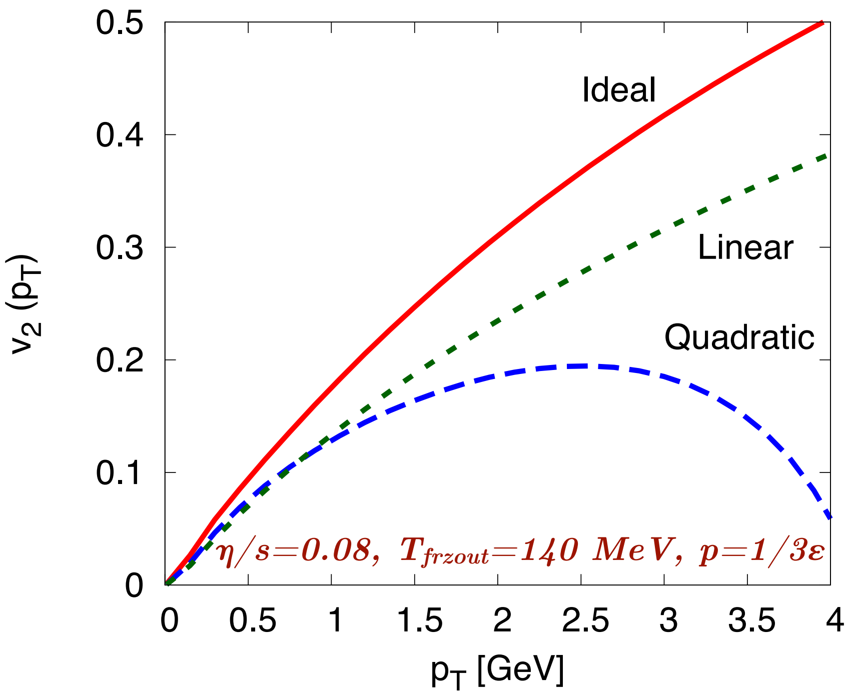
First moment of δf determines shear viscosity.

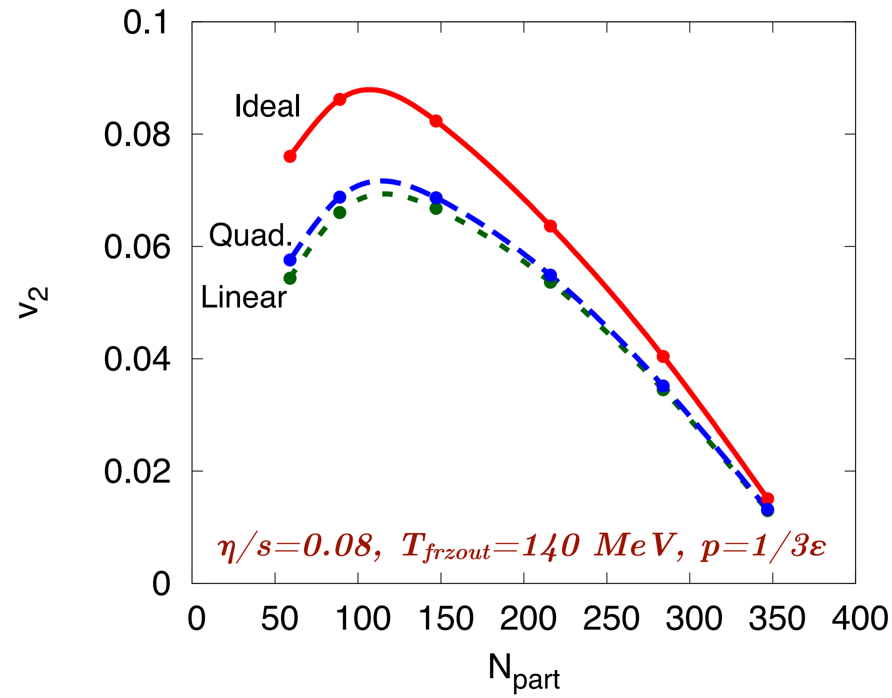
$$\delta f = -\chi(\tilde{p}) \times f_0 \hat{p}^i \hat{p}^j \langle\partial_i u_j\rangle \longrightarrow \eta = \frac{1}{15}\int_{\mathbf{p}} f_o \chi(p) p$$

$$\chi(\tilde{p}) = \frac{120}{\Gamma(6-\alpha)} \times \frac{\eta}{sT} \times \tilde{p}^{2-\alpha}$$

So the form of δf is partially constrained by viscosity.

Two Extreme Limits





$$\eta \propto \int_{\mathbf{p}} p \; f_0 \chi(p)$$

$$\delta\overline{v_2} \propto \int_{\mathbf{p}} p^2 \; f_0 \chi(p)$$

Weakly coupled pure-glue QCD

- Boltzmann equation

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{2\leftrightarrow 2}[f] - \mathcal{C}^{1\leftrightarrow 2}[f]$$

- Substitute $f(p) = f_o(p) + \delta f(p)$ and find

$$f_o \frac{p^i p^j}{T E_p} \langle \partial_i u_j \rangle = -\mathcal{C}^{2\leftrightarrow 2}[\delta f] - \mathcal{C}^{1\leftrightarrow 2}[\delta f]$$

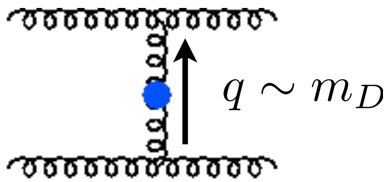
- This integral equation can be inverted to obtain δf.

Weakly coupled pure-glue QCD

- Three different modes of energy loss

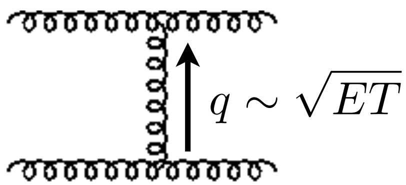
Asymptotic Forms

1. Soft Scattering



$$\frac{dp}{dt} \propto g^4 \log\left(\frac{T}{m_D}\right) \qquad \chi(p) \propto p^2$$

2. Collisional



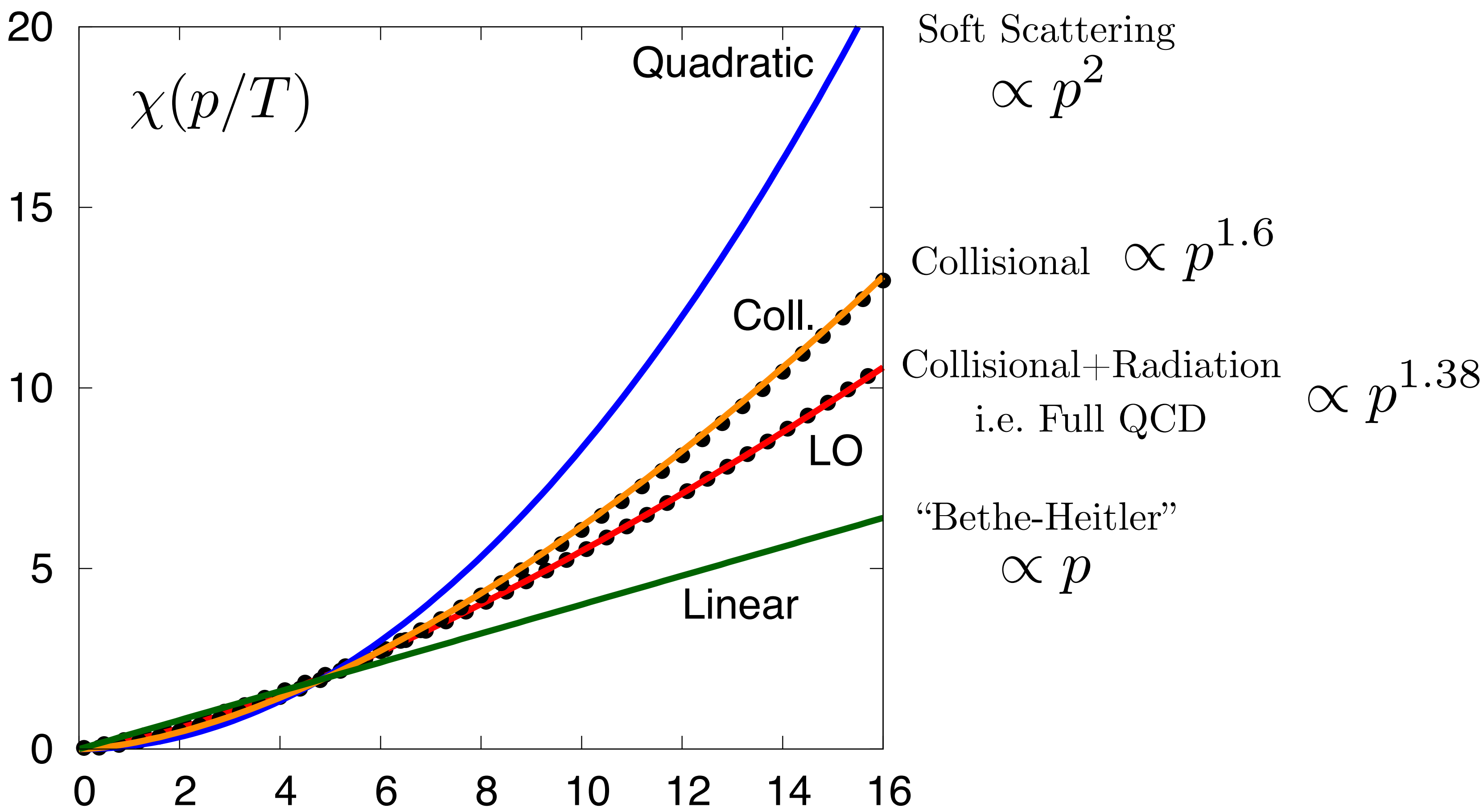
$$\frac{dp}{dt} \propto g^4 \log\left(\frac{p}{m_D}\right) \qquad \chi(p) \propto \frac{p^2}{\log p}$$

3. Radiative

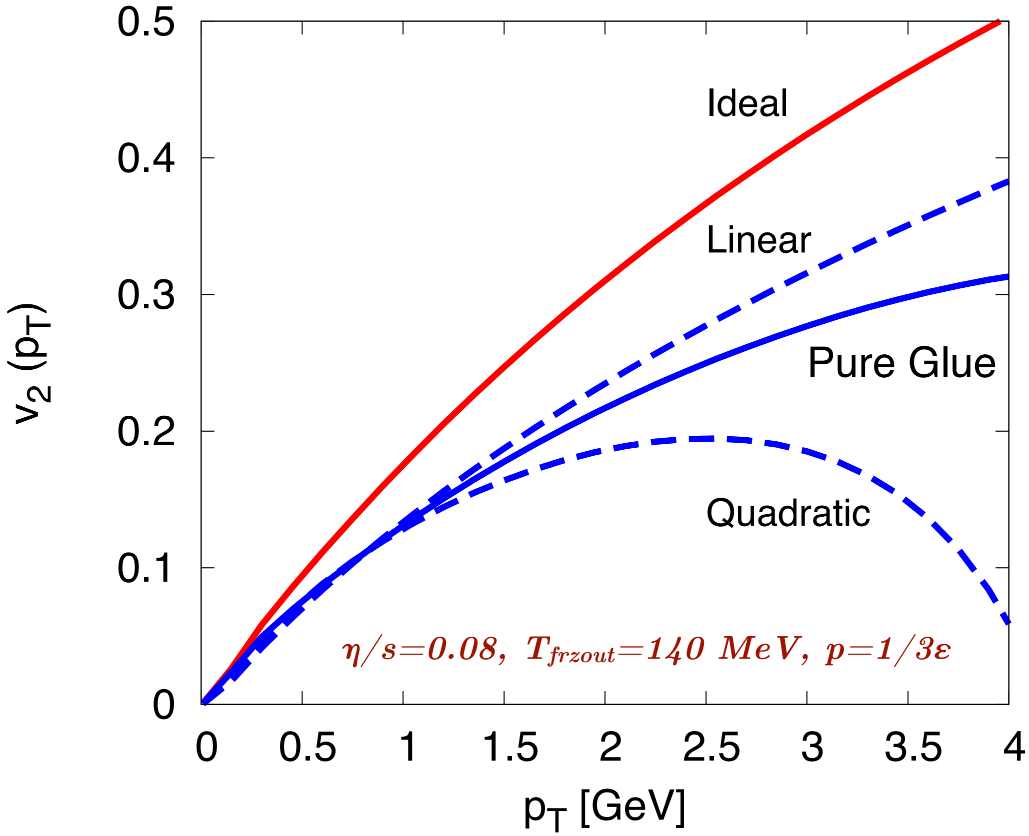
$$\frac{\Delta p}{\Delta t} \propto g^2 \sqrt{\hat{q} E_p} \qquad \chi(p) \propto p^{3/2}$$

The forms of χ(p) at large momentum (including the constant) can be found analytically from the Boltzmann equation.

Weakly coupled pure-glue QCD



Weakly coupled pure-glue QCD

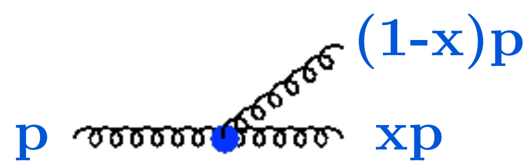


Connection to energy loss

- Let's look at large energies where radiation dominates

$$\partial_t f + v_{\mathbf{p}} \cdot \partial_{\mathbf{x}} f = -\mathcal{C}^{1\to 2}[f]$$

- The collision operator is



$$\mathcal{C}^{1\to 2} \propto \int_0^1 dx\ \gamma(p; xp, (1-x)p) \left[\chi_p - \chi_{xp} - \chi_{(1-x)p}\right]$$

- with splitting function at large p, $\ln^{-1}(\tilde{p}) \ll 1$

$$\gamma \propto \alpha_s C_A d_A \sqrt{p\hat{q}} \frac{[1 - x(1-x)]^{5/2}}{[x(1-x)]^{3/2}}$$

P. Arnold, C. Dogan, arXiv:0804:3359

Connection to energy loss

- Linearize Boltzmann equation

$$\delta f = -\chi(\tilde{p}) \times f_o(1+f_o)\hat{p}^i\hat{p}^j\langle\partial_i u_j\rangle$$

- in the high momentum limit

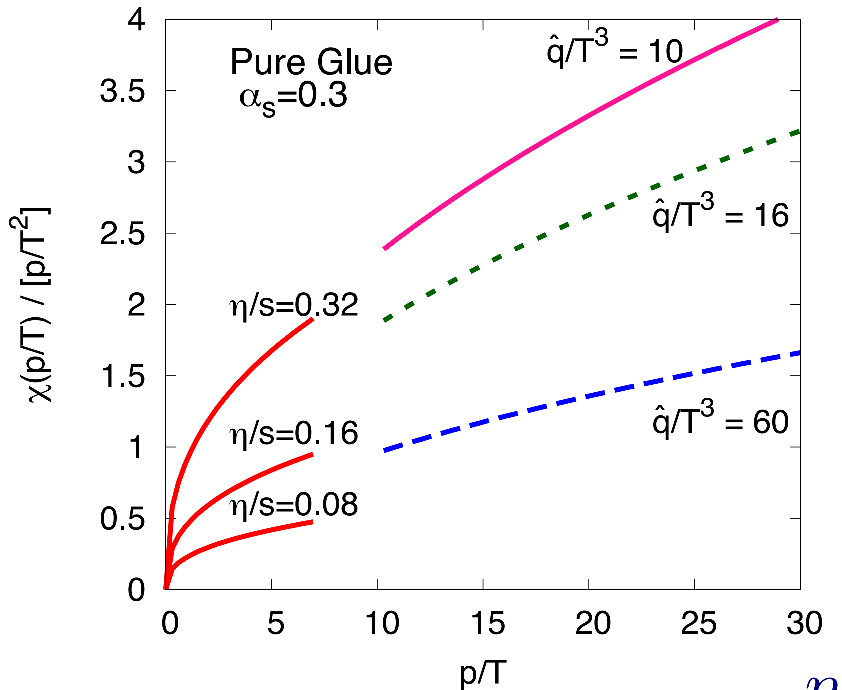
$$\frac{p^2}{T} = -\frac{(2\pi)^3}{32}\int_0^\infty dx\, \gamma(p;xp;(1-x)p)\ \left[\chi_p - \chi_{xp} - \chi_{(1-x)p}\right]$$

- Remember, the splitting function went like $\gamma \propto \alpha_s\sqrt{p\hat{q}} \times F(x)$

$$\chi(p) \approx 0.704778 \times \frac{p^{3/2}}{\alpha_s T\sqrt{\hat{q}}}$$

Connection to energy loss

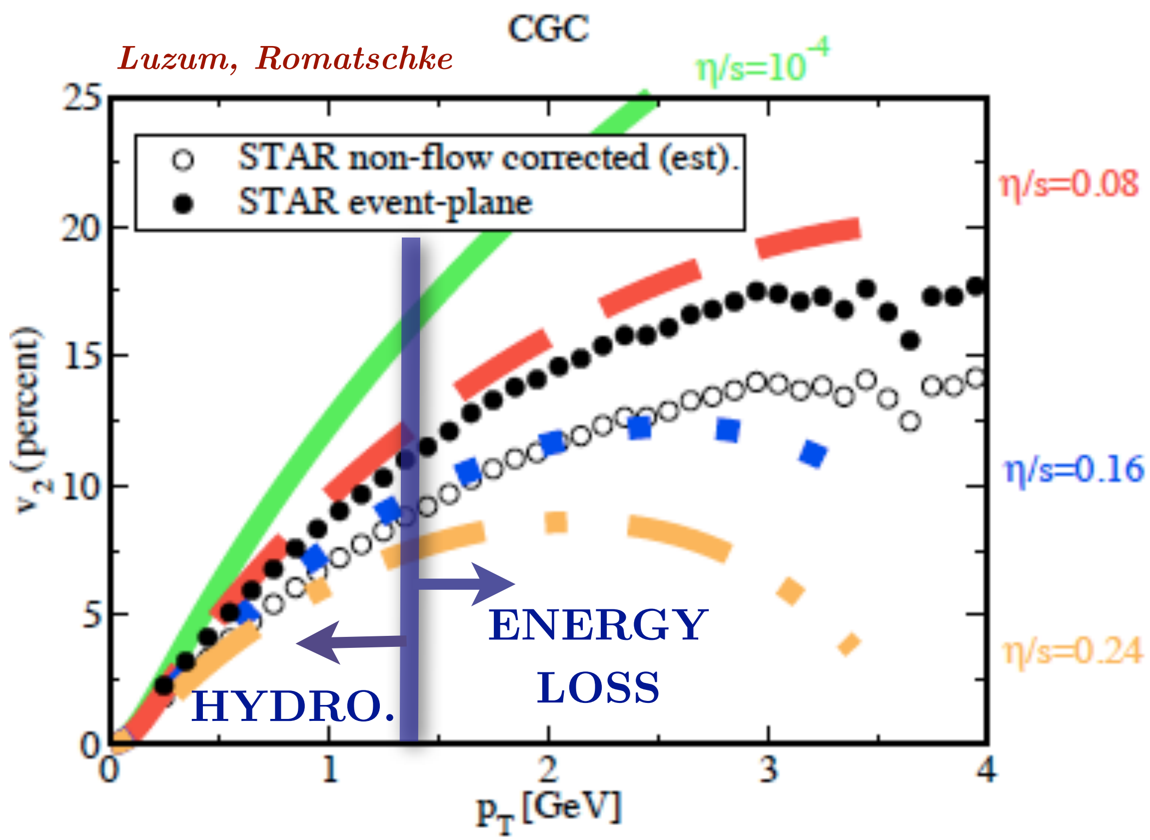
1. At low momentum $\chi(\mathrm{p})$ controlled by shear viscosity η/s
2. At high momentum $\chi(\mathrm{p})$ controlled by $\hat{q}$



$$\chi_g(p) \approx \frac{0.7}{\alpha_s T\sqrt{\hat{q}}} p^{3/2}$$

There must be some consistency between $\dfrac{\eta}{s}$ and $\hat{q}$.

Connection to energy loss



So far only a single component plasma (pure-glue QCD).
Now we will come to multi-component plasmas.

Quark and Gluons

- Quarks and Gluons have different δf

$$\begin{aligned} \delta f_g(p) &= \chi_g(\tilde{p}) f_o \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle \\ \delta f_q(p) &= \chi_q(\tilde{p}) f_o \hat{p}^i \hat{p}^j \langle \partial_i u_j \rangle \end{aligned}$$

- One constant provided by shear viscosity

$$\eta = \frac{1}{15} \sum_{a=q,g} \nu_a C_a \int \frac{d^3p}{(2\pi)^3} p^{3-\alpha_a} n \left(1 \pm n\right)$$

- Second constant and momentum dependence comes from Boltzmann equation.

Quark and Gluons

- Boltzmann equation schematically written as

$$\begin{bmatrix} f_o^g \tilde{p}_g^i \hat{p}_g^j \langle \partial_i u_j \rangle \\ f_o^q \tilde{p}_q^i \hat{p}_q^j \langle \partial_i u_j \rangle \end{bmatrix} = \begin{bmatrix} \Gamma_{gg} & \Gamma_{gq} \\ \Gamma_{qg} & \Gamma_{qq} \end{bmatrix} \begin{bmatrix} \delta f_g \\ \delta f_q \end{bmatrix}$$

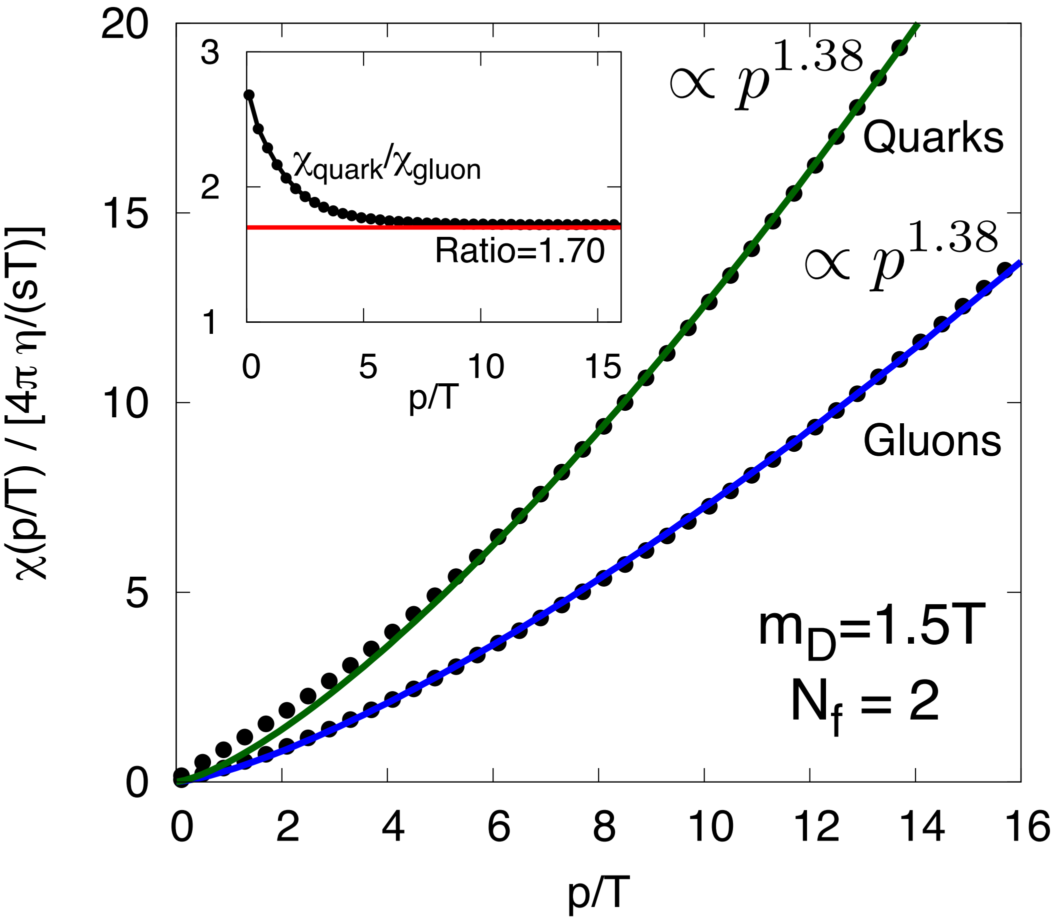
- at asymptotically high momentum

$$\nu_g \frac{p^2}{(2\pi)^3} = -\frac{1}{2} \int_0^\infty dx\, \gamma_{gg}^g(p; xp; (1-x)p) \left[\chi_p^g - \chi_{xp}^g - \chi_{(1-x)p}^g \right]$$

$$- \int_0^\infty dx\, \gamma_{qq}^g(p; xp; (1-x)p) \left[\chi_p^g - \chi_{xp}^q - \chi_{(1-x)p}^q \right]$$

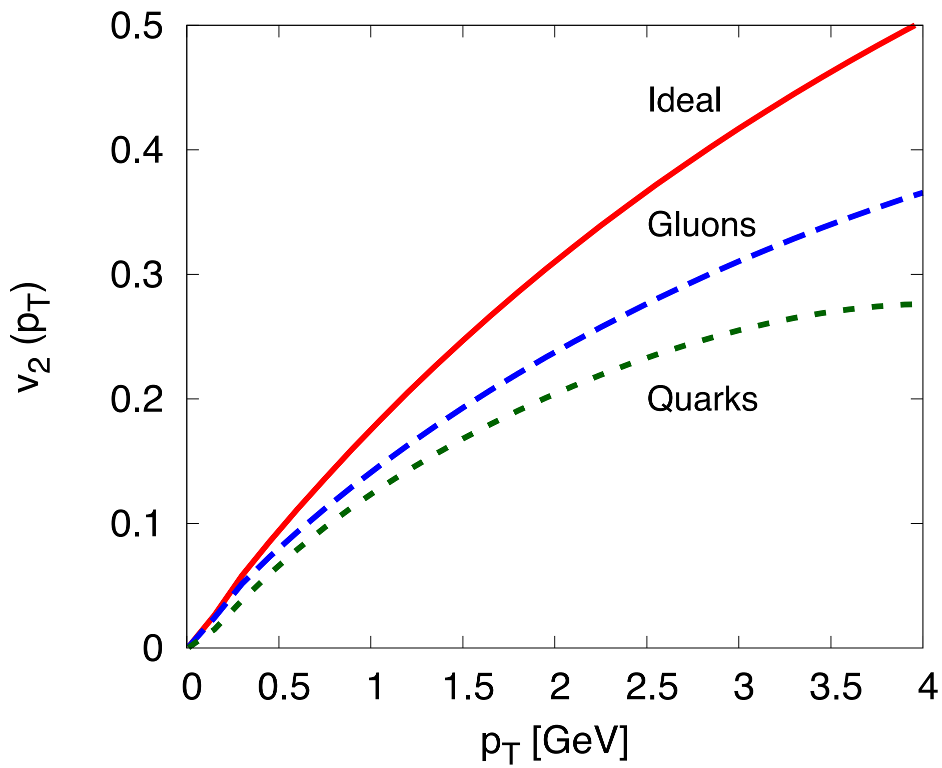
$$\nu_q N_f \frac{p^2}{(2\pi)^3} = -\int_0^\infty dx\, \gamma_{qg}^q(p; xp; (1-x)p) \left[\chi_p^q - \chi_{xp}^g - \chi_{(1-x)p}^q \right]$$

Quark and Gluons



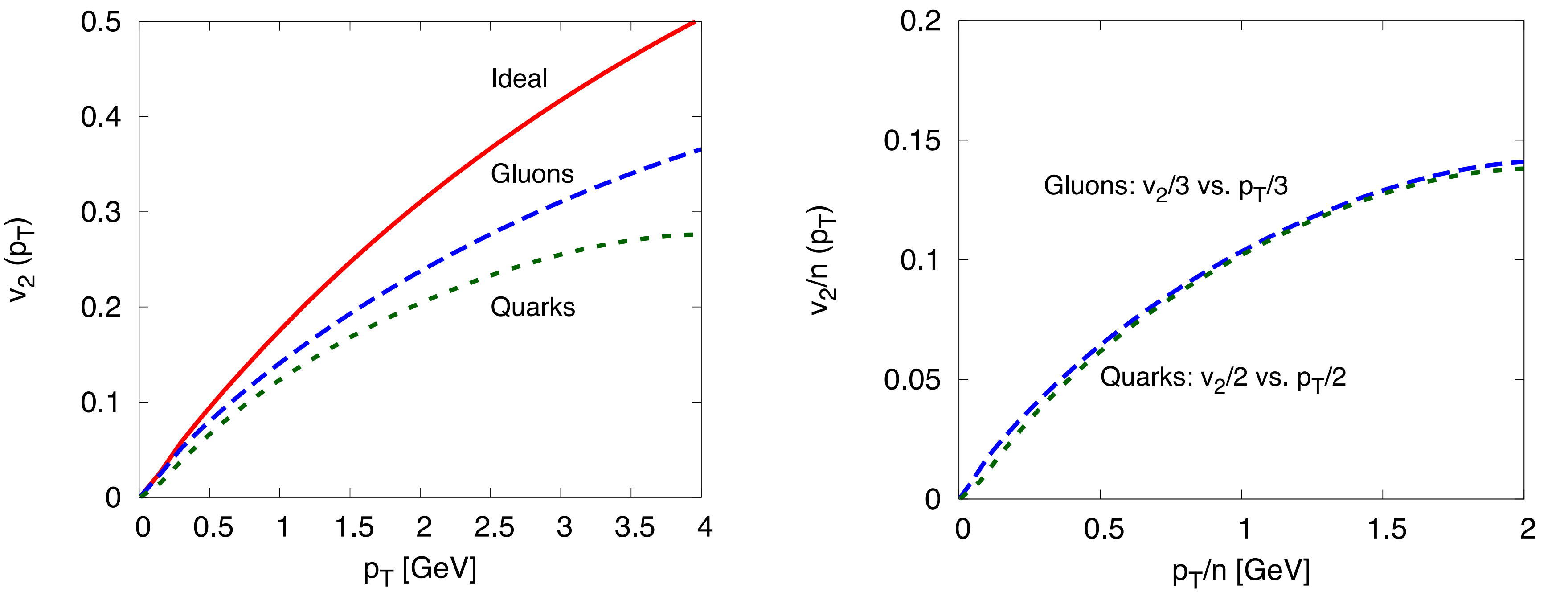
$$\frac{\chi_{quark}}{\chi_{gluon}} \sim \frac{\tau_R^Q}{\tau_R^G} \sim 1.7$$

Quarks and Gluons



Quarks and Gluons have different relaxation time and therefore different flows.

Scaling



In this case scaling is simply an artifact of the two different relaxation times.

Two Component Meson / Baryon gas

$$
\begin{aligned}
\delta f_m(p) &= n_p(1+n_p)\chi_m(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle \\
\delta f_b(p) &= n_p(1-n_p)\chi_b(\tilde{p})\hat{p}^i\hat{p}^j \langle \partial_i u_j \rangle
\end{aligned}
$$

- Lets start with simple quadratic ansatz

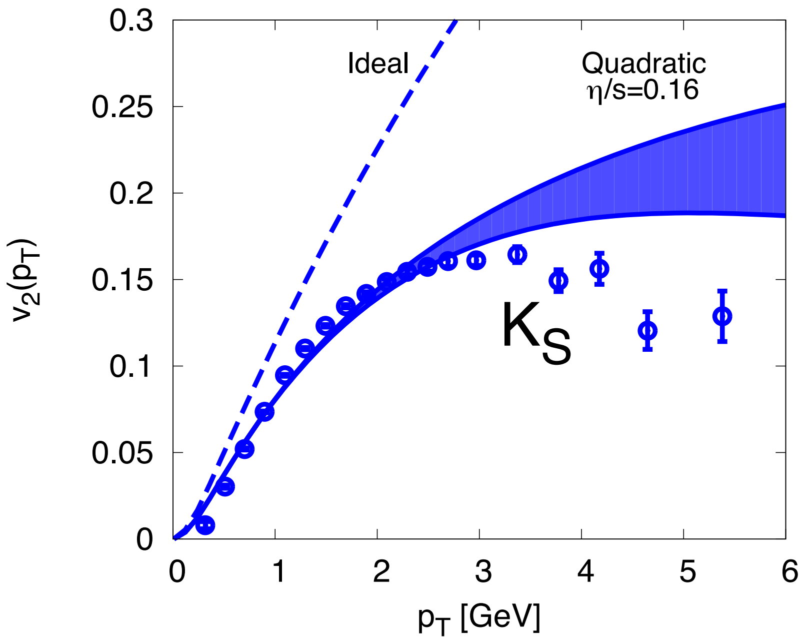
$$
\begin{aligned}
\chi_m(\tilde{p}) &= C_m \tilde{p}^2 \\
\chi_b(\tilde{p}) &= C_b \tilde{p}^2
\end{aligned}
$$

- Fit $\dfrac{C_m}{C_b} = 1.6$

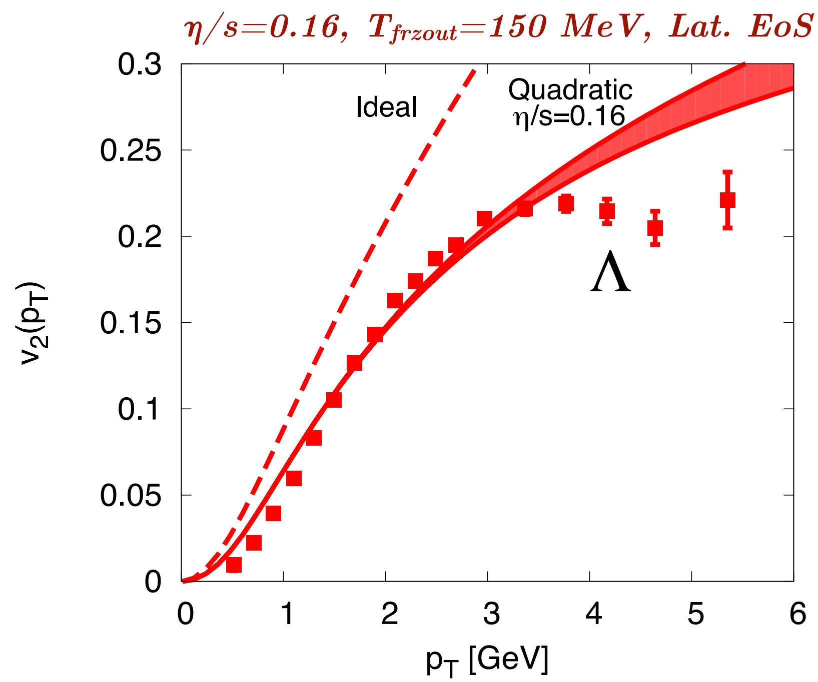
- And constrain to shear viscosity

$$
\eta = \frac{1}{15} \sum_{a=\pi,K,\ldots} \nu_a C_{m/b} \int \frac{d^3p}{(2\pi)^3 E_a} p^{4-\alpha} n(E_a) \left[1 \pm n(E_a)\right]
$$

Results

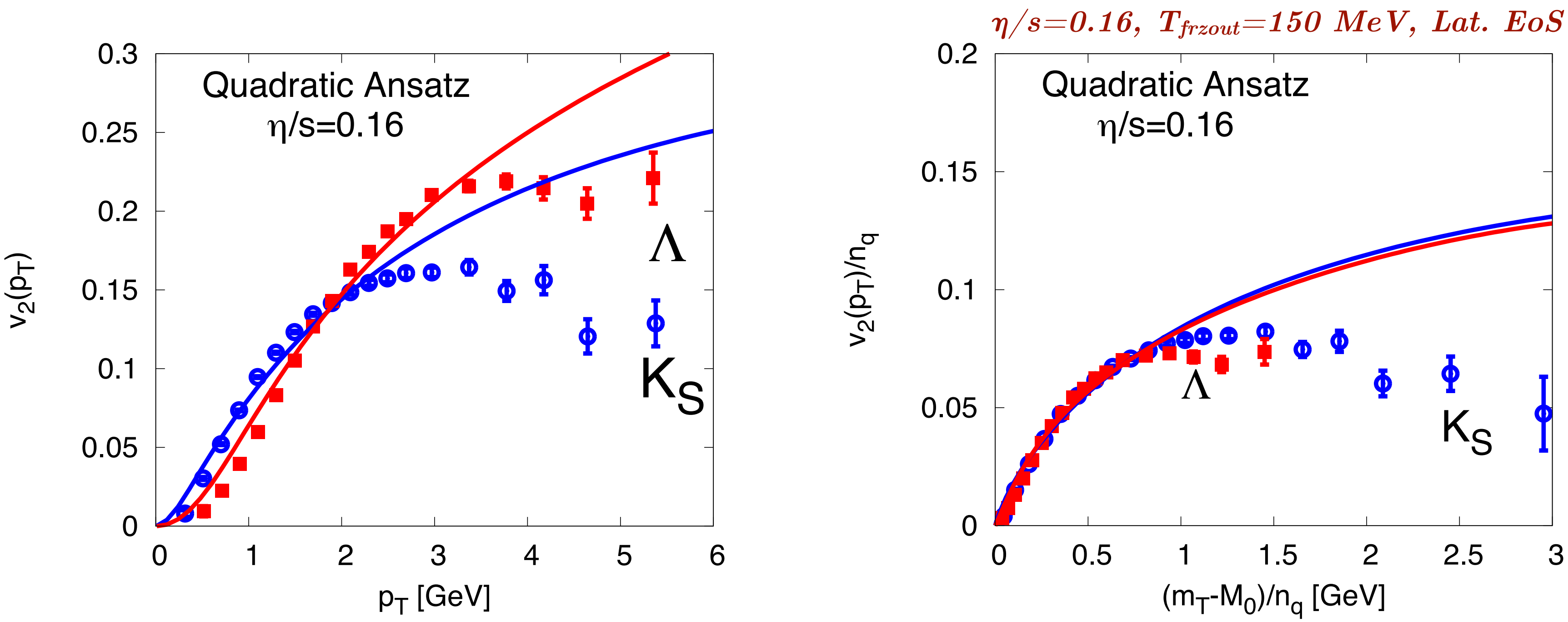


Ideal
Quadratic
η/s=0.16
K_S
$v_2(p_T)$
p_T [GeV]
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6



η/s=0.16, T_{frzout}=150 MeV, Lat. EoS
Ideal
Quadratic
η/s=0.16
Λ
$v_2(p_T)$
p_T [GeV]
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6

Scaling



We find constituent quark scaling without constituent quarks.

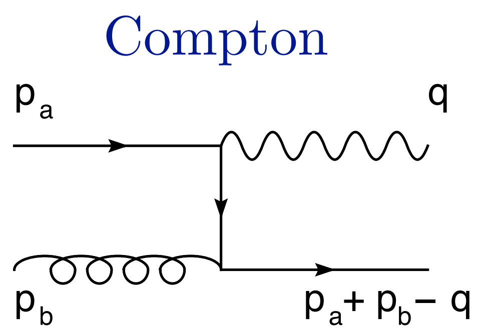
In our case we simply have Relaxation Time Scaling (RTS).

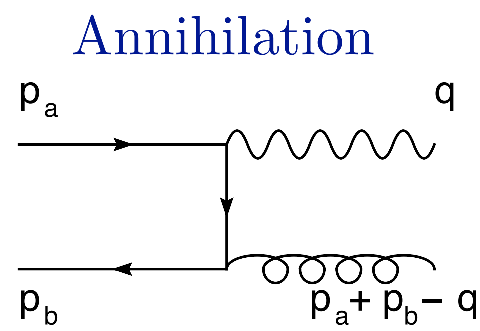
How does viscosity affect photons and dileptons?

Photons: K.D. arXiv:0903.1764

Dileptons: K.D., Shu Lin NPA 809:246-258, 2008.

Photon production at leading log





1. Photons are completely out of equilibrium
   - Their spectra only appears thermal since the quarks creating the photons are thermal
2. At leading log we have

$$p^\mu_{\rm quark} \approx q^\mu_{\rm photon}$$

so distribution of quarks "matches" spectra of photons

Photons from a viscous medium

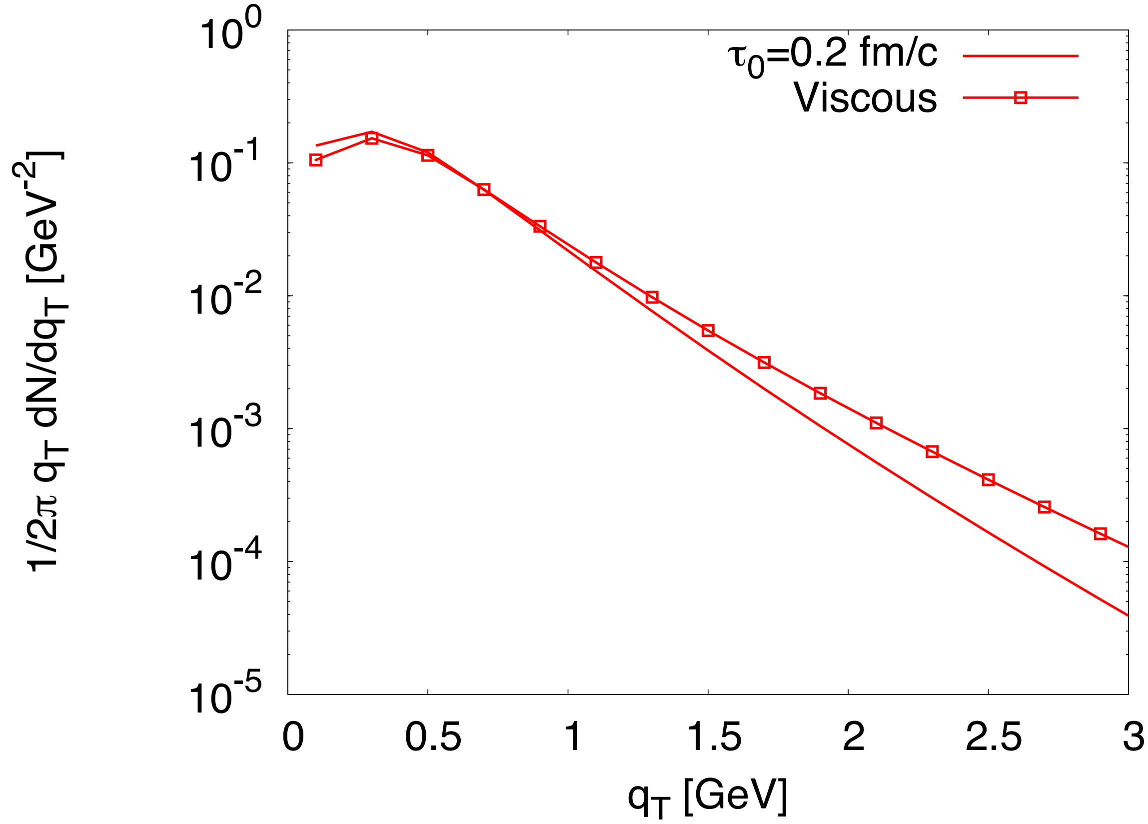
- At leading log

$$E_\gamma \frac{dN_\gamma}{d^3 q_\gamma} = \frac{5}{9} \frac{\alpha_e \alpha_s}{2\pi^2} f_q(q_\gamma) T^2 \ln\left(\frac{3.7 E_\gamma}{g^2 T}\right)$$

- where f_q is the quarks' distribution and at finite viscosity takes the form

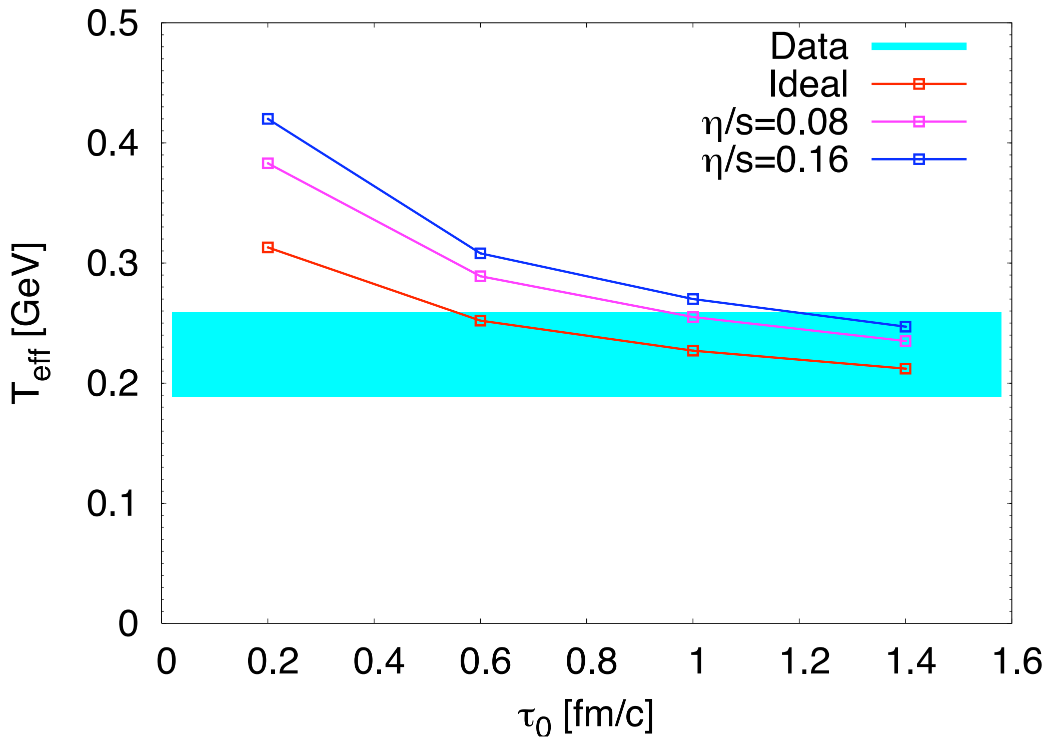
$$f_q(q) = f_0(q) + 1.3 \frac{\eta}{2sT^3} f_0(q) q^i q^j \partial_{\langle i} u_{j\rangle}$$

Photons from a viscous medium



Viscosity makes photon spectra harder.

Photons from a viscous medium



Photons can in principal constrain η/s and τ_0.

Conclusions

1. Showed how the kinetics of quarks and gluons influence v_2
2. Made a precise connection between v_2 and energy loss
3. Observed a Relaxation Time Scaling (RTS) in elliptic flow
4. Showed the imprint of quark kinetics on photon spectra

Backup

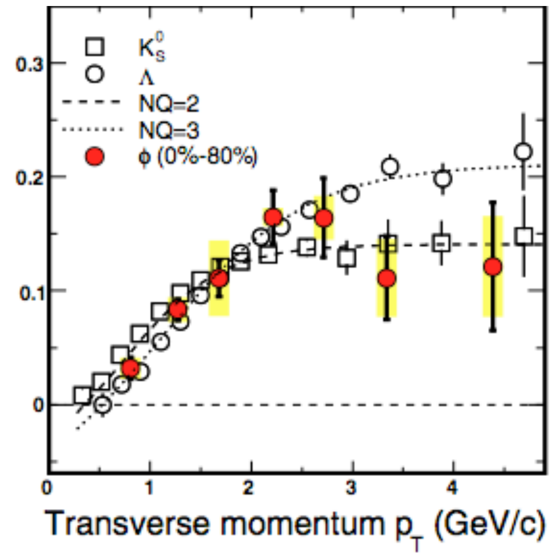
Transition Region

- Long Lived ≈3 fm/c
- At these momentum, interaction very inelastic
- Suggests Additive Quark Model *AQM: Levin, Frankfurt, Lipkin, Sheck*

$$\frac{C_m}{C_b} \approx \frac{\sigma_b}{\sigma_m} \approx \frac{3}{2}$$

- Transition region (high T) also approx. SU(3) symmetric

- $\underbrace{\pi, K}_{\tau_1} \quad \underbrace{p, \Lambda, \Sigma, \Xi}_{\tau_2} \quad \underbrace{\phi}_{\tau_3} \quad \underbrace{\Omega^-}_{\tau_4}$



Quark and Gluons

- Summary of analytic results for radiative energy loss
  - $N_f = 0$

$$\chi(p) \approx 0.704778 \times \frac{p^{3/2}}{\alpha_s T\sqrt{\hat{q}}}$$

  - $N_f = 2$

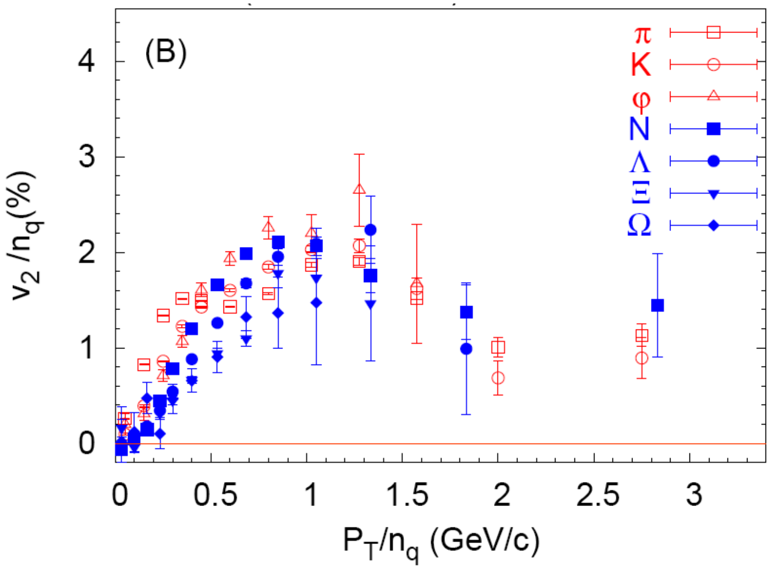
$$\chi_g(p) \approx 0.759158 \times \frac{p^{3/2}}{\alpha_s T\sqrt{\hat{q}}} \qquad \chi_q(p) \approx 1.257913 \times \frac{p^{3/2}}{\alpha_s T\sqrt{\hat{q}}}$$

- Ratios
  - $N_f = 1$ $\quad \frac{\chi_q}{\chi_g} = 1.702$ $\qquad N_f = 2$ $\quad \frac{\chi_q}{\chi_g} = 1.657$
  - $N_f = 3$ $\quad \frac{\chi_q}{\chi_g} = 1.618$ $\qquad N_f = \infty$ $\quad \frac{\chi_q}{\chi_g} = 1.128$

Scaling of v_2 in URQMD

From talk of Marcus Bleicher, HQ06

Y. LU, M. Bleicher nucl-th/0602009



AQM cross sections:

Nπ: 26 mb
Λπ: 23 mb
Ξπ: 20 mb
Ωπ: 16 mb

ππ: 18 mb
Kπ: 14 mb

~3:2

Except magnitude is off by a factor of ≈4.

Simple Scattering

- The "full" 22 collision operator is

$$C[f, \boldsymbol{p}] = \int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'} \Gamma_{\boldsymbol{pk}\to\boldsymbol{p}'\boldsymbol{k}'} \left[f_{\boldsymbol{p}} f_{\boldsymbol{k}}(1+f_{\boldsymbol{p}'})(1+f_{\boldsymbol{k}'}) - f_{\boldsymbol{p}'} f_{\boldsymbol{k}'}(1+f_{\boldsymbol{p}})(1+f_{\boldsymbol{k}})\right]$$

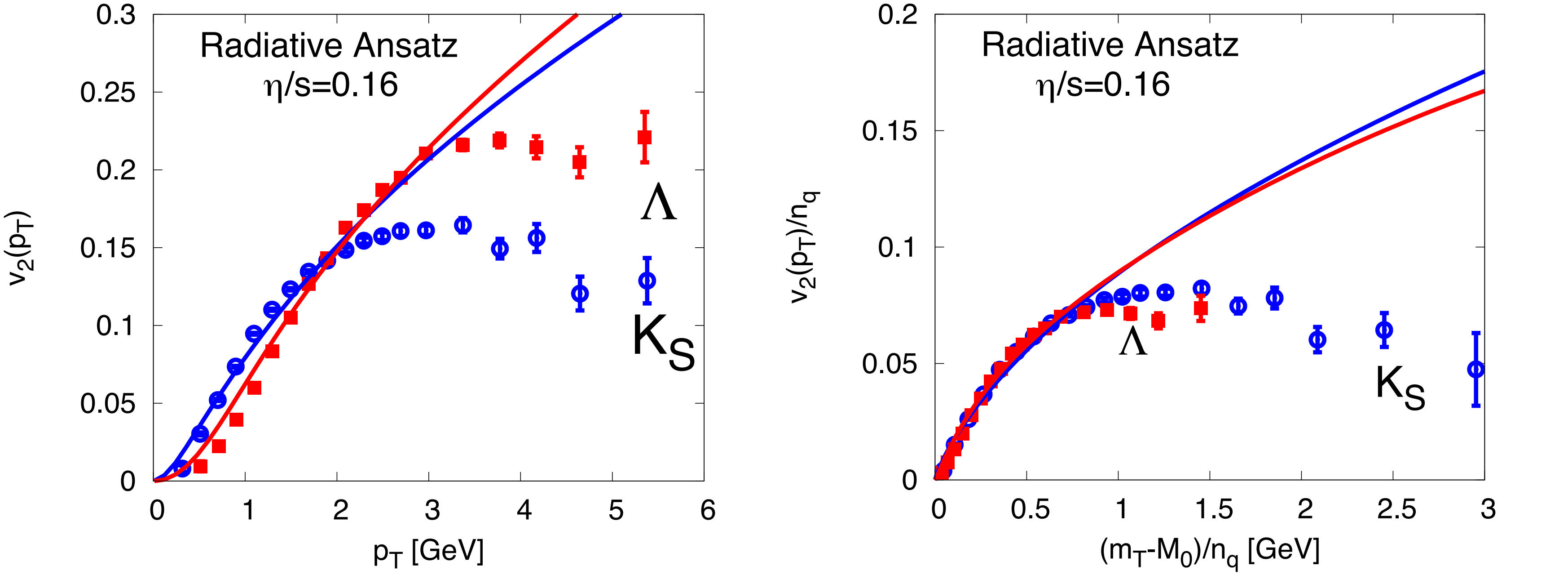
- where the transition rate is

$$\Gamma_{\boldsymbol{pk}\to\boldsymbol{p}'\boldsymbol{k}'} = \frac{1}{2}\,\frac{|\mathcal{M}|^2}{(2E_{\boldsymbol{p}})(2E_{\boldsymbol{k}})(2E_{\boldsymbol{p}'})(2E_{\boldsymbol{k}'})}(2\pi)^4\delta^4(P+K-P'-K')$$

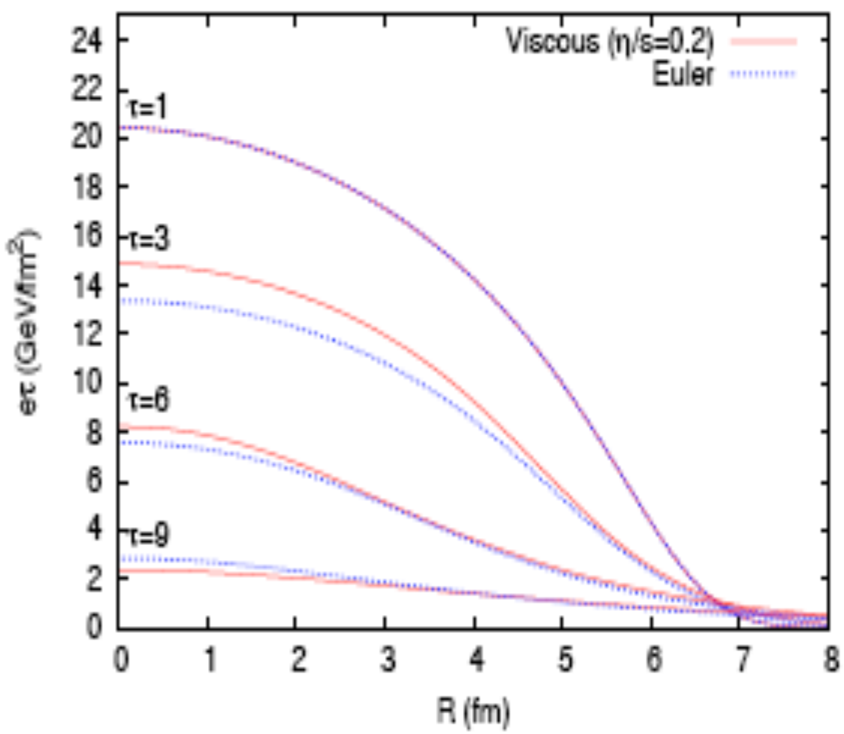
- After linearizing the Boltzmann equation becomes

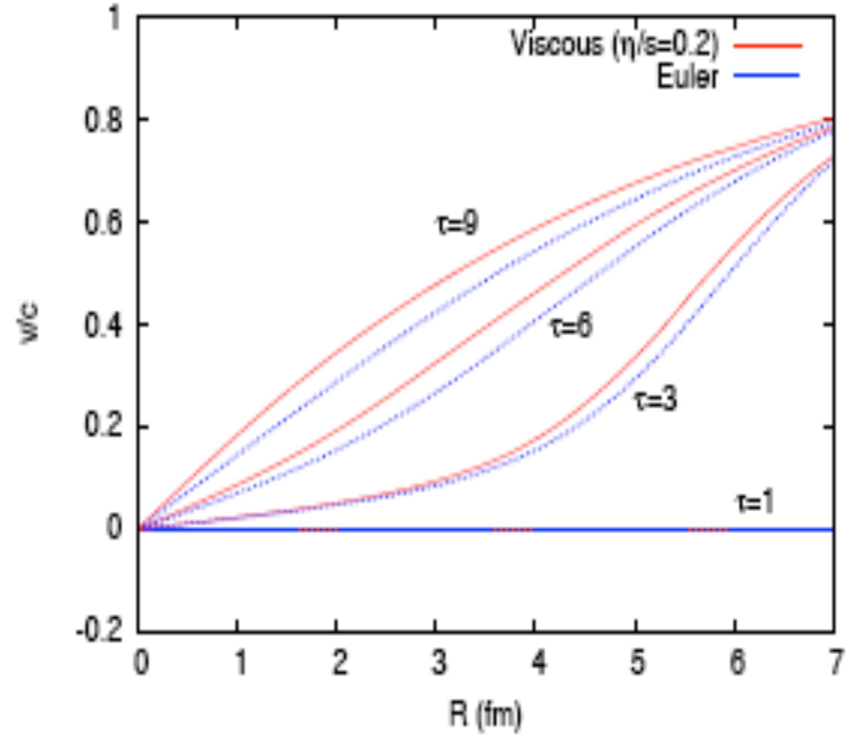
$$\frac{p^i p^j}{TE_p}\langle\partial_i u_j\rangle = \int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'} \Gamma_{\boldsymbol{pk}\to\boldsymbol{p}'\boldsymbol{k}'}\; n_p n_k (1+n_{p'})(1+n_{k'})\; [\chi(\boldsymbol{p}) + \chi(\boldsymbol{k}) - \chi(\boldsymbol{p}') - \chi(\boldsymbol{k}')]$$

Scaling with radiative ansatz



Viscous Correction to EoM





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