QCD thermodynamics on the lattice: (approaching the continuum limit with physical quark masses)

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Outline

Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Coll. (+MILC='hotQCD'):

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from both $\chi_{\bar{\psi}\psi}$ and Polyakov loop:

T_c=192(7)(4) MeV

Wuppertal-Budapest group (WB):

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

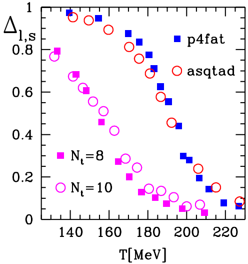
chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

Literature: discrepancies between T dependences

Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted



for chiral quantities ≈35 MeV; for confinement ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

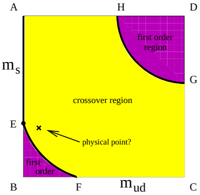
150 MeV transition temperature: isn't it a bit too small?
lattice works in V$\rightarrow\infty$, which gives much smaller T_c

Safe and unsafe lattice approaches

three very important issues for a safe lattice result:

i. discretization effects; ii. continuum limit; iii. physical quark masses

i. discretization effects: example of the m_{ud}–m_s phase diagram



F.Karsch et al., Nucl.Phys.Proc. 129 ('04) 614; G.Endrodi et al. PoS Lat'07 182('07); de Forcrand et al., ibid 178

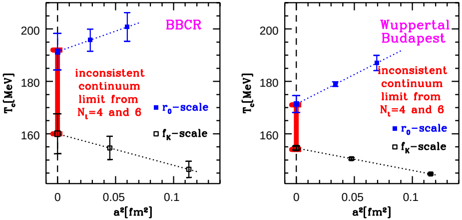
n_f=3 case (standard action, N_t=4): critical $M_{ps}\approx$300 MeV
different discretization effects (stout or p4 action or N_t=6):
critical M_{ps} drops even below 100 MeV (physical M_{ps} is between)
$\Longrightarrow$ huge uncertainties: safe only in the continuum limit

note: state of the art T=0 physics a$\lesssim$0.06 fm; T>0 physics a$\gtrsim$0.1 fm

Safe and unsafe lattice approaches

ii. continuum limit: is there any way to show that it is safe?

a. monitor asymptotic scaling (a^2) b. with different scale settings



N_t=4,6: inconsistent, but N_t=6,8,10 consistent continuum limit independently of the quantity used to set the physical scale (MeV)

iii. physical quark masses are needed

(non-physical masses: only dimensionless ratios in the continuum)

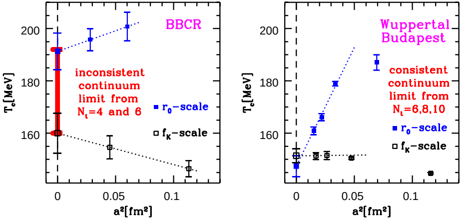
pion splitting (only one π instead of three) $\propto a^2 \Rightarrow$ mimics large M_π

some actions are quite good at reducing this non-physical effect

Safe and unsafe lattice approaches

ii. continuum limit: don't depend on the quantity which fixes the scale

monitor asymptotic scaling (a^2) with different scale settings



N_t=4,6: inconsistent, but N_t=6,8,10 consistent continuum limit independently of the quantity used to set the physical scale (MeV)

iii. physical quark masses are needed to give results in MeV

(non-physical masses: only dimensionless ratios in the continuum)

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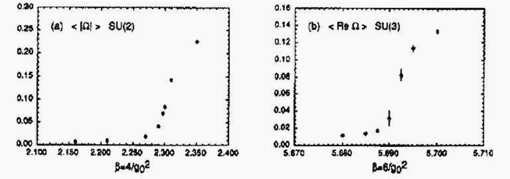
Finite-size scaling theory

problem with phase transitions in Monte-Carlo studies

Monte-Carlo applications for pure gauge theories ($V = 24^3 \cdot 4$)

existence of a transition between confining and deconfining phases:

Polyakov loop exhibits rapid variation in a narrow range of β



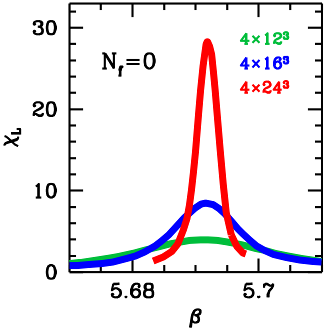
• theoretical prediction: SU(2) second order, SU(3) first order

$\Longrightarrow$ Polyakov loop behavior: SU(2) singular power, SU(3) jump

data do not show such characteristics!

Finite size scaling in the quenched theory

look at the susceptibility of the Polyakov-line
first order transition (Binder) $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V



finite size scaling shows: the transition is of first order

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

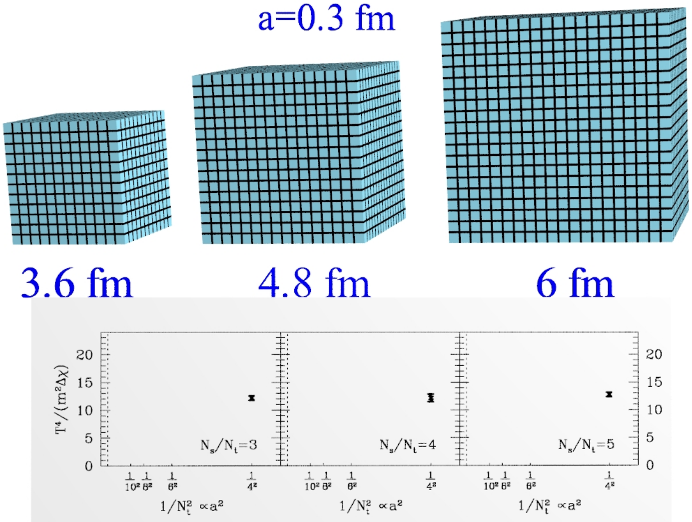
finite size scaling study of the chiral condensate (susceptibility)

$$\chi=(T/V)\partial^2\log Z/\partial m^2$$

phase transition: finite V analyticity $V\rightarrow\infty$ increasingly singular
(e.g. first order phase transition: height $\propto$ V, width $\propto$ 1/V)
for an analytic cross-over χ does not grow with V

two steps (three volumes, four lattice spacings):
a. fix V and determine χ in the continuum limit: a=0.3,0.2,0.15,0.1fm
b. using the continuum extrapolated χ_{max}: finite size scaling

Approaching the continuum limuit



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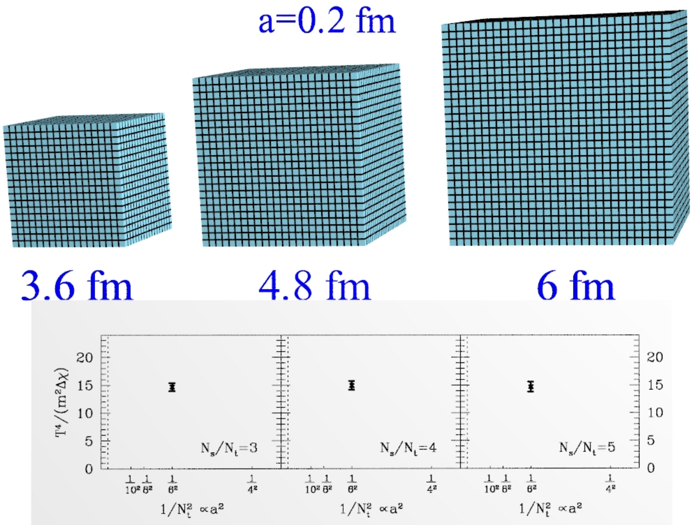
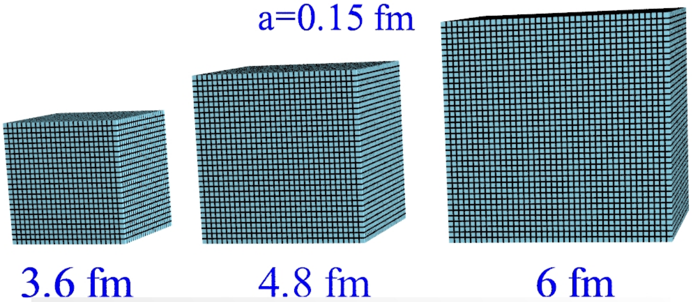
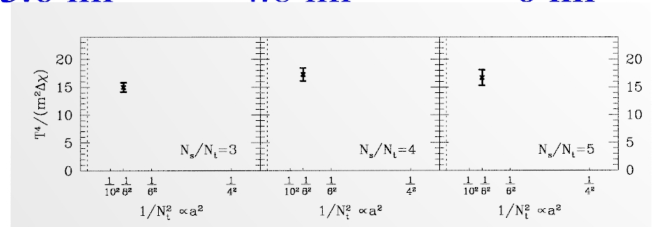


Approaching the continuum limuit





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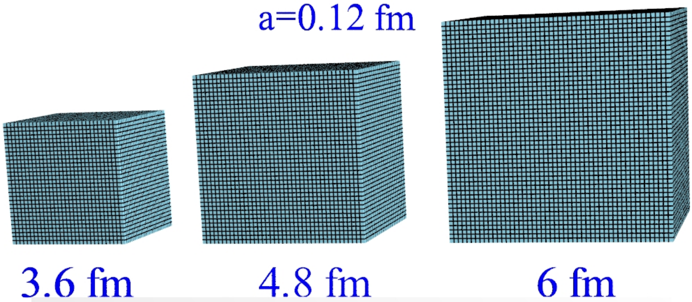

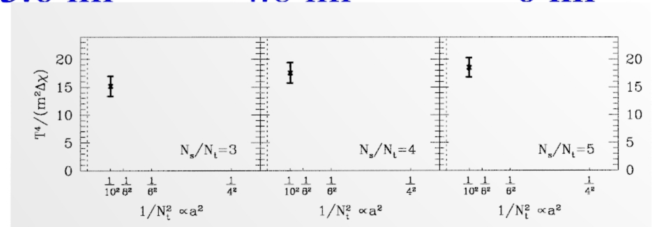

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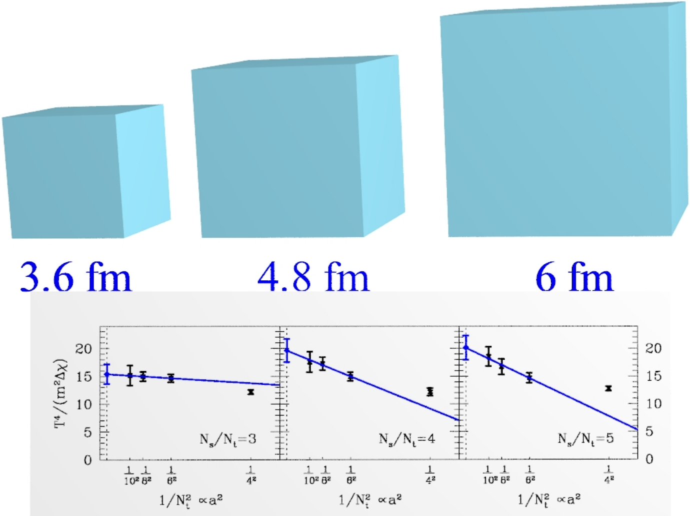
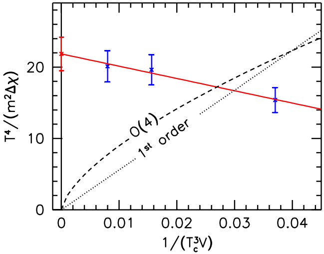


The nature of the QCD transition: result

- finite size scaling analysis with continuum extrapolated $T^4/m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

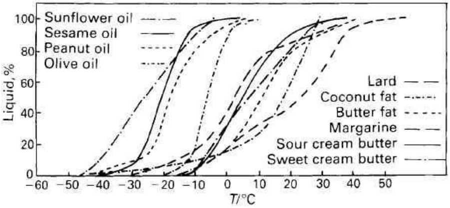
continuum result with physical quark masses in staggered QCD:

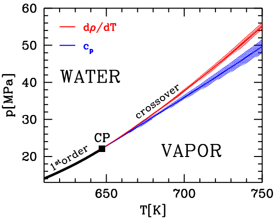
the QCD transition is a cross-over

Analytic QCD transition (cross-over): consequences

highly non-trivial a→0 result Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

analytic transition (cross-over) ⇒ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition





above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values ⇒ physical difference

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

various types of discretization errors $\Rightarrow$ we improve on them (costs)

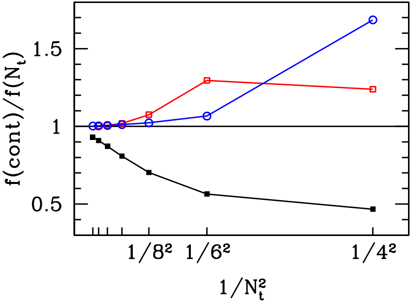
we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$<T_c$

example: 'expansion' around a Stefan-Boltzman gas (van der Waals)
for water: it is a fairly good description for T$\gtrsim 300^o$
claculate the boiling point: more accuracy needed for the liquid phase

Examples for improvements, consequences

analytic: how to reach the continuum free energy density at T=∞?



Bielefeld: p4 action is essentially designed for this quantity T$\gg T_c$

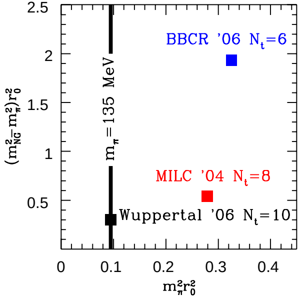
MILC: asqtad is designed mostly for T=0 (but good at T$\gg T_c$, too)

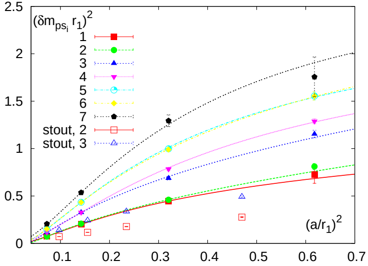
BW: stout-smeared link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

Chiral symmetry breaking and pions

transition temperature for remnant of the chiral transition:
balance between the chirally broken and chirally symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact ⇒ disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact

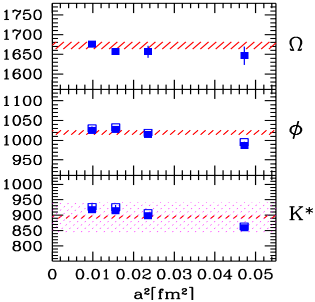


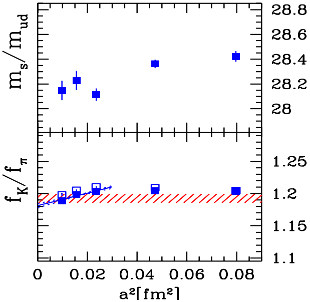


Setting the scale in lattice QCD

in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that M_Ω=1672 MeV we obtain 'a' and T=1/N_ta

Y.Aoki et al. [Wuppertal-Budapest Collaboration] JHEP 0906:088,2009



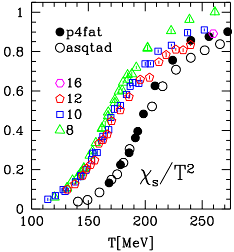


independently which quantity is taken (we used physical masses)

$\Rightarrow$ one obtains the same 'a' and T, result is safe

T>0 results: strange susceptibility

Compare the Wuppertal-Budapest results [Y.Aoki et al. JHEP 0906:088,2009] with 'hotQCD'

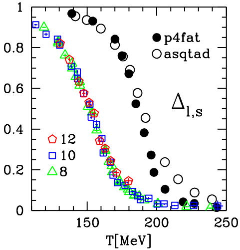


'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12,(16)
'hotQCD': results with two different actions are almost the same
WB: for large T one extrapolates according to the known a^2 behaviour
WB: no change in the lattice results compared to our 2006 paper
note, that the experimental value of f_K decreased by 3% since 2006

about 20 MeV difference between the results

T>0 results: chiral condensate

Compare the Wuppertal-Budapest results [Y.Aoki et al. JHEP 0906:088,2009] with 'hotQCD'



'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12

'hotQCD': results with two different actions are almost the same

WB: no lattice spacing dependence observed for N_t=8,10,12

WB: no change in the lattice results compared to our 2006 paper

about 35 MeV difference between the results

transition temperatures for various observables

	$\chi_{\bar\psi\psi}/T^4$	$\chi_{\bar\psi\psi}/T^2$	$\chi_{\bar\psi\psi}$	$\Delta_{l,s}$	L	χ_s
WB'09	146(2)(3)	152(3)(3)	157(3)(3)	155(2)(3)	170(4)(3)	169(3)(3)
WB'06	151(3)(3)	-	-	-	176(3)(4)	175(2)(4)
BBCR	-	192(4)(7)	-	-	192(4)(7)	-

renormalized chiral susceptibility, renormalized chiral condensate
Polyakov loop and strange quark number susceptibility

no change compare to our 2006 data (errors are reduced)
note, that the experimental value of f_K decreased by 3% since 2006
Particle Data Group now gives f_K=155.5(2)(8)(2) MeV (error 0.5%)

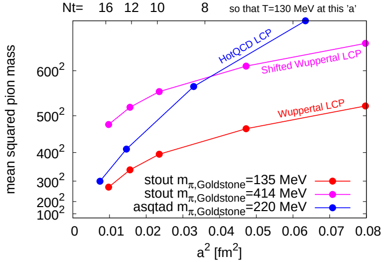
r_0 is not directly measurable:

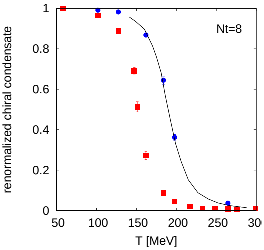
ETM:0.444(4) fm, QCDSF:0.467(6) fm,
HPQCD&UKQCD:0.469(7) fm, PACS-CS:0.492(6)(+7) fm

Illustration: lattice artefacts due to pion splitting

we have seen: our action (WB) has less unphysical pion splitting than the asqtad (MILC) and far less than the p4 (Bielefeld) action

in the continuum limit: no problem; at $a \neq 0$ it mimics larger M_π
"reproduce" the result of hotQCD with larger M_π (asqtad is better)

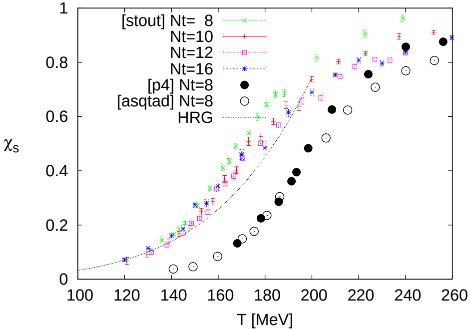




$M_\pi \approx$220 MeV (hotQCD) "corresponds" to M_π $\approx$410 MeV (WB)
asqtad (MILC) needs finer p4 (Bielefeld) needs much finer lattices in order to handle physical quark masses (suboptimal improvement)

Illustration: lattice artefacts due to pion splitting

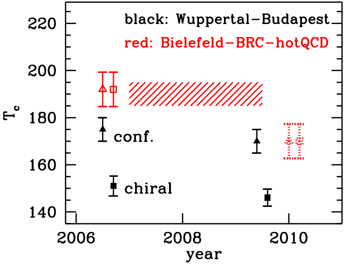
go even closer to the continuum limit: N_t=16 (preliminary)
compare WB and hotQCD with the hadron resonance gas model



Wuppertal-Budapest: nice agreement upto 160 MeV (almost T_c)
hot-QCD: several sigma discrepancy
reason: hotQCD action has large π-splitting $\Rightarrow$ large hadron masses

Story of T_c (last 4 years) hotQCD versus Wuppertal

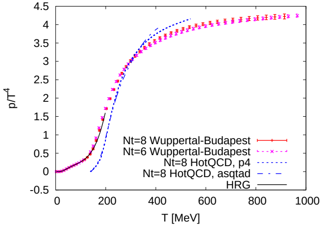
results in physical units (MeV) are meaningful only in the
continuum limit (extrapolation) with physical quark masses



2006 claims: continuum limit results with physical quark masses
many standard deviation discrepancy: statistically "impossible"
our interpretation since 2006: BBRC result on T_c is "non-continuum"
hotQCD preliminary: closer to continuum ⇒ more than 20 MeV drop
trust a result: scaling behavior & independence of scale setting

Equation of state: preliminary results

our improved action: Symanzik gauge & stout smeared fermions
physical quark masses and our line of constant physics can be used
renormalization: G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, PoS Lat07 228 (2007)
leading order (SB limit) normalization is also applied
full temperature dependence upto 1 GeV is given for N_t=8
(monitor the continuum limit: soon 3 sensitive points with N_t=10,12)



hotQCD result has a larger charcteristic temperature (similar to T_c)
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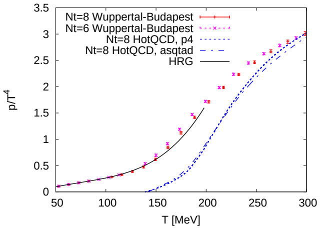

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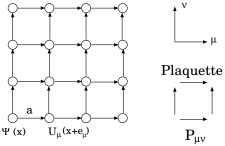
Summary

- QCD transition: analytic cross-over (continuum with M_π=135 MeV)
- new (2009) results for the transition temperature
- three improvements since 2006
  a. at T=0 all simulations are done with physical quark masses
  b. to verify that the results are independent of the scale setting
  we use 5 experimentally well-known quantites: f_K,f_π,m_{K^*},m_Ω,m_Φ
  c. even smaller lattice spacings: N_t=12 (in one case N_t=16)
- all findings are in complete agreement with our 2006 results
- Particle Data Group reduced the experimental value of f_K: 3%
- discrepancy between Wuppertal-Budapest & 'hotQCD' results
  a. for the remnant of the deconfinement transition: about 20 MeV
  b. for the remnant of the chiral transition: about 35 MeV
  possible explanation: hotQCD has large discretization effects
- preliminary equation of state results N_t=8(10,12)

Lattice Lagrangian: gauge fields



$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}(D_\mu\gamma^\mu + m)\psi$$

anti-commuting $\psi(x)$ quark fields live on the sites
gluon fields, $A^a_\mu(x)$ are used as links and plaquettes

$$U(x,y) = \exp\left(ig_s\int_x^y dx'^\mu\, A^a_\mu(x')\lambda_a/2\right)$$

$$P_{\mu\nu}(n) = U_\mu(n)U_\nu(n+e_\mu)U^\dagger_\mu(n+e_\nu)U^\dagger_\nu(n)$$

$S = S_g + S_f$ consists of the pure gluonic and the fermionic parts

$$S_g = 6/g_s^2 \cdot \sum_{n,\mu,\nu}[1 - \mathrm{Re}(P_{\mu\nu}(n))]$$

Lattice Lagrangian: fermionic fields

quark differencing scheme:

$$\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x) \to \bar{\psi}_n\gamma^\mu(\psi_{n+e_\mu} - \psi_{n-e_\mu})$$

$$\bar{\psi}(x)\gamma^\mu D_\mu\psi(x) \to \bar{\psi}_n\gamma^\mu U_\mu(n)\psi_{n+e_\mu} + \ldots$$

fermionic part as a bilinear expression: $S_f = \bar{\psi}_n M_{nm}\psi_m$
we need 2 light quarks (u,d) and the strange quark: $n_f = 2+1$

(complication: fermion doubling $\Rightarrow$ staggered/Wilson)

Euclidean partition function gives Boltzmann weights

$$Z = \int \prod_{n,\mu}[dU_\mu(x)][d\bar{\psi}_n][d\psi_n]e^{-S_g-S_f} = \int \prod_{n,\mu}[dU_\mu(n)]e^{-S_g}\det(M[U])$$

Importance sampling

$$Z = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability $\propto$ its weight

importance sampling, Metropolis algorithm:
(all other algorithms are based on importance sampling)

$$P(U \to U') = \min\left[1, \exp(-\Delta S_g) \det(M[U'])/\det(M[U])\right]$$

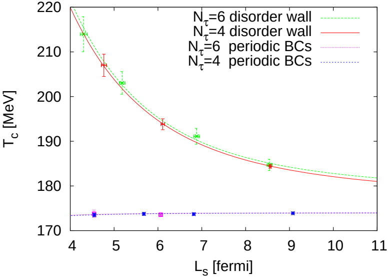
gauge part: trace of 3$\times$3 matrices (easy, without M: quenched)
fermionic part: determinant of $10^6 \times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation (Mx=a), but still hard

T_c strongly depends on the geometry

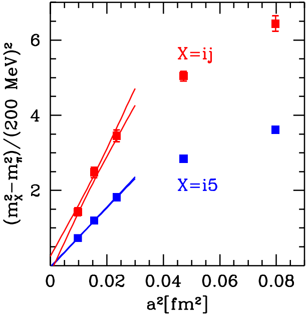
nanotube-water doesn't freeze, even at hundred degrees below 0^oC

exploratory study: A. Bazavov and B. Berg, Phys.Rev. D76 (2007) 014502
use 'confined' spatial boundary conditions: more like experiments



large deviation (upto 30 MeV) from the infinite volume limit
if $V \to \infty$ is 150 MeV a 100 fm^3 system might have 170 MeV

Scaling for the pion splitting



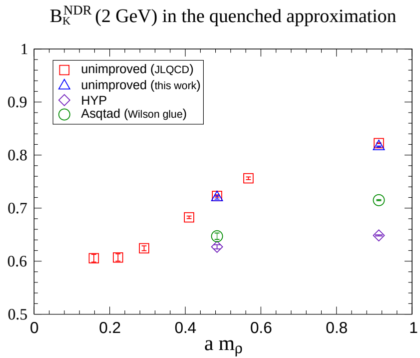
scaling regime is reached if a^2 scaling is observed
asymptotic scaling starts only for N_t>8 (a$\lesssim$0.15 fm): two messages

a. N_t=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

b. stout-smeared improvement is designed to reduce this artefact
most other actions need even smaller 'a' to reach scaling

Scaling of B_K in quenched simulations

HPQCD and UKQCD Collaborations, Phys. Rev. D73 (2006) 114502



unimproved action has large scaling violations

asqtad action is somewhat better

HYP smeared improvement $\Rightarrow$ almost perfect scaling