Rheology of the Quark Gluon Plasma
Measuring the shear viscosity with Pt Correlations

Claude A. Pruneau, for the STAR Collaboration

Acknowledgements
• This talk based on a STAR Analysis carried out by Monika Sharma
• Thanks to S. Gavin for many discussions

The perfect fluid!?!
Perfect Fluid?

- Superfluid Helium
- Ultra Cold Gasses
  - few nK.
- Quark Gluon Plasma
  - T~200 MeV~10^{12} K
  - Temperature of early universe at ~1us

Conjectured low bound of shear viscosity/entropy:

Supersymmetric Yang Mill Theory (Ads/CFT duality)
Kovtun, Son, & Starinets, PRL94(2005)

\[ \frac{\eta}{s} \geq \frac{1}{4\pi} \]

M. Luzum & P. Romatschke, 0804.4015
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Can we measure the viscosity by other means at RHIC?

viscosity/entropy:

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$$\frac{\eta}{\hat{h}s} \geq \frac{1}{4\pi}$$

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C. A. Pruneau, 26th Workshop on Nuclear Dynamics, Ocho Rios, Jamaica, Jan 2-9, 2010
Perfect Fluid?

- Superfluid Helium
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- Quark Gluon Plasma
  - $T \sim 200 \text{ MeV} \sim 10^{12} \text{ K}$
  - Temperature of early universe at $\sim 1\text{us}$

Can we measure the viscosity by other means at RHIC?

Yes: Use ptpt 2-particle correlations

Kovtun, Son, & Starinets, PRL94(2005)

$$\frac{\eta}{\hbar s} \geq \frac{1}{4\pi}$$
Rheometry: Measurement of Shear Viscosity

- Stress vs Deformation $\tau = \eta \frac{du}{dy}$
- Velocity Gradient (m/s): $du/dy$
- Shear Stress (Pa): $\tau$
- Dynamic viscosity (Pa s): $\eta$
- Kinematic Viscosity (m$^2$/s): $\nu = \frac{\eta}{\rho}$
- Density (kg/m$^3$): $\rho$
- Relation to the Mean Free Path (m), $\lambda$: $\nu = \frac{1}{2} \bar{u} \lambda$
- In terms of the stress energy tensor: $T_{yx} = -\eta \frac{dv_x}{dy}$
Reynolds number, $Re$

**Definition**

$$Re = \frac{\rho VL}{\mu} = \frac{\rho VL}{\nu}$$

where:
- $V$ is the mean fluid velocity (SI units: m/s)
- $L$ is a characteristic length, (traveled length of fluid) (m)
- $\mu$ is the **dynamic viscosity** of the fluid (Pa·s or N·s/m² or kg/m·s)
- $\nu$ is the **kinematic viscosity** ($\nu = \mu / \rho$) (m²/s)
- $\rho$ is the **density** of the fluid (kg/m³)

**In the context of HI collisions:**

An effective Reynolds number

$$Re = \frac{3}{4} \frac{\tau_0 Ts}{\eta}$$

where:
- $T$ is the system temperature
- $s$ the entropy of the system
- $\eta$ shear viscosity
- $\tau_0$ formation time

Small $Re$ implies laminar flow
Large value implies turbulent flow


$Re \sim 5$
Measurement of viscosity based on $p_t p_t$ Correlations

Gavin and Abdel-Aziz, nucl-th/0606061 (2006)

- Viscous friction arises as fluid elements flow past each other thereby reducing the relative velocity: damping of radial flow.
- $T_{zr}$ changes the radial momentum current of the fluid $T_{0r} = \gamma^2 (\epsilon + p) v_r$
- Diffusion equation for the momentum current $\left( \frac{\partial}{\partial t} - \nu \nabla^2 \right) g_t = 0$
- Viscosity reduces fluctuations, distributes excess momentum density over the collision volume: broadens the rapidity profile of fluctuations
- Width of the correlation grows with diffusion time (system lifetime) relative to its original/initial width $r_g = \langle g_t (x_1) g_t (x_2) \rangle - \langle g_t (x_1) \rangle \langle g_t (x_2) \rangle$

$$\sigma^2 = \sigma_o^2 + 2 \Delta V(\tau_f)$$
$$\Delta V(\tau) \equiv \left( \langle \eta - \langle \eta \rangle \rangle \right)^2 = \frac{2\nu}{\tau_o} \left( 1 - \frac{\tau_o}{\tau} \right)$$

$\tau_o$ Formation Time
$\tau_f$ Freeze-out Time

STAR Range based on 2-part correlations
Reometry of the QGP with $p_t$-$p_t$ Correlations

- **Integral** Correlation Function (S. Gavin, M. Abdel-Aziz, nucl-th/060606)

\[
C(\Delta \eta) = \langle p_{\perp,1} p_{\perp,2} \rangle - \langle p_{\perp} \rangle^2
\]

\[
\langle p_{i1} p_{i2} \rangle \equiv \frac{1}{\langle N \rangle^2} \left( \sum_{\text{pairs } i \neq j} p_{ij} p_{ij} \right)
\]

\[
\langle p_t \rangle \equiv \frac{1}{\langle N \rangle} \langle \sum p_{ti} \rangle
\]

\[
\sigma_c^2 - \sigma_p^2 = 4\nu \left( \tau^{-1}_{f,p} - \tau^{-1}_{f,c} \right)
\]

\[
\nu = \frac{\eta}{T_c s}
\]

\[
\tau_{f,p}, \tau_{f,c} \quad \text{Freeze out Times}
\]
Estimate by Gavin, PRL 97, 162302 (2006) 
(Integral) Transverse Momentum Correlations

0.08 < ∊/s < 0.3

based on

p_T correlations

η/∊ ≈ 0.08

Number density correlations
STAR, PRC 73, 064907, 2006 (AuAu 130 GeV)

η/∊ ≈ 0.3

But, ...

Proper estimation of ∊/s requires an observable with contributions from number density & pT correlations

C = <p_{1t}p_{2t}> - <p_t>^2

<\sum_{pairs \neq j} p_i p_j> ≈ \frac{1}{<N>^2} <\sum_p n_p>
This work: Differential $p_t$ $p_t$ Correlations

\[
C(\Delta \eta, \Delta \varphi) = \frac{\left\langle \sum_{i=1}^{n_1} \sum_{j \neq i=1}^{n_2} p_i p_j \right\rangle}{\left\langle n_1 \right\rangle \left\langle n_2 \right\rangle} - \left\langle p_{t,1} \right\rangle \left\langle p_{t,2} \right\rangle
\]

\[
\Delta \eta = \eta_1 - \eta_2
\]

\[
\Delta \varphi = \varphi_1 - \varphi_2
\]

Inclusive average $p_t$:
\[
\left\langle p_{t,i} \right\rangle(\eta_i, \varphi_i) = \frac{\left\langle \sum_{k=1}^{n_i} p_{t,k} \right\rangle}{\left\langle n_i \right\rangle}
\]

Transverse momentum of particles in bin $i$:
\[
P_{t,i}
\]

Number of particles in bin $i$
\[
n_i \equiv n_i(\eta_i, \varphi_i), \quad i = 1, 2
\]

Broadening
\[
\sigma_c^2 \approx \sigma_{Diffusion}^2 + \sigma_0^2
\]
Theoretical/Physics Caveats

- The system temperature, viscosity, and Reynolds number vary through the lifetime of the collision system.
  - Our measurement will yield time averaged quantities.
- Freeze out times must be inferred from other data and model.
- Other effects may contribute to the longitudinal shape of the correlation function:
  - Decays, thermal broadening, jets, radial flow, CGC, etc.
  - Jet expected to have minor impact in the momentum range considered in this analysis.
    - Diffusion expected to dominate the broadening (see next few slides).
- A detailed interpretation of the measurements requires collision models that provide comprehensive understanding of HI data.
Dynamical Effects (1): Radial Flow

- Based on PYTHIA p+p collisions at $\sqrt{s} = 200$ GeV
  
  $0.2 < p_T < 2.0$ GeV/c

  $|\eta| < 1$

- PYTHIA Simulation including radial flow (transverse boost) with v/c=0.3

- Near-side kinematic focusing, formation of ridge-like structure,
- Different shapes
- Narrowing of near side


Dynamical Effects (2): Resonance Decays

• An increase in system temperature and/or radial flow implies kinematical focusing of the decay products: **narrowing of the correlation function**.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Impact</th>
</tr>
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<tbody>
<tr>
<td>Parent at rest</td>
<td>Low temperature or radial velocity</td>
</tr>
<tr>
<td>Low temperature or radial velocity</td>
<td>Medium temperature or radial velocity</td>
</tr>
<tr>
<td>Medium temperature or radial velocity</td>
<td>High temperature or radial velocity</td>
</tr>
</tbody>
</table>

- Note however that re-scattering after decay implies causes **thermal diffusion**, and **correlation broadening**. --- Needs modeling to properly assess its impact...
Dynamical Effects (3): Core vs Corona

- Simulation of Au Au @ $\sqrt{s}=200$ GeV based on the **EPOS-1 Not HYDRO** (courtesy of K. Werner)

<table>
<thead>
<tr>
<th>90-80%</th>
<th>60-50%</th>
<th>5-0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Diagram of 3D projections within $</td>
<td>\Delta \phi</td>
<td>&lt; 1$, offset subtracted, normalized](#)</td>
</tr>
</tbody>
</table>

No broadening
No ridge

Momentum conservation and elliptic flow
• Data from STAR TPC, $2\pi$ coverage
• Dataset: RHIC Run IV: AuAu 200 GeV
• Events analyzed: 10 Million
• Minimum bias trigger
• Track Kinematic Cuts
  • Goal: Measure medium properties i.e. Bulk Correlation
  • $|\eta| < 1.0$
  • $0.2 < p_T < 2.0$ GeV/c
• Analysis done vs. collision centrality measured based on multiplicity in $|\eta| < 1.0$
• Centrality bins: 0-5%, 5-10%, 10-20%…….

Note: Using STAR usual track and event quality cuts
Results: C vs Collision Centrality

- Prominent near side peak in peripheral collisions
- Ridge-like structure on the away-side (momentum conservation) in peripheral collisions
- Monotonic reduction of the correlation amplitude with increasing $N_{\text{part}}$.
- Evidence of elliptic flow component in mid-central collisions.
- Emergence of a near side ridge with increasing $N_{\text{part}}$.
- Monotonic elongation in $\Delta \eta$ of the near side peak with increasing $N_{\text{part}}$. 
C --- Near Side Projection

Fit Function: \[ C(b, a_w, \sigma_w, a_n, \sigma_n) = b + a_w \exp\left(-\Delta \eta^2 / 2\sigma_w^2\right) + a_n \exp\left(-\Delta \eta^2 / 2\sigma_n^2\right) \]
Correlation Width vs. Collision Centrality

- Width approximately constant (decreasing actually) in most peripheral bins
  - Incomplete thermalization?
  - Radial flow effect?
  - Corona dominated
  - Event centrality selection technique?
- Linear increase with $N_{\text{part}} > \sim 100$
- Saturation for most central collisions?
- Freeze-out expected to increase with $N_{\text{part}}$. Observed width does not change much, what does that mean? Require theoretical model improvements!
Estimation of the shear viscosity

From S. Gavin:

\[
\sigma_c^2 - \sigma_p^2 = 4 \nu \left( \tau_p^{-1} - \tau_c^{-1} \right)
\]

\[
\sigma_{p+p} \approx \sigma_{70-80\%} = 0.54 \pm 0.02^* \quad \tau_{p+p} \approx 1 \, \text{fm/c}
\]

\[
\sigma_{w,0-5\%} = 1.0 \pm 0.2 \quad \tau_c = 20 \, \text{fm/c}
\]

\[
\frac{\eta}{s} = 0.17 \pm 0.08
\]

Non Gaussian shape observed in most central collisions suggests broadening could have contributions from other phenomena as well diffusion (viscosity).

The above value is thus an upper limit of the time averaged viscosity.

* statistical errors only at this stage, systematic errors under study.
Estimation of the Reynolds Number

- Neglect central collision freeze out time contribution, and approximate the peripheral freeze out time as the formation time.

\[ \sigma_c^2 - \sigma_p^2 = 4 \frac{v}{\tau_p} \left( 1 - \frac{\tau_p}{\tau_c} \right) \approx 3 \text{Re}^{-1} \quad \text{for} \quad \tau_o \approx \tau_{f,p} \ll \tau_{f,c} \]

\sigma_p = 0.54 \pm 0.02
\sigma_c = 1.0 \pm 0.2

Estimate

\[ \text{Re} \approx \left( \frac{\sigma_c^2 - \sigma_p^2}{3} \right)^{-1} = 5 \pm 2 \]

**What does it mean?**

* statistical errors only at this stage, systematic errors under study.
Summary

- First measurement of the differential observable C advocated by Gavin et al. for measurements of shear viscosity in Au + Au collisions at $\sqrt{s_{NN}} = 200 GeV$
- C behave as expected with collision centrality
  - “Strong” flow component in mid-central collisions
  - Emergence of a ridge on the near side for large $N_{\text{part}}$
  - Longitudinal broadening of the near side correlation peak
- Estimate of the Reynolds number $Re = 5 \pm 2$  
- Estimate of the shear viscosity (Upper limit?) $\frac{\eta}{s} \leq 0.17 \pm 0.08$
Additional Material
Analysis: Technical Details

- In order to mitigate efficiency dependencies on the z-vertex position, field polarity, and detector occupancy, ...
- Reported correlation functions are a weighted average of values measured for
  - specific z-vertex bins of 2.5 cm in the range $|z| < 25$ cm.
  - forward (F) and reverse (R) full field data
- Offset correction: Average correlation offsets differences vs z-bin and F/R field are set to zero: dispersion provides estimate of systematic error assoc. w/ pt dependence of efficiency.
- Statistical errors based on the variance of the measurements in different z bins and field polarity, after offset correction.
- Include track merging correction

“S”, shows up in azimuth as a point where tracks have merged.
Experimental Caveat: Observable Robustness(?)

Study with PYTHIA, p+p collisions at $\sqrt{s} = 200$ GeV

Twelve fold angular efficiency dependence, and linear dependence on pT

$$\epsilon(\varphi, p_\perp) = \epsilon_0 (1 - ap_\perp) \left[ 1 + \sum_{n=1}^{12} \epsilon_i \cos(n\varphi) \right]$$

Efficiency = 100%

Efficiency = 80%

Difference

$\epsilon_0 = 0.8$, $a = 0.05$

Statistical error = 0.001, difference = 0.0005 => Robust Observable if efficiency has small dependence on pt.

In practice, a measurement 'near' detection threshold in $p_t$, implies the observable is not perfectly robust (Simulation in progress)