Relativistic 3D Viscous Hydrodynamics

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Outline

- Hydrodynamic Theory
- Computational Solution and Its Verification
- Is 3D Necessary at Midrapidity?

The Hydrodynamic Problem

- Given:
  - An Initial Condition
  - Equation of State
- Produce:
  - Evolution
  - Observables

Problem Description

- Conservation of Stress-Energy:

$$\partial^\mu T_{\mu\nu} = 0$$
$$T_{\mu\nu} = (\epsilon + P) u_\mu u_\nu - P g_{\mu\nu} + \Pi_{\mu\nu}$$

- Effective Pressure Altered by Velocity Gradients:

$$\Pi_{ij}^{(NS)} = -\eta[\partial_i n_j + \partial_j n_i - \delta_{ij}\partial_k n_k] - \zeta \partial_k n_k$$

- Rapid Longitudinal Expansion yields Reduced Effective Longitudinal Pressure.

Israel-Stewart Theory

- Israel-Stewart Allows Dynamical Effective Pressure.
  - Deviations from NS decay exponentially

$$\partial_t(a_i/\alpha) = \frac{-(a_i - a_i^{(NS)})}{\tau_\eta \alpha} + \Omega_{ij}(a_j/\alpha)$$

$$\alpha^2 = \eta T s$$

Problem Description

- Projected Pressure Corrections:

$$
\begin{aligned}
a_1 &= \Pi_{xx} - \Pi_{yy} \\
\sqrt{3}a_2 &= \Pi_{xx} + \Pi_{yy} - 2\Pi_{zz} \\
a_3 &= \Pi_{xy}, \quad a_4 = \Pi_{xz}, \quad a_5 = \Pi_{yz} \\
b &= (1/3)(\Pi_{xx} + \Pi_{yy} + \Pi_{zz})
\end{aligned}
$$

Features

- 3d Reflection Symmetry (up to 8x speed up).
- Multiple Time Integration Methods.
- Optional Bjorken Expansion.
- 40^3 grid integrates in 2 hours.

Basic Testing

- Total Entropy Conservation
  - $\sim 10^{-5}$
  - Scales with Grid Density/Time Step.
- Test Problems
  - Exponential Energy Density
  - TECHQM Standard Solutions (soon)

Exponential Test

- For initial condition: $\epsilon(x) = \epsilon_0 \cdot exp[-x/R]; v(x) = 0$
- Energy Conservation: $\partial_\mu T^{\mu 0} = 0$

$$2\epsilon(1+c_s^2)u\dot{u} + [u_0^2 + c_s^2(u_0^2 - 1)]\dot{\epsilon} = (1+c_s^2)u_0 u \frac{\epsilon}{R}$$

- Momentum Conservation: $\partial_\mu T^{\mu x} = 0$

$$\frac{(1+c_s^2)\epsilon}{u_0}[u^2 + u_0^2]\dot{u} + (1+c_s^2)u_0 u\dot{\epsilon} = \left[u^2(1+c_s^2) - c_s^2\right]\frac{\epsilon}{R}$$

Exponential Test (u)

- Separable for u:

$$\dot{u} = \frac{u_0 c_s^2}{R(1+c_s^2)(u_0^2 - c_s^2 u^2)}$$

- Analytically Integrable:

$$\frac{c_s^2}{R(1+c_s^2)}t \quad = \quad \frac{1}{2}\left[(1+c_s^2)sinh^{-1}(u) - (c_s^2-1)u\sqrt{u^2+1}\right]$$

$$\frac{c_s^2}{R(1+c_s^2)}t \quad \approx \quad u$$

- But not invertible.

Exponential Test (u)

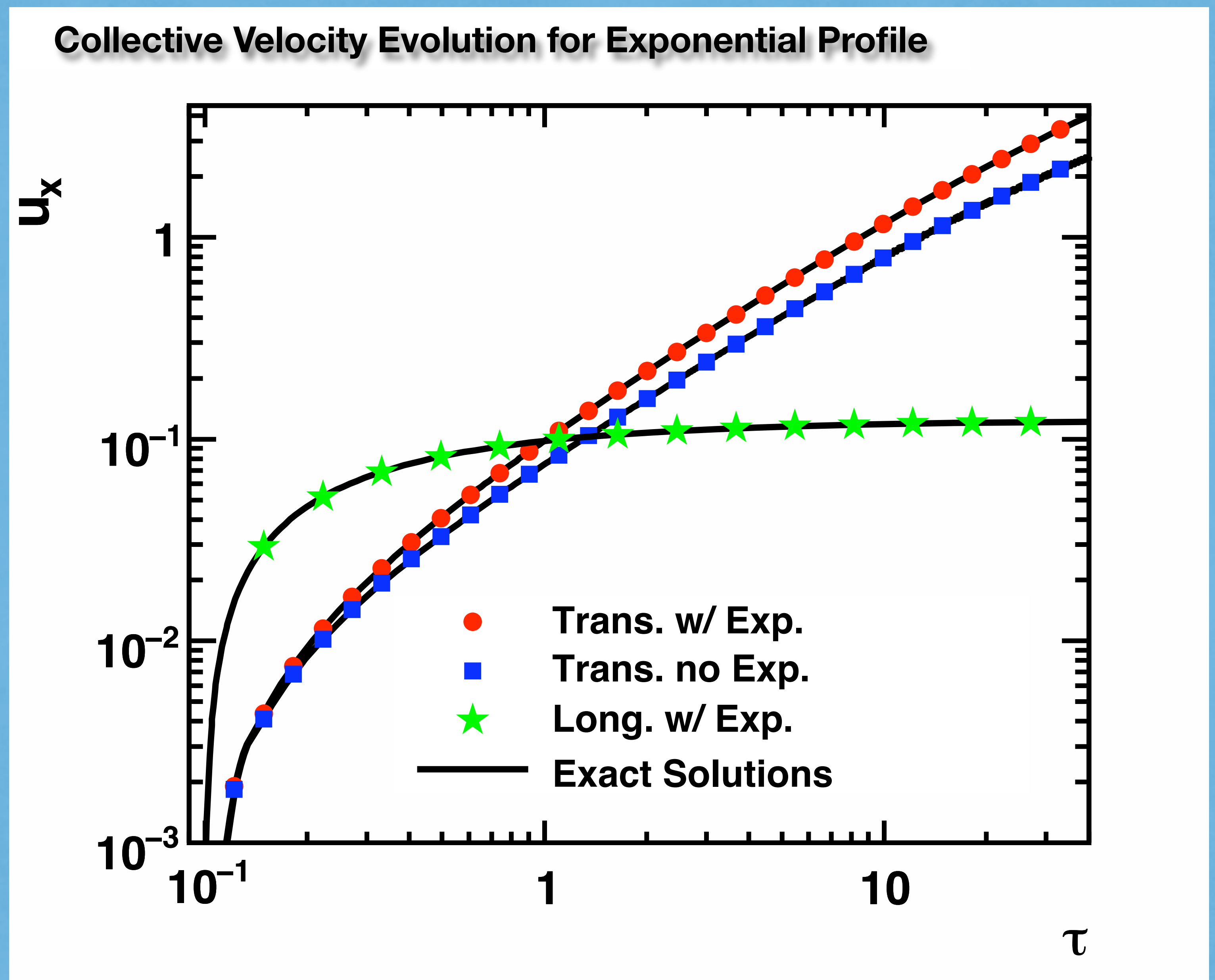
- Standard Collective Acceleration:

$$\dot{u} = \frac{u_0 c_s^2}{R(1 + c_s^2)(u_0^2 - c_s^2 u^2)}$$

- Modified by longitudinal expansion:

$$\dot{u} = \frac{u_0 c_s^2}{u_0^2 - c_s^2 u^2} \cdot \left[\frac{u_0 u}{\tau} + \frac{1}{R(1 + c_s^2)} \right]$$

Exponential Test (u)

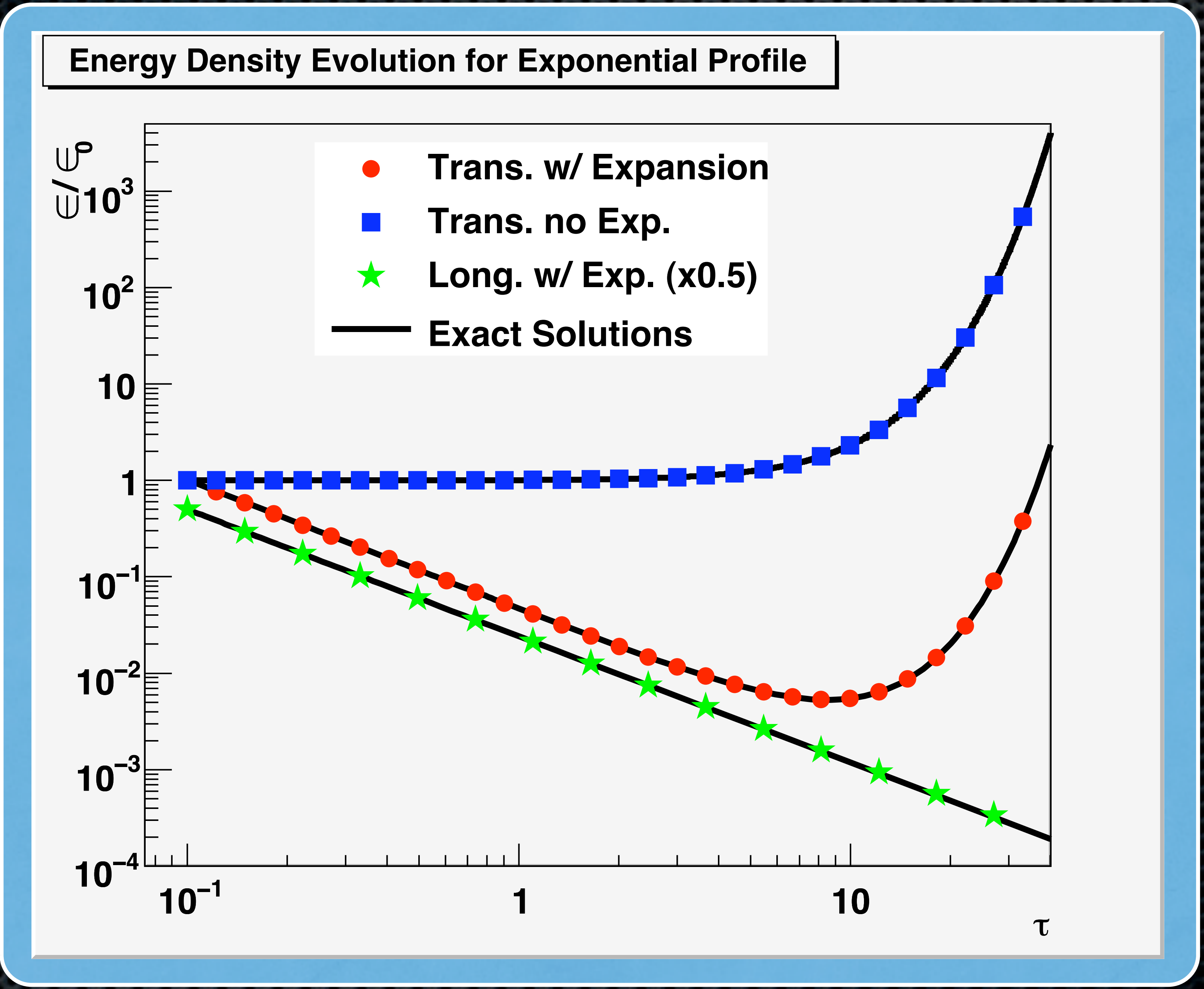


Exponential Test (ε)

- We could have instead solved conservation equations for energy.
- With no Bjorken expansion: $$\frac{\dot{\epsilon}}{\epsilon} = \frac{u u_0 (1 - c_s^2)}{R(u^2(1 - c_s^2) + 1)}$$
  - Which may be integrated: $$\epsilon = \epsilon_0 \cdot exp\left[\frac{1 - c_s^4}{2c_s^2} u^2\right]$$
  - Or with Bjorken expansion:

$$\frac{\dot{\epsilon}}{\epsilon} = \frac{u_0}{u^2(1 - c_s^2) + 1} \cdot \left[\frac{(1 - c_s^2)u}{R} - \frac{(1 + c_s^2)u_0}{\tau}\right]$$

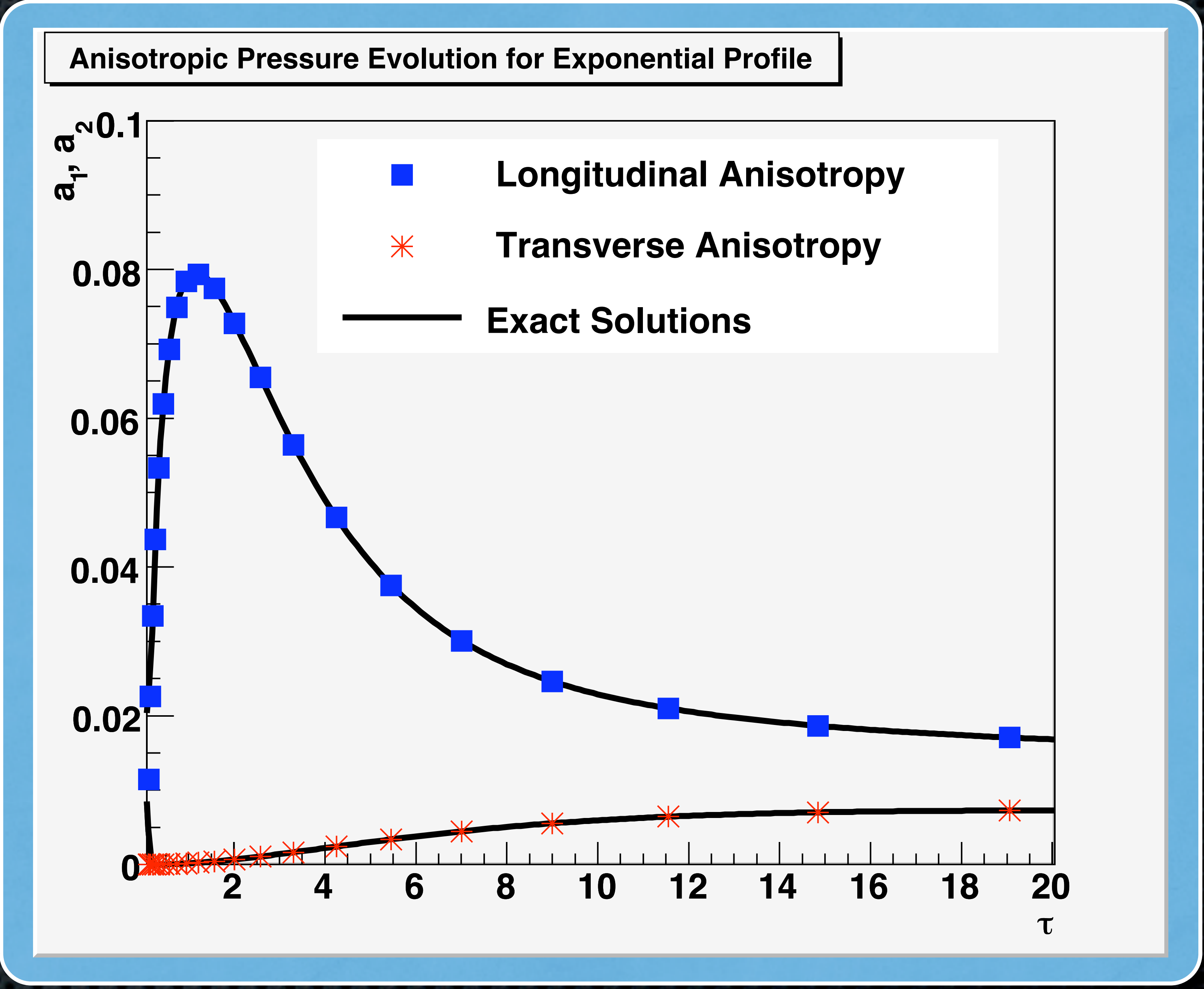
Energy Density Test



Exponential Test ($\pi_{\alpha\beta}$)

- Similar testing of the viscous deviations.
- For simplicity:

$$\eta = \epsilon / 4\pi$$



Is the 3rd dimension necessary?

- Used Ideal EOS - constant sound speed.
- $4\pi \cdot \frac{\eta}{s} = 2$.
- Initial conditions:
  - Woods-Saxon; quick fit to TECHQM tests.
  - Sharper Woods-Saxon in Eta.

Effects of (3+1) - Long. Velocity Gradient

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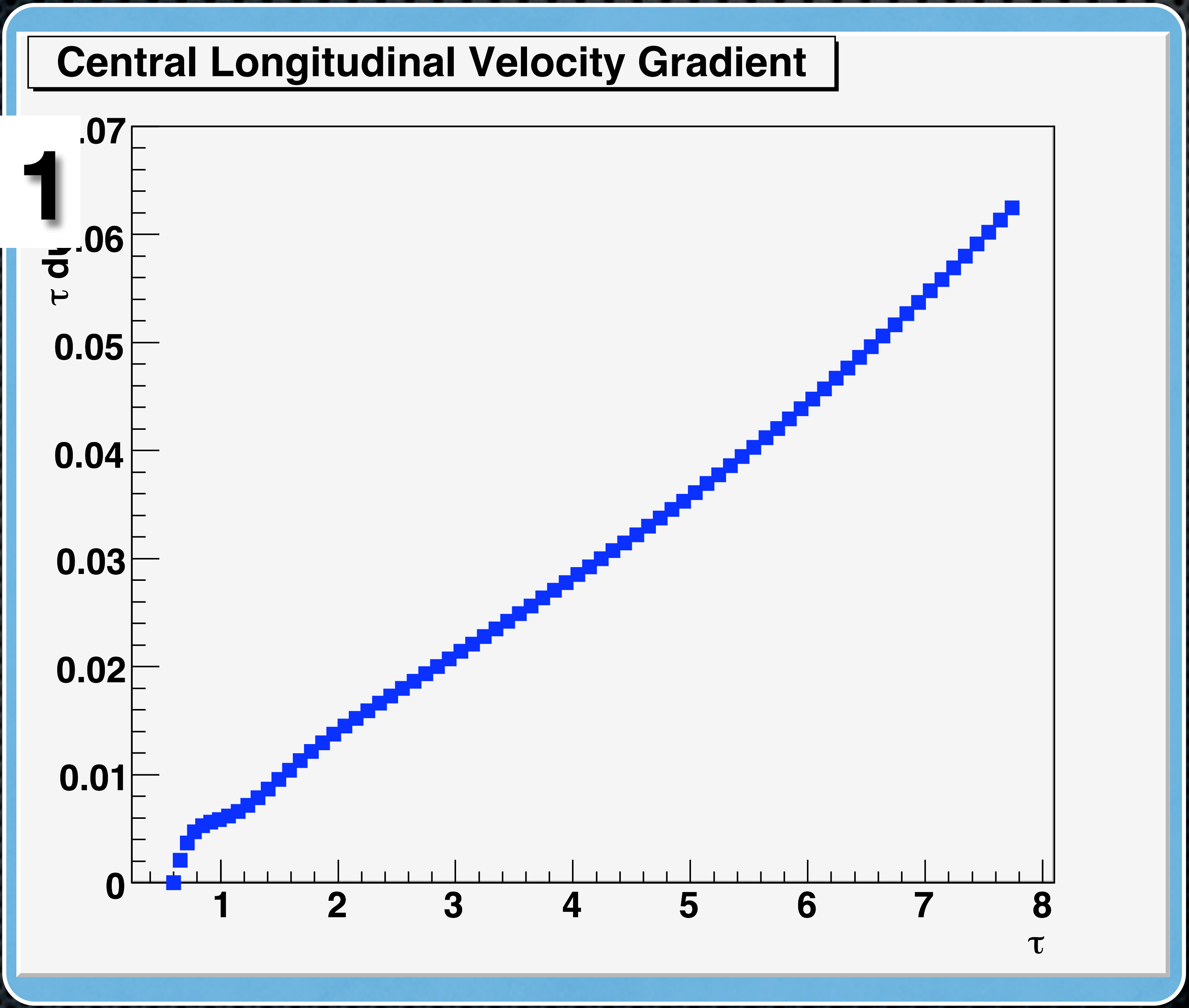
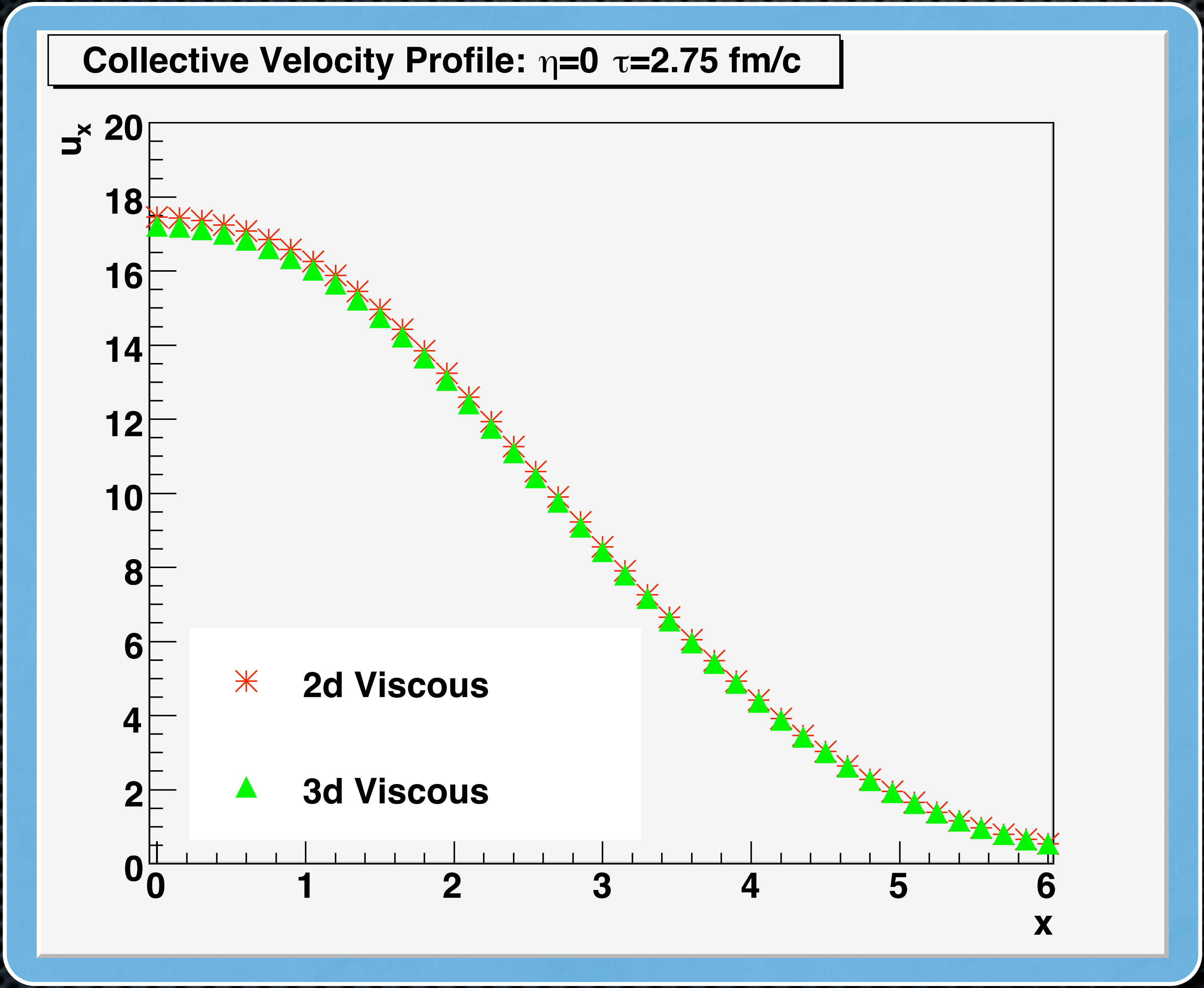


Effects of (3+1) - Energy Density Profiles



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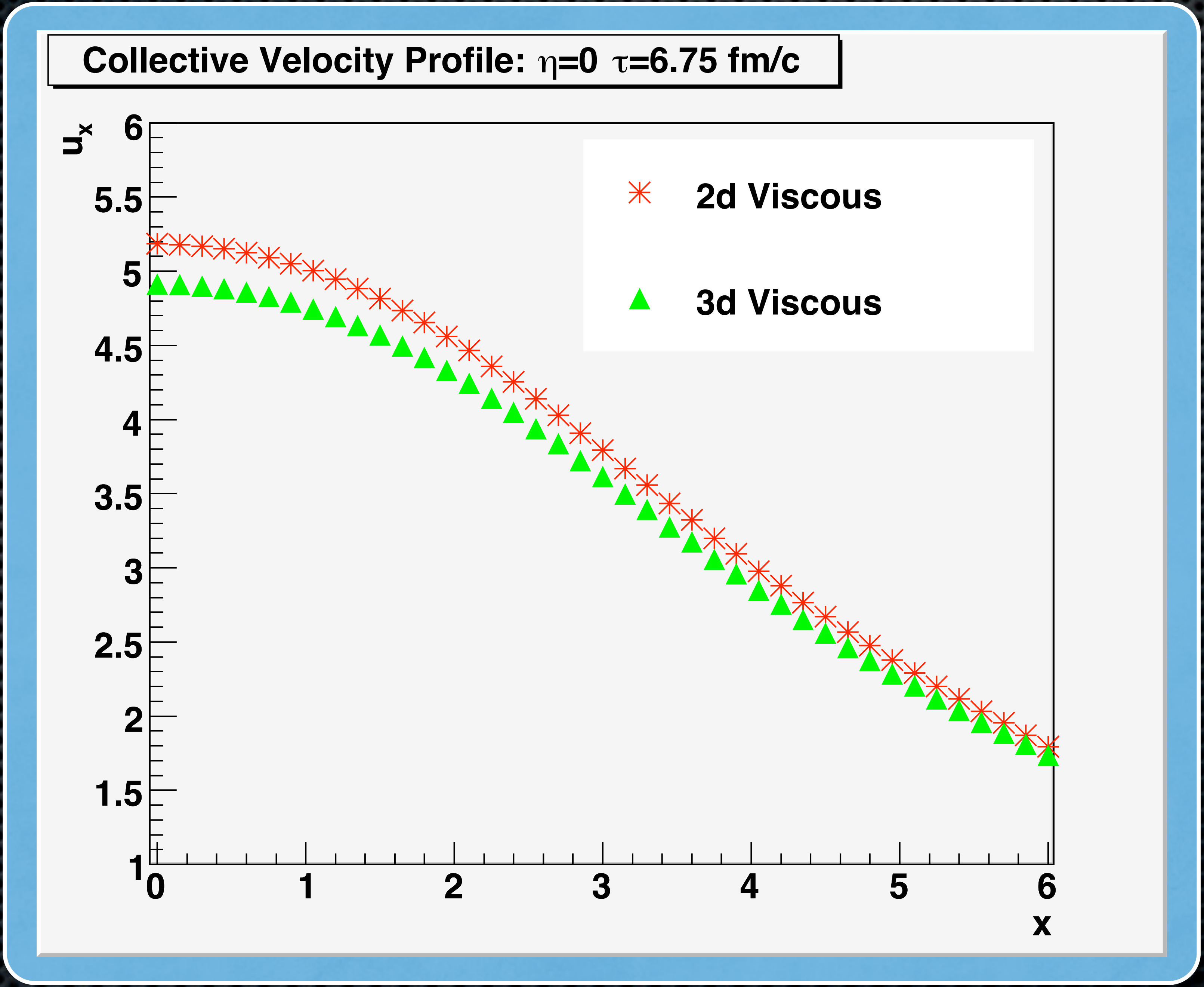
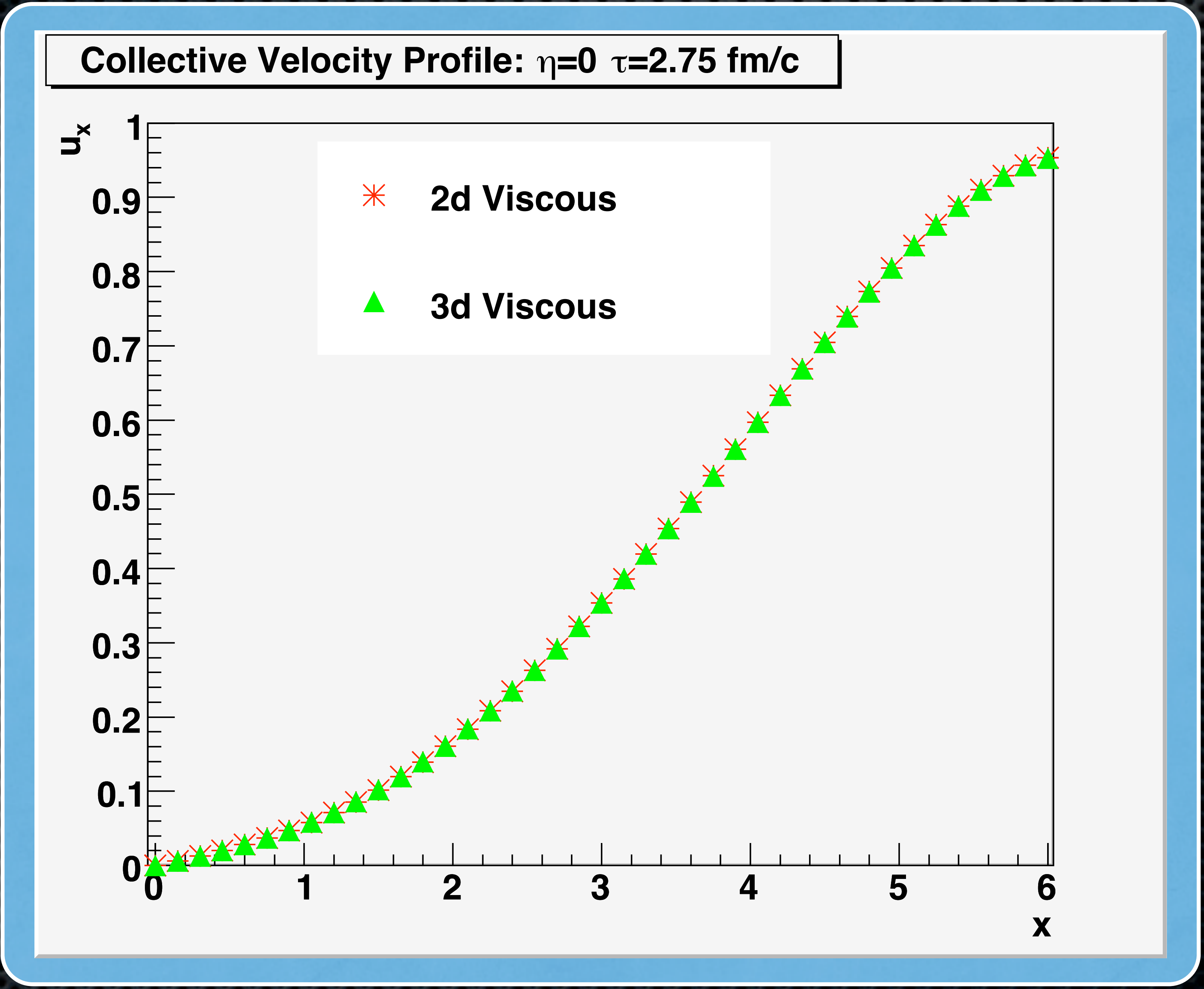


Effects of (3+1) - Velocity Profiles



Moving Forward

- Significant progress has been made toward a (3+1) dimensional Israel-Stewart code.
  - Additional testing required
    - TechQM tests
    - Shocks (Travel to Frankfurt in Spring)
  - Lattice EOS (R. Soltz, J. Newby and A. Glenn).