CAPACITORS

- The potential difference between two charged plates A and B is related to the E-field via: $V = Ed$.
- Definition: The *capacitance* C is a proportionality constant that relates q to V: $q = CV$ or $C = q/V$, SI Unit: $1Farad = 1F = 1C/V$
- Parallel capacitor: (q is the total charge on one plate): $4\pi kq = EA = (V/d)A \Rightarrow$
  $C = q/V = A/(4\pi kd) = \epsilon_0 A/d$
- Cylindrical capacitor: $C = 2\pi\epsilon_0 L/(ln(b/a)$, where a and b correspond to the radius of the inner and outer plate, respectively.
- Spherical capacitor: $C = 4\pi\epsilon_0 ab/(b-a)$, where a and b correspond to the radius of the inner and outer spheres, respectively.
- Isolated Sphere: $C = 4\pi\epsilon_0 R = kR$.

CAPACITORS IN PARALLEL

- The sides of two capacitors connected together are on the same potential: $V_1 = V_2 = V$. The charges on each capacitor are: $q_1 = C_1 V$ and $q_2 = C_2 V$. The total charge is : $q = q_1 + q_2$.
  Therefore, the capacitance of the equivalent capacitor is:
  $C_p = q/V = (q_1 + q_2)/V = C_1 + C_2 \Rightarrow C_p = C_1 + C_2$, or for n capacitors in parallel: $C = \sum_i^n C_i$.

CAPACITORS IN SERIES

- If the capacitors are connected in series the two connected plates in the middle have zero net charge. Therefore the charges on both capacitors are equal: $q_1 = q_2 = q$.
  The total voltage is then $q/C_s = V = V_1 + V_2 = q_1/C_1 + q_2/C_2 = q/C_1 + q/C_2 \Rightarrow$
  $1/C_s = 1/C_1 + 1/C_2$, or for n capacitors in series: $1/C = \sum_i^n 1/C_i$.

ENERGY STORED IN A CAPACITOR

- Potential energy differences are created by transporting charge from one plate to the other.
  $W = Vq/2$ with $q = CV \Rightarrow W = q^2/(2C) = CV^2/2$.

DIELECTRICS

- Dielectrics are materials composed of permanent dipoles which can be reoriented but cannot move. The dipoles align and partially compensate the external field. With fixed charges in the capacitor, the E-field between the plates is reduced with the dielectric present as compared to without.
- Definition: *Dielectric constant*, κ, $\kappa = C_\kappa/C_0 = E_0/E_\kappa = V_0/V_\kappa$ where E_0 is the electric field without the dielectric present, and E_κ is the field with it present.
- With the permittivity of the dielectric material $\epsilon = \epsilon_0\,\kappa$: $C_\kappa = \kappa\, C_0 = \epsilon_0\,\kappa A/d = \epsilon\, A/d$.

CURRENT AND CIRCUITS

ELECTRIC CURRENT

- Amount of current = amount of charge that passes per time unit through an area perpendicular to the flow: $i = dq/dt$, Unit: C/s = A (Ampere).
- The current density is defined as $J = i/A$, where A is the cross sectional area.
- Convention: Direction of current = direction of positive charge flow. Since e^- are the moving charges, the defined direction of the current is opposite to the direction of the physical current.
- The current is related to the electron drift velocity which is amazingly small: $v_d = J/(ne)$
  It is typically $/sim$ mm/s. ne is the charge carrier density.
- Why does current flow instantaneously? Because of the E-field which moves with speed of light, which causes all electrons in wire to drift at the same time.

RESISTANCES

- If one connects a wire between both terminals of a battery, a current flows. The resistance is defined as $R = V/i$ (Ohm's law), Unit: Ω.
- Physical reason for resistance: Scattering of the conduction electrons off obstacles in the conductor. Ohm's law is not generally valid, but it is a good empirical rule for most systems.
- The resistivity is defined as $\rho = E/J$.
- The overall resistance of a wire should be proportional to its length and inversely proportional to its cross sectional area The proportionality constant is called resistivity, ρ.
  $R = \rho\, L/A \Rightarrow \rho = R\, A/L$, Unit: Ωm
- The resistivities are temperature dependent due to thermal vibrations in the material:
  $\rho - \rho_0 = \rho_0 \alpha (T - T_0)$.
- Resistivities are usually tabulated at room temperature, 20 °C.
- The conductivity is: $\sigma = 1/\rho$.

POWER DISSIPATION IN A SIMPLE CIRCUIT

- Potential drop across resistor (Ohm's law): $V = i\, R$.
- Since charge is transported from the positive end of the load resistor to the negative end across a potential V, it loses potential energy $PE_{elec} = V\,\Delta q = V\, i\,\Delta t$. This potential energy is converted into some other form of energy, here: heat (Joule heating).
- The power dissipated is the change in potential energy (work) per time unit:
  $P = \Delta W/\Delta t = V\, i\,\Delta t/\Delta t = V\, i$.
- Using $V = R\, i$, we can also write for the power $P = i^2\, R = V^2/R$.