CAPACITORS

- The potential difference between two charged plates A and B is related to the E-field via: \( V = Ed \).
- Definition: The capacitance \( C \) is a proportionality constant that relates \( q \) to \( V \): \( q = CV \) or \( C = q/V \). SI Unit: \( 1 \text{ Farad} = 1F = 1C/V \)
- Parallel capacitor: \( q \) is the total charge on one plate: \( 4\pi kq = EA = (V/d)A \Rightarrow C = q/V = A/(4\pi kd) = \epsilon_0 A/d \)
- Cylindrical capacitor: \( C = 2\pi \epsilon_0 L/(ln(b/a)) \), where \( a \) and \( b \) correspond to the radius of the inner and outer plate, respectively.
- Spherical capacitor: \( C = 4\pi \epsilon_0 ab/(b - a) \), where \( a \) and \( b \) correspond to the radius of the inner and outer spheres, respectively.
- Isolated Sphere: \( C = 4\pi \epsilon_0 R = kR \).

CAPACITORS IN PARALLEL

- The sides of two capacitors connected together are on the same potential: \( V_1 = V_2 = V \). The charges on each capacitor are: \( q_1 = C_1 V \) and \( q_2 = C_2 V \). The total charge is: \( q = q_1 + q_2 \). Therefore, the capacitance of the equivalent capacitor is: \( C_p = q/V = (q_1 + q_2)/V = C_1 + C_2 \Rightarrow C_p = C_1 + C_2 \), or for \( n \) capacitors in parallel: \( C = \sum_i^n C_i \).

CAPACITORS IN SERIES

- If the capacitors are connected in series the two connected plates in the middle have zero net charge. Therefore the charges on both capacitors are equal: \( q_1 = q_2 = q \). The total voltage is then \( q/C_s = V = V_1 + V_2 = q_1/C_1 + q_2/C_2 = q/C_1 + q/C_2 \Rightarrow 1/C_s = 1/C_1 + 1/C_2 \), or for \( n \) capacitors in series: \( 1/C = \sum_i^n 1/C_i \).

ENERGY STORED IN A CAPACITOR

- Potential energy differences are created by transporting charge from one plate to the other. \( W = Vq/2 \) with \( q = CV \Rightarrow W = q^2/(2C) = CV^2/2 \).

DIELECTRICS

- Dielectrics are materials composed of permanent dipoles which can be reoriented but cannot move. The dipoles align and partially compensate the external field. With fixed charges in the capacitor, the E-field between the plates is reduced with the dielectric present as compared to without.
- Definition: Dielectric constant, \( \kappa \), \( \kappa = C_\kappa/C_0 = E_0/E_\kappa = V_0/V_\kappa \) where \( E_0 \) is the electric field without the dielectric present, and \( E_\kappa \) is the field with it present.
- With the permittivity of the dielectric material \( \epsilon = \epsilon_0 \kappa \) : \( C_\kappa = \kappa C_0 = \epsilon_0 \kappa A/d = \epsilon A/d \).
CURRENT AND CIRCUITS

ELECTRIC CURRENT

- Amount of current = amount of charge that passes per time unit through an area perpendicular to the flow: \( i = \frac{dq}{dt}, \) Unit: C/s = A (Ampere).

- The current density is defined as \( J = i/A, \) where \( A \) is the cross sectional area.

- Convention: Direction of current = direction of positive charge flow. Since \( e^- \) are the moving charges, the defined direction of the current is opposite to the direction of the physical current.

- The current is related to the electron drift velocity which is amazingly small: \( v_d = J/(ne) \)
  It is typically \( \sim \) mm/s. \( ne \) is the charge carrier density.

- Why does current flow instantaneously? Because of the \( E \)-field which moves with speed of light, which causes all electrons in wire to drift at the same time.

RESISTANCES

- If one connects a wire between both terminals of a battery, a current flows. The resistance is defined as \( R = V/i \) (Ohm’s law), Unit: \( \Omega. \)

- Physical reason for resistance: Scattering of the conduction electrons off obstacles in the conductor. Ohm’s law is not generally valid, but it is a good empirical rule for most systems.

- The resistivity is defined as \( \rho = E/J. \)

- The overall resistance of a wire should be proportional to its length and inversely proportional to its cross sectional area. The proportionality constant is called resistivity, \( \rho. \)
  \( R = \rho L/A \Rightarrow \rho = RA/L, \) Unit: \( \Omega \)m

- The resistivities are temperature dependent due to thermal vibrations in the material:
  \( \rho - \rho_0 = \rho_0 \alpha(T - T_0). \)

- Resistivities are usually tabulated at room temperature, 20 °C.

- The conductivity is: \( \sigma = 1/\rho. \)

POWER DISSIPATION IN A SIMPLE CIRCUIT

- Potential drop across resistor (Ohm’s law): \( V = iR. \)

- Since charge is transported from the positive end of the load resistor to the negative end across a potential \( V, \) it loses potential energy \( P E_{\text{elec}} = V \Delta q = V i \Delta t. \) This potential energy is converted into some other form of energy, here: heat (Joule heating).

- The power dissipated is the change in potential energy (work) per time unit:
  \( P = \Delta W/\Delta t = V i \Delta t/\Delta t = V i. \)

- Using \( V = R i, \) we can also write for the power \( P = i^2 R = V^2/R. \)