BATTERIES

- Batteries supply a potential difference which is called an *electromotive force* (emf). This emf is defined by the work dW done on a charge dq: $\mathcal{E} = dW/dq$.
- Real batteries have an internal resistance r when current is drawn from the battery. This internal resistance reduces the nominal *emf* according to Ohm's law to the *terminal voltage* (TV) which is given by $TV = \mathcal{E} - ir$.
- Ideal batteries do not have any internal resistances.

KIRCHHOFF'S RULES

- *Kirchhoff's first rule: (Junction Rule)* The sum of the currents flowing into a junction is equal to the sum of the currents flowing out of the junction (conservation of charge).
- *Kirchhoff's second rule: (Loop Rule)* The sum of the potential drops is equal to the sum of the potential rises within a closed loop (conservation of energy).
- *Resistance rule:* For a move through a resistor in the direction of the current, the change in potential is $-iR$, in the opposite direction it is $+iR$.
  emf rule: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathcal{E}$, in the opposite direction it is $-\mathcal{E}$.

RESISTORS IN SERIES AND IN PARALLEL

- For two resistors in series the current that flows through the circuit is the same everywhere and the voltage drops across the resistors add up:
  $\mathcal{E} = V_1 + V_2 = i_1\, R_1 + i_2\, R_2 = i\, R_1 + i\, R_2 = i\,(R_1 + R_2) = i\, R_s \Rightarrow R_s = R_1 + R_2$
  or for n resistors in series: $R_s = \sum_i^n R_i$.
- For two parallel resistors the potential drop across each resistor is constant: $\mathcal{E} = V_1 = V_2$, and the current that flow through the individual resistors add up to the total current:
  $i = i_1 + i_2 = V_1/R_1 + V_2/R_2 = \mathcal{E}/R_1 + \mathcal{E}/R_2 = \mathcal{E}\,(1/R_1 + 1/R_2) = \mathcal{E}/R_p \Rightarrow 1/R_p = 1/R_1 + 1/R_2$
  or for n resistors in parallel: $1/R_p = \sum_i^n 1/R_i$.

CHARGING AND DISCHARGING A CAPACITOR

- It takes time to put charges onto a capacitor. Initally it is fast, but as more charges are on the capacitor it becomes harder: $q = C\mathcal{E}(1 - e^{-t/RC})$, $i = (\mathcal{E}/R)e^{-t/RC}$. RC is called the time constant of the circuit.
- Discharging the capacitor is again fast at the beginning, getting slower the fewer charges are left on the capacitor: $q = q_0 e^{-t/RC}$, $i = -(q_0/RC)e^{-t/RC}$.

MAGNETISM

MAGNETIC FIELD

- Magnets have two *poles*, named North (N) and South (S). Magnets have *magnetic field lines* extending from the North pole to the South pole. Two like poles repel, two unlike poles attract each other.
- The magnetic field exerts a force on a **moving** charge: $\vec{F}_B = q\vec{v} \times \vec{B}$. The magnitude of this force is $F_B = |q|vB \sin\Phi$, where Φ is the angle between the directions of the velocity and the field.
- The direction of the $\vec{B}$-field is given by a Right-hand-rule:
  When the fingers sweep $\vec{v}$ into $\vec{B}$, the thumb points for positive charges in the direction of the force $\vec{F}_B$. Units of B: 1 $Tesla$ $(T) = Ns/Cm = Vs/m^2$ Other unit: 1 $Gauss(G) : 1G = 10^{-4}T$

TRAJECTORY OF CHARGES IN CONSTANT B-FIELDS

- The magnetic force F_B on a charge moving perpendicular to a magnetic field causes a circular motion. It is a centripetal force: $F_c = m\,v^2/r$. Therefore: $F_m = F_c \Rightarrow q\,v\,B = m\,v^2/r$.
- Solving for the radius of the orbit: $r = m\,v/(q\,B)$. The frequency is then $f_{osc} = 1/T = q\,B/(2\pi\,m)$.

HALL EFFECT

- Moving electrons in a wire (= current) can be deflected by a magnetic field which produces a (Hall) potential difference. In the equilibrium ($F_E = F_B$) it is possible to measure the charge density $n = Bi/(Vle)$, where l is the thickness of the strip of wire.

FORCE ON A CURRENT

- Current consists of moving charges. The magnitude of the magnetic force on a wire of length L with current i due to a magnetic field B is given by $\vec{F}_B = i\vec{L} \times \vec{B}$, $F_B = iLB\sin\theta$, where θ is the angle between the wire (current) and the magnetic field.

TORQUE ON A CURRENT CARRYING COIL

- Assume a coil in a magnetic field $\vec{B}$. For one loop of the coil: The forces on the current in pivoted part of the loop are equal and in opposite directions ($\vec{B} \perp \vec{i}$) and thus there is no net torque.
- The force on the current I in the non-pivoted side of the loop (width a) is $F = iaB$.
- The torque $\vec{\tau}$ on one side of the loop is $\tau = (b/2)F\sin\theta$,
  where θ is the angle between the normal to the loop and the magnetic field.
- Thus, the total torque on the loop (the two opposite sides add up) is
  $\tau = (b/2)iaB\sin\theta + (b/2)iaB\sin\theta = iAB\sin\theta$ where the area of the loop is $A = ab$.
- When the coil consists of N loops the total torque is $\tau = NiAB\sin\theta$.
- The magnetic moment of the coil is defined as: $\mu = NiA$ and thus the torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$.