MAGNETIC FIELD DUE TO A CURRENT

- The magnetic field $d\vec{B}$ at a distance r from a wire due to a current i inside a piece $d\vec{s}$ of the wire is given by Bio-Savart's law: $d\vec{B} = \mu_0 i d\vec{s} \times \vec{r}/(4\pi r^3)$
  The constant: $\mu_0 = 4\pi \cdot 10^{-7}\, Tm/A$ is called the permeability of free space.
- Magnetic field of a long straight wire at a distance r from the wire: $B = \mu_0 i/(2\pi r)$
- Magnetic field inside of a long straight wire with radius R at a distance r from the center: $B = \mu_0 i r/(2\pi R^2)$
- Magnetic field at a center of a circular arc: $B = \mu_0 i \Phi/(4\pi R)$, where Φ is measured in radians.
  Thus the magnetic field at the center of a current loop is: $B = \mu_0 i/(2R)$
- Magnetic field of a coil along the symmetry axis z through the center: $B = \mu_0 \vec{\mu}/(2\pi z^3)$,
  where $\mu = NiA$ is the magnetic dipole moment of the coil and N is the number of turns in the coil.
- Right Hand Rule: The direction of the field curles around with the fingers of the right hand when the thumb points in the direction of the current.

FORCE BETWEEN TWO WIRES

- The force on a wire which carries a current i_1 due to the magnetic field of another (parallel) wire with current i_2 ($B = \mu_0 i_2/(2\pi d)$) is given by: $F = \mu_0 L i_1 i_2/(2\pi d)$.
  d is the distance between the two wires and L is the length of the wires.
- If the current in both wires are in the same direction, the force is attractive, and if the currents are in opposite directions the force is repulsive.

MAGNETIC FIELD OF A SOLENOID AND TOROID

- A solenoid is a long straight coil of tightly wound wire. The magnetic field inside the solenoid is directed along the center of the coil: $B = \mu_0 i n$, where n is the number of turns per unit length. Thus, if N is the total number of turns of the solenoid of length L, then $n = N/L$.
- A toroid is a solenoid bent into the shape of a doughnut. The magnetic field at the center of the toroid is: $B = \mu_0 i N/(2\pi r)$, where N is the total number of turns and r is the radius of the toroid.

AMPERE'S LAW

- For certain situations which involve symmetries it is easier to use Ampere's law rather than Biot-Savart's law. The integral of the scalar product of the magnetic field $\vec{B}$ and pathsegment $d\vec{s}$ over a closed imaginary loop is proportional to the enclosed current: $\oint \vec{B} d\vec{s} = \mu_0 i_{enc}$.

INDUCTION

- A current I generates a magnetic field B. Can a magnetic field also generate a current? Yes and No. A constant (in time) magnetic field does not generate a current, but changes in the field do.
- Faraday's observations: An emf can be generated in a loop of wire by:
  (i) holding it close to a coil (solenoid) and changing the current in the coil.
  (ii) keeping the current in the coil steady, but moving the coil relative to the loop.
  (iii) moving a permanent magnet in or out of the loop.
  (iv) rotating the loop in a steady magnetic field.
  (v) changing the shape of the loop in the field.
- The Magnetic flux is defined as $\Phi_B = \int \vec{B} d\vec{A}$. If B is constant over the area, then the flux is given by $\Phi_B = BA\cos\theta$. θ is the angle between $\vec{B}$ and $\vec{A}$, the "normal" to the surface.
- A change in the magnetic flux produces a potential difference (and via Ohm's law a current) in a coil: $\mathcal{E} = -N d\Phi_B/dt$.
  Φ is the flux through the coil and N is the number of turns of the coil.
- Thus, if B is constant within the coil: $\mathcal{E} = -Nd(BA\cos\theta)/dt$. In most cases, only one variable depends on the time:
  (1) A, θ constant: $\mathcal{E} = -NA\cos\theta dB/dt$
  (2) B, θ constant: $\mathcal{E} = -NB\cos\theta dA/dt$
  (3) A, B constant: $\mathcal{E} = -NABd(\cos\theta)/dt$
- Why the negative sign? **Lenz's law:** An induced emf gives rise to a current whose magnetic field opposes the change in flux that produced it.
- The magnitude of the emf of a moving conductor in a perpendicular magnetic field is given by: $\mathcal{E} = BLv$, where L is the length of the conductor and v is the velocity (perpendicular to the field) of the conductor ($v = dx/dt$).
- Faraday's law of induction can also be expressed in terms of an electric field: $\oint \vec{E} d\vec{s} = -d\Phi_B/dt$.

INDUCTANCE AND SELFINDUCTION

- The inductance is defined as $L = N\Phi/i$, Units: 1 henry = 1 H = 1 Vs/A.
- The inductance per unit length of a solenoid is $L/l = \mu_0 n^2 A$.
- A changing current in a coil generates a self-induced emf in the coil: $\mathcal{E}_L = -Ldi/dt$.
- An RL-circuit consists of an inductor and a resistor in series: $Ldi/dt + Ri = \mathcal{E}$. The current rises according to $i = \mathcal{E}/R(1 - exp(-t/\tau_l))$ where $\tau_L = L/R$ is the inductive time constant. After a connection is broken the current decreases as $i = i_0 exp(-t/\tau_l)$.
- The energy stored in an inductor is given by $U_B = Li^2/2$.
  The energy density of a coil is given by $u_B = B^2/(2\mu_0)$.
- The mutual inductance between two coils is defined as M and $E_2 = -Mdi_1/dt$ and $E_1 = -Mdi_2/dt$.