MAGNETIC MATERIALS

- Microscopic origin: Atoms have small magnetic moments (elementary magnetic dipoles). In ferromagnetic materials (iron, nickel ...) these magnetic moments interact so strongly that the dipoles spontaneously align. Thus the material forms one large magnetic dipole → magnet.
- Inserting materials into a magnetic field can polarize the material and create a magnetic dipole moment. The total magnetic field can then be much stronger as it is the sum of the external field and the magnetisation of the ferromagnetic material: $B = B_{ext} + B_M$.

AMPERE-MAXWELL'S LAW AND DISPLACEMENT CURRENT

- Just as a changing magnetic flux creates an electric field (Faraday's law, see page 10 of the lecture notes) a changing electric flux can create a magnetic field: $\oint \vec{B} d\vec{s} = \mu_0 \epsilon_0 d\Phi_E/dt$ (Maxwell's law).
- Thus, a magnetic field can be created by a current or by a changing electric field, which combines Ampere's and Maxwell's law: $\oint \vec{B} d\vec{s} = \mu_0 \epsilon_0 d\Phi_E/dt + \mu_0 i_{enc}$.
- A displacement current can be defined as $i_d = \epsilon_0 d\Phi_E/dt$, and then the Ampere-Maxwell law can be written as $\oint \vec{B} d\vec{s} = \mu_0 i$, with $i = i_{enc} + i_d$.

GAUSS' LAW FOR MAGNETIC FIELDS

- In analogy to Gauss' law for electric fields, it also can be written for magnetic fields: $\Phi_B = \oint \vec{B} d\vec{A} = 0$.
- This means that the net magnetic flux through an enclosed surface is always zero, or: A magnetic monopole does not exist.

MAXWELL'S EQUATIONS

- The basic laws combining electricity and magnetism are Maxwell's equations:

$\oint \vec{E} d\vec{A} = q/\epsilon_0$	$\oint \vec{B} d\vec{A} = 0$
$\oint \vec{E} d\vec{s} = -d\Phi_B/dt$	$\oint \vec{B} d\vec{s} = \mu_0 \epsilon_0 d\Phi_E/dt + \mu_0 i_{enc}$

LC OSCILLATOR

- The total energy stored in an LC oscillator is the sum of the energy stored in the capacitor $U_E = q^2/(2C)$ and the energy stored in the inductor $U_B = i^2 L/2$.
- The total energy is conserved and thus $dU/dt = 0$.
- The solution to the resulting differential equation is: $q = Qcos(\omega t + \phi)$ and $i = Isin(\omega t + \phi)$, where $I = \omega Q$ and $\omega = 1/\sqrt{LC}$.
- The total energy is $U = Q^2/(2C)$, $U_E = Q^2/(2C)\cos^2(\omega t + \phi)$ and $U_B = Q^2/(2C)\sin^2(\omega t + \phi)$. circuit.

RLC OSCILLATOR

- Now energy is dissipated in the resistor and thus $dU/dt = -i^2 R$.
  The solution is now: $q = Qe^{-Rt/2L}\cos(\omega' t + \phi)$ with $\omega' = \sqrt{\omega^2 - (R/2L)^2}$.
  The total energy decreases as $U = Q^2/(2C)e^{-Rt/L}$.

R, L, OR C IN AN AC CIRCUIT

- If we connect a resistor, a capacitor, or an inductor to an external ac (alternating current) emf, $\mathcal{E} = \mathcal{E}_m \sin(\omega_d t)$, ω_d is called the driving frequency. The voltage drop across the element (x = R, C or L) is then $v_x = V_x \sin(\omega_d t)$.
- The current can then be calculated with $i_x = I_x \sin(\omega_d t - \phi)$, where ϕ corresponds to the phase difference between the voltage and the current.
- The equivalence of a resistance for a capacitor and an inductor is the reactance. The capacitive reactance is $X_C = 1/(\omega_d C)$ and the inductive reactance is $X_L = \omega_d L$.

Element	Symbol	Resistance or Reactance	Phase of Current	Phase angle ϕ	Amplitude Relation
Resistor	R	R	in phase	$0°$	$V_R = I_R R$
Capacitor	C	$X_C = 1/(\omega_d C)$	leads v_C (ICE)	$-90°$	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	lags v_L (ELI)	$90°$	$V_L = I_L X_L$

RLC CIRCUIT

- In an RLC circuit the instantaneous voltages have to add up to the emf: $\mathcal{E} = v_R + v_L + v_C$.
- The amplitudes can be calculated from the vector sum of the phasors. Thus the maximum current is given by $I = \mathcal{E}_m/Z$ where $Z = \sqrt{R^2 + (X_L - X_C)^2}$ is the impedance of the circuit.
- The phase constant is defined as $\tan\phi = (X_L - X_C)/R$.
- The current has a maximum when the impedance has a minimum, i.e. $Z = R$. This occurs at the resonance frequency $\omega_d = \omega = 1/\sqrt{LC}$.
- The average power dissipated in an ac circuit is $P_{av} = I_{rms}^2 R$, where the rms current is $I_{rms} = I/\sqrt{2}$. The definition of an rms emf as $\mathcal{E}_{rms} = \mathcal{E}_m/\sqrt{2}$ yields also: $P_{av} = \mathcal{E}_{rms} I_{rms} R/Z = \mathcal{E}_{rms} I_{rms} \cos\phi$.

TRANSFORMERS

- Transformers consist of two coils (primary and secondary) wound on the same iron core with different number of turns. The same magnetic flux is then in both coils:
  $V_p = -N_p \Delta\phi_B/\Delta t$ and $V_s = -N_s \Delta\phi_B/\Delta t \rightarrow V_s/V_p = N_s/N_p$.
  The currents are then related via: $I_s/I_p = N_p/N_s$.
- The equivalent resistance of the coil in the primary circuit is given by $R_{eq} = (N_p/N_s)^2 R$, where R is the resistance in the circuit of the second coil.