INTRODUCTION TO ONE-DIMENSIONAL COLLISIONS

(Elastic and Inelastic collisions)

The following two experiments deal with two different types of one-dimensional collisions. Below is a discussion of such collisions, and the principles and equations which will be used in analyzing them.

For a single particle, momentum is defined as the product of the mass and the velocity:

$$p = mv \qquad (1)$$

Momentum is a vector, making its direction a necessary part of the data (for the one-dimensional case the direction can only be the +x direction or the -x direction). For a system of more than one particle, the total linear momentum is the vector sum of the individual momenta:

$$p = p_1 + p_2 + ... = mv_1 + mv_2 + ... \qquad (2)$$

One of the most fundamental laws of physics is that the total momentum of any system of particles is conserved (constant), provided that the net external force on the system is zero. Assume we have two particles with masses m_1, m_2 and speeds v_1 and v_2 which collide, without any external force, resulting in speeds of v_{1f} and v_{2f} after the collision. Conservation of momentum then states that the total momentum before the collision ($P_{initial} = P_i$) is equal to the total momentum after the collision (P_{final}=P_f):

$$P_i = m_1v_1 + m_2v_2 \quad , \quad P_f = m_1v_{1f} + m_2v_{2f} \quad \text{and} \quad P_i = P_f \qquad (3)$$

In a given system, the total energy is generally the sum of several different forms of energy. Kinetic energy is the form associated with motion, and for a single particle:

$$KE = \frac{mv^2}{2} \qquad (4)$$

In contrast to momentum, kinetic energy is NOT a vector; for a system of more than one particle the total kinetic energy is the algebraic sum of the individual kinetic energies of each particle:

$$KE = KE_1 + KE_2 + \ldots \qquad (5)$$

Another fundamental law of physics is that the total energy of a system is always conserved. However within a given system, one form of energy may be converted to another (such as potential energy converted to kinetic in the experiment entitled Analysis of a Freely Falling Body). Therefore, kinetic energy alone is not often conserved.

There are two basic kinds of collisions, elastic and inelastic:

In an elastic collision, two or more bodies come together, collide, and then move apart again with no loss in total kinetic energy. An example would be two identical "superballs", colliding and then rebounding off each other with the same speeds they had before the collision. Given the above example conservation of kinetic energy then implies:

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 \quad \text{or} \quad KE_{\text{initial}} = KE_{\text{final}} \qquad (6)$$

In an inelastic collision, the bodies collide and come apart again, but now some kinetic energy is lost (converted to some other form of energy). An example would be the collision between a baseball and a bat. If the bodies collide and stick together, the collision is called completely inelastic. In this case, all of the kinetic energy relative to their center of mass is lost in the collision (converted to other forms).

In these experiments you will be dealing with a) a completely inelastic collision (this lab) in which all kinetic energy relative to the center of mass is lost in the collision, but momentum is still conserved, and b) a nearly elastic collision (next lab) in which both momentum and kinetic energy are conserved to within a few percent.

Objectives for collision experiments:

PRIMARY OBJECTIVES:

1) To measure the momentum and kinetic energy of two objects before and after a one-dimensional collision.

2) To observe that the concept of conservation of momentum is independent of conservation of energy, i.e., that the total momentum remains constant in an inelastic collision where kinetic energy is lost as well as in a nearly elastic collision where very little kinetic energy is lost .

SECONDARY OBJECTIVE: To try to account for any change in KE in the nearly elastic collision.

ADVANCED OBJECTIVE: To calculate the percentage of KE which will be lost (converted to other forms of energy, notably, heat) in a completely inelastic collision between an initially stationary mass, m_1 and an initially moving mass, m_2; and to compare this calculation with the result of the elastic collision.

EXPERIMENT: ONE-DIMENSIONAL COLLISIONS: INELASTIC

OBJECTIVES: See previous pages

APPARATUS:

One-dimensional air track, Photogate timing circuit, Analytical balance

PROCEDURE:

In this and the next experiment we will use two carts of different masses, with one initially at rest. The carts move on an air track, so the first item to check is whether the air track is level.

In this lab we study the totally inelastic collision, by arranging that when two carts collide they will stick together and move with some velocity common to both masses. Thus, we have to measure the velocity of cart 1 before the collision and the common velocity of the carts 1 and 2 after the collision. For this purpose, we use two photogates (see Figure 1). Each of them allows to measure the time it takes the cart to go through it. The velocities are calculated by dividing the length of the plate on the cart 1 by the measured time (speed = length/time).

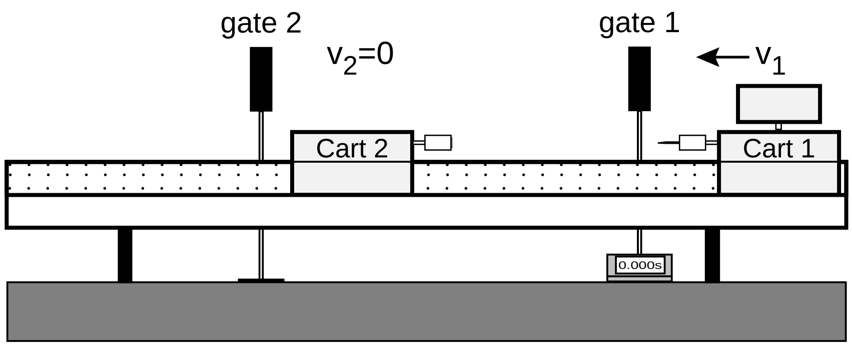

Figure 1: The initial state of the carts before collision.

Position cart 2 close to the gate 2 and set the photogate timer to "GATE" mode and the memory switch in “ON” position. In this mode the photogate display unit will display the first time interval measured. Subsequent measurements will not be displayed (only the first one is), but the times are added in the memory. By pushing the “READ” switch you can display the memory contents, which is the sum of all measurements. Example: the initial reading for cart 1 (the time which it took to pass through the gate 1) is 0.300 seconds. Cart 1 collides with cart2 and they go together through the photogate 2 (Figure 2). Suppose it now takes 0.513 seconds. The display will remain at 0.300, but the memory will contain 0.300 + 0.513=0.813 seconds. To find the second time, You have to subtract the first time from the contents of the memory. Try this out by moving the cart through the gate by hand a few times.

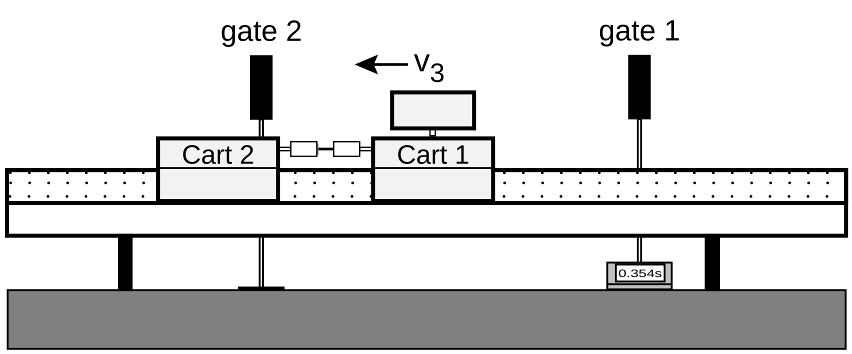

Figure 2: The final state of the carts after collision.

The mass of the carts can be varied by adding masses (small metal disks) to them. The experiment will be done with cart 2 initially at rest. So in the initial state we have :

Initial state: Cart1: has a mass m_1 and initial velocity v_1
Cart 2: has mass m_2 and is at rest, so v =0 cm/s

After the collision(final state): The two carts will stick together
The total mass of the carts is (m_1+m_2) and the velocity is v_3.

You will do 6 trials with the following m_1 and m_2 choices:

Trial 1+2: no mass disks on cart 1, 4 mass disks on cart 2;
Trial 3+4: 2 mass disks on cart 1, 2 mass disk on cart 2;
Trail 5+6: 2 mass disks on cart1 , no mass disks on cart2.

Enter the data into the spreadsheet and calculate the momenta and kinetic energies before and after the collisions, as well as the percent change in the total momentum and the kinetic energy.

It is possible to calculate the percentage of the kinetic energy lost in a completely inelastic collision; you will find that this percentage depends only on the masses of the carts used in the collision, if one of the carts starts from rest.

The initial *KE* is given by:

$$KE_i = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \qquad \text{but since } v_2 = 0$$

$$KE_i = \frac{m_1 v_1^2}{2} \tag{5}$$

The final *KE* is given by:

$$KE_f = v_3^2 \frac{m_1 + m_2}{2} \tag{6}$$

From conservation of momentum:.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v_3 \quad \text{or, since } v_2 = 0$$

$$m_1 v_1 = (m_1 + m_2) v_3 \tag{7}$$

Combine equations (5), (6), and (7) to obtain an expression for KE_f in terms of KE_i. Compare this result with the results of the experiment. Do they agree?

CHECKLIST:

Your lab report should include the following:

1) Several runs with the momentum and kinetic energy calculations for the system before and after the collision.

2) Calculations for the percent changes in the momentum and kinetic energy through the collision. Calculations for the errors in v, P, KE, P_{diff} and KE_{diff}.

3) A calculation of the % loss of KE in a totally inelastic collision.

4) Comment on the uncertainties of the measurement of the total momenta and kinetic energies. HINT: If the errors in the measurement of P_{diff} and KE_{diff}. are large compared to P_{diff} and KE_{diff} themselves, that would mean that our measurement instruments (the timer, the ruler) are not precise enough to pinpoint the change in the total momentum and kinetic energy. Is this the situation with Your measurements?

Summary of formulas for momentum and kinetic energy:

Initial state:

Measure photogate time t1, calculate v1 and error in v1 (called δv1 on spreadsheet)*****

Momentum: $P_i = m_1 v_1$ and kinetic energy: $KE_i = \frac{m_1 v_1^2}{2}$

Final state:

Measure photogate time t3, calculate v3 and error in v3 (called δv3 on spread sheet)*****

Momentum: $P_f = m_3 v_3 = (m_1 + m_2) v_3$ and kinetic energy: $KE_f = \frac{m_3 v_3^2}{2} = \frac{(m_1 + m_2) v_3^2}{2}$

The difference between initial and final state is:

Momentum: $P_{diff} = P_i - P_f$ which is called Pdiff in spreadsheet.
Kinetic energy: $KE_{diff} = KE_i - KE_f$ which is called KEdiff in the spreadsheet.

*** Summary of errors:**

You have to determine(= choose) the errors in the length measurement δL and the time δt. Here we will neglect the error in the mass measurement. The error is the velocity is given by:

$\frac{\delta v}{v} = \left(\frac{\delta L}{L} + \frac{\delta t}{t} \right)$ resulting in $\delta v = v\left(\frac{\delta L}{L} + \frac{\delta t}{t} \right)$

The error in momentum is: $\delta P = P \frac{\delta v}{v}$ and in kinetic energy is: $\delta KE = KE \frac{2\delta v}{v}$

The error in the differences for momenta and kinetic energies is then:
$\delta P_{diff} = \delta P_i + \delta P_f$ and $\delta KE_{diff} = \delta KE_i + \delta KE_f$