EXPERIMENT: MOMENT OF INERTIA

OBJECTIVES:

1) To familiarize yourself with the concept of the moment of inertia, I, which plays the same role in the description of the rotation of the rigid body as the mass plays in the description of its steady motion .

2) To measure the moments of inertia of several objects by studying their accelerating rotation under the influence of unbalanced torque.

APPARATUS: See Figure 3

THEORY

If we apply a single unbalanced force, F, to an object, the object will undergo a linear acceleration, a, which is determined by the force and the mass of the object. The mass is a measure of the object's resistance to being accelerated, its inertia. The mathematical relationship which says this is

$$F = ma.$$

If we consider rotational motion we find that a single unbalanced torque

$$\tau = (Force)(lever\ arm)^{\#}$$

produces an angular acceleration, α, which depends not only on the mass of the object but on how that mass is distributed. The equation which is analogous to
$F = ma$ is:

$$\tau = I\,\alpha. \tag{1}$$

where τ is the torque in Newton-Meters, α is the rotational acceleration in radians/sec^2 [*] and I is the MOMENT OF INERTIA in kg-m^2. It is a measure of the way the mass is distributed on the object and determines its resistance to rotational acceleration.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are several examples of the expression for I for a few rather special objects.

[#] In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i. e., perpendicular to the radius (see your instructor for the case when the lever arm is not perpendicular to the force).

[*] A radian is an angle measure based upon the circumference of a circle $C = 2\pi r$ where r is the radius of the circle. A complete circle (360˚) is said to have 2π radians (or radiuses). Therefore, 90^o (1/4 circle) is $\pi/2$ radians, etc. Angular accelerations (α) are measured in units of radians/sec^2

One point mass m on a weightless rod of radius r $(I = mr^2)$:

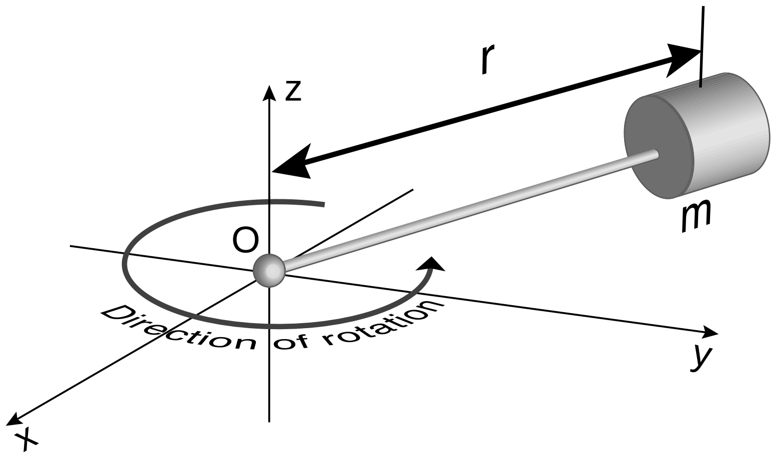

Figure 1

Two point masses on a weightless rod ($I = m_1 r_1^2 + m_2 r_2^2$):

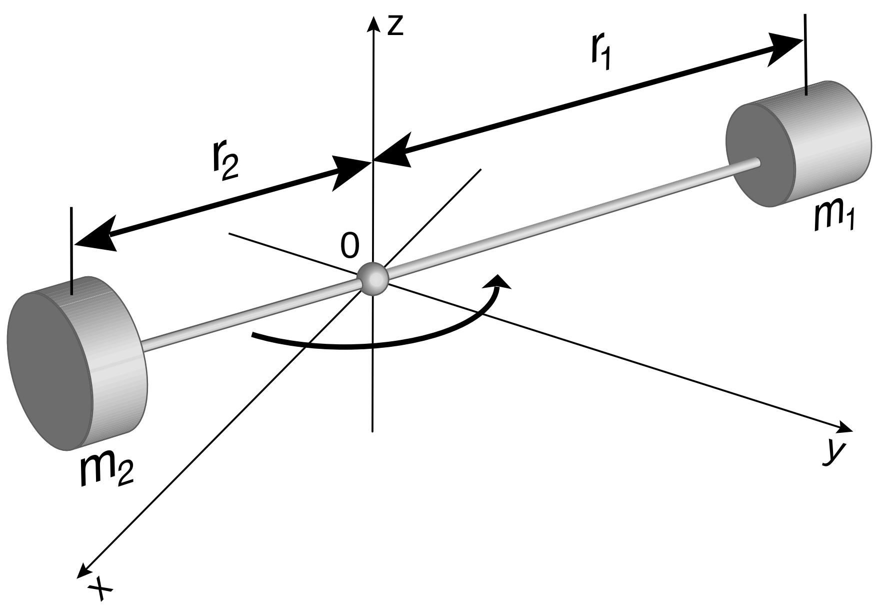

Figure 2

To illustrate we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod 2 meters in length (object #1 above):

$$I = mr^2 = (2\ \text{kg})\ (2\ \text{meters})^2 = 8\ \text{kg meter}^2$$

If a force of 5 N were applied to the mass perpendicular to the rod (to make the lever arm = r) the torque is given by:

$$\tau = Fr = (5\ \text{N})\ (2\ \text{meter}) = 10\ \text{N meter}$$

By equation 1 we can now calculate the angular acceleration:.

$$\alpha = \frac{\tau}{I} = \frac{10\ Nmeter}{8kg\ meter^2} = 1.25\left(\frac{radians}{\sec^2}\right).$$

It will be important to note that we obtain the moment of inertia of a complicated object by adding up the moments of each individual piece (object 2 above is the sum of two object 1 components). We will use these concepts in this lab, where, by measuring the torque and angular acceleration of various objects, we will determine their moments of inertia.

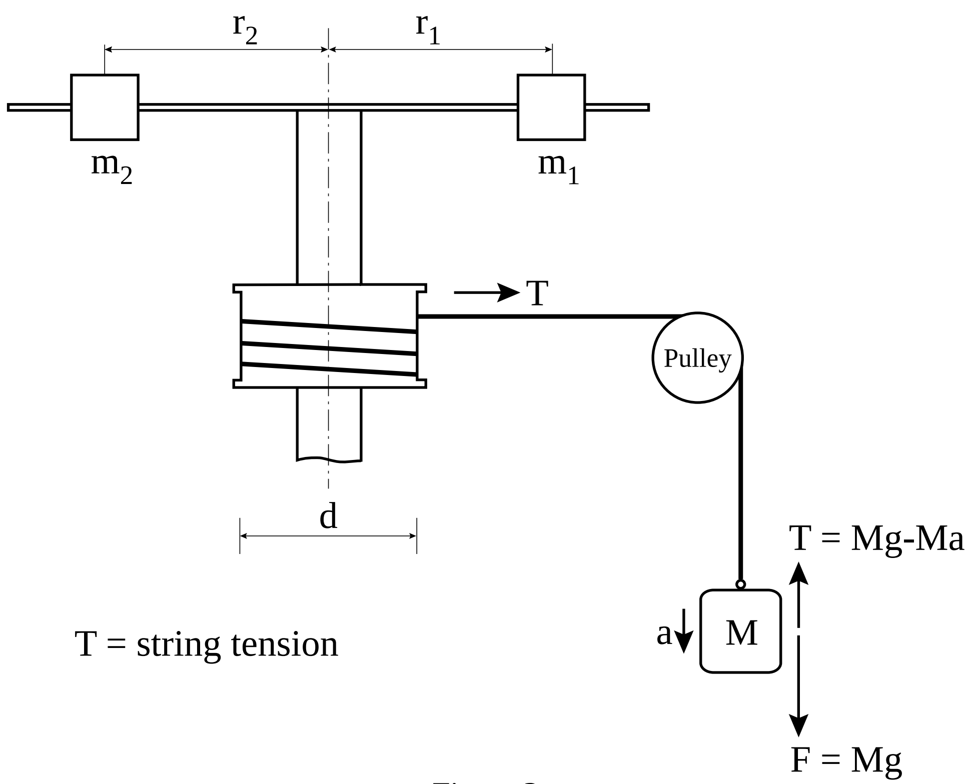

Figure 3

EXPERIMENTAL APPARATUS

In our case the rigid body consists of 2 cylinders, which are placed on the metallic rod at different radii from the axis of rotation (Figure 3). Note, that we can't ignore the mass of the rod and the supporting structure in our measurements, so that their moment of inertia isn't equal to zero. At the same time, we can measure this quantity both for the supporting structure + the rod alone and with the cylinders attached at various distances from the axis.

To set up your rigid body, wrap the string around the axle several times, run it over the pulley to a known weight as shown in Figure 3.

Consider the following series of steps:

a) If we release the weights from rest and measure how long it takes to fall a distance y then from

$$y = \frac{at^2}{2}$$

we can solve for a, the linear acceleration of the weights, the string and a point on the side of the axle.

b) Using α = angular acceleration $= \dfrac{\textit{Linear acceleration}}{\textit{radius of axle}} = \dfrac{a}{\left(\frac{d}{2}\right)}$

we obtain the angular acceleration.

c) The torque acting on the axle is given by

$$\tau = \textit{(Force)(lever arm)}$$

which is

$$\tau = T\left(\frac{d}{2}\right) = (Mg - Ma)\frac{d}{2}$$

Since we now have α and τ, we can calculate I from equation 1. Before class, be sure you know how to use equation 1 and the above 3 steps to obtain the expression:

$$I = \frac{Mgd^2t^2}{8y} - \frac{Md^2}{4} \tag{2}$$

which allows calculation of I from measurements of t with no intermediate steps. In the measurements made in this experiment the constant term $\frac{Md^2}{4}$ is always quite small compared to the t^2 term. We will therefore ignore the constant term when calculating I.

PROCEDURE

Remove the masses m_1 and m_2 and measure the moment of inertia of the support structure alone. Do this 5 times and use the standard deviation of the mean value for t as the uncertainty (δt) for the measurements of t in this experiment. Calculate $I_{support}$ using the mean time you obtained.

Make a series of measurements of I, the moment of inertia of the rigid body, with the masses m_1 and m_2 placed an equal distance r $(r_1 = r_2)$ from the axis of rotation. Take measurements for at least 6 different r values spanning the length of the rod. Make sure that You measure the radius r from the center of mass of the cylinder to the axis, and not from either of its edges. Again use the formula (2) to calculate I_{meas}.

Since the moment of inertia is the sum of the moments of the individual pieces we may write

$$I_{meas} = I_{support} + I_{masses} = I_{support} + (m_1+m_2)r^2 ,$$

where I_{meas} is the moment of inertia you calculated and recorded in your data sheet. So the graph of I_{meas} vs. r^2 should be a straight line. Make a plot of your measurements of I_{meas} vs. r^2. Compare the slope and intercept of this data with the values previously measured for $I_{support}$ and m_1+m_2. Do they agree?

QUESTIONS

1. In the plot I vs. r^2, why did we use r^2 and not r in the plot? What are the units of the slope of the plot I vs. r^2?

2. Explain why putting the masses at $r = 0$ (if we could) is the same or is not the same as removing them from apparatus