

## EXPERIMENT: THE PENDULUM

**OBJECTIVES:** To investigate the functional dependence of the period of a pendulum ( $\tau$ ) on the length of a pendulum ( $L$ ), the mass of the bob ( $m$ ) and the starting angle ( $\theta_0$ ).

To check for systematic errors by calculating  $g$ .

### APPARATUS:

Point masses and string, Digital timer and period gate, Meter stick

The period gate uses the light and photodetector in the following fashion: When the beam is interrupted the first time, it starts the timer, keeps it going when it is interrupted the second time but stops the timer on the third interruption of the beam -- thus measuring the time for one complete oscillation of the pendulum, its period. (It can also start on the 1st and stop on the 5th break of the beam thus measuring two periods.)

### THEORY

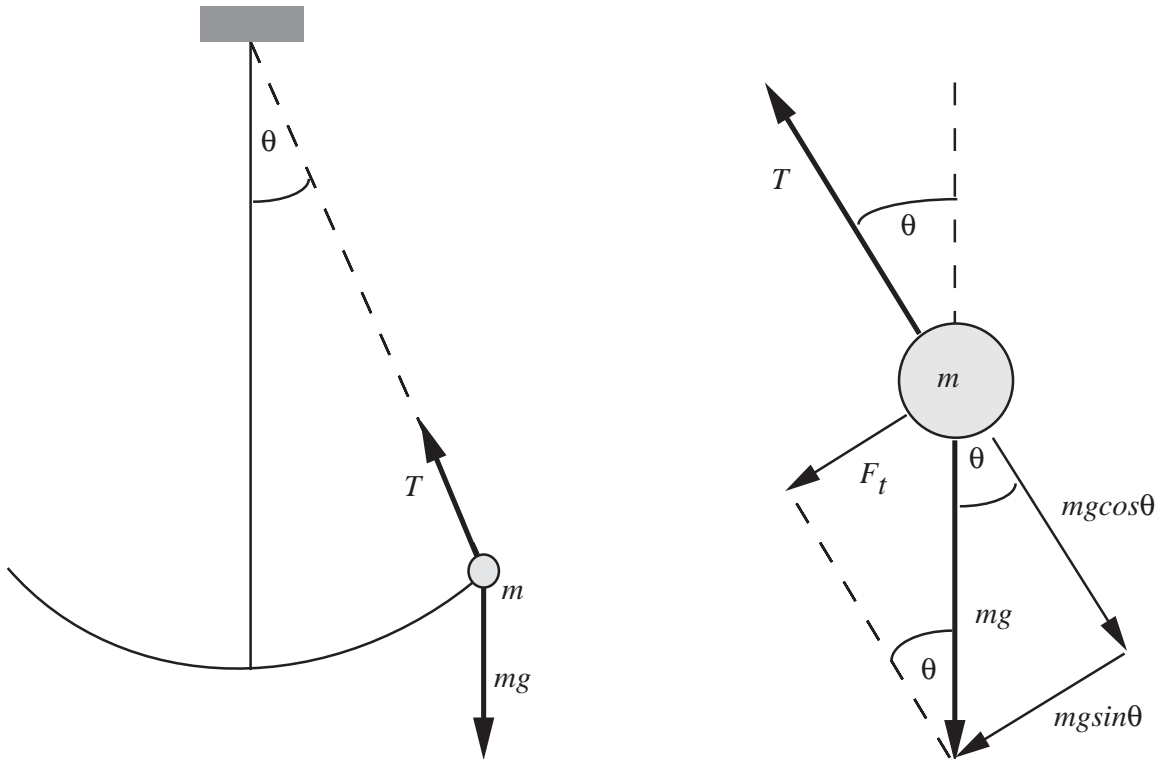
For regions where there exists a constant net force ( $F$ ), Newton's Law  $F = ma$  tells us that the acceleration ( $a$ ) is a constant and therefore the position of the object can be written as  $s = s_o + v_o t + \frac{at^2}{2}$ . This is the formula we have tested in the Free Fall labs.

In the analysis of the motion of a pendulum we have to pay attention that:

- 1) the motion is part of a circle so that angular acceleration ( $\alpha$ ) is the most useful variable, and
- 2) the angular acceleration will not be a constant throughout the motion.

Consider the pendulum shown on the next page. The acceleration of the bob tangential to the circle,  $a_t$ , is determined by  $F_t$  - the force tangential to the circle. Since the tension in the string always acts along the radius, it doesn't contribute to  $F_t$ . Decomposing the gravitational force  $mg$  into components perpendicular and parallel to the string as shown in the above diagram, we find that  $F_t = mg \sin \theta$ . Thus, the acceleration tangent to the circle is given by

$$a_t = \frac{F_t}{m} = g \sin \theta.$$



The angular acceleration  $\alpha$  is then found by the relationship for circular motion:

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L} \sin \theta \quad \left( \frac{\text{radians}}{\text{sec}^2} \right)^*$$

Thus, as we had suggested, the angular acceleration  $\alpha$  is not a constant but varies as the sine of the displacement angle of the pendulum.

For small angles (say  $\theta < 0.5$  radian) angular accelerations, which depend on the sine of the displacement angle  $\theta$ , can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle  $\theta$  by

$$\theta = \theta_o \cos\left(\frac{2\pi t}{\tau}\right)$$

where  $\theta_o$ , is the angle at time  $t = 0$  (when we release the pendulum) and  $\tau = 2\pi\sqrt{\frac{L}{g}}$  is the period of the motion.

This formula describes the motion of a pendulum. In this lab we will investigate the validity of this formula.

### PROCEDURE

The three parameters of the system are  $L$ , the length of the pendulum from the support to the center of mass of the "point mass";  $m$ , the mass; and  $\theta_o$ , the angle through which you displace the mass as you pull time pendulum back to start it. The quantity you measure is the period  $\tau$ .

For a pendulum with starting angle  $\theta_o = 0.5$  radians, make a graph showing the angle of the pendulum at  $t = 0, 0.1\tau, 0.2\tau, \dots, 1.0\tau$  (every  $0.1\tau$ ). Draw a smooth curve through the points and comment on the values of  $t$  when  $\theta = 0$  (this should be done before lab). The displacement is zero when  $t = \frac{\tau}{4}$  and  $\frac{3\tau}{4}$ .

**Before starting the measurements write your introduction and write what YOU think the period of the pendulum depends on. Forget for the moment the formulas you learned and simply use your intuition to estimate whether the period depends on the pendulum length, mass and our initial angle. Your measurements will verify which one is correct.**

Using the period gate measure the period several times for several different combinations of  $L$ ,  $m$  and  $\theta_o$ . You should normally keep two parameters fixed while you vary the third. **When you vary  $L$ , do it over the largest range of values that you can with your apparatus. Always keep starting angles  $\theta_o$ , less than 0.5 radian.**

Make graphs of:

- $\tau$  vs.  $m$  (for fixed  $\theta_o$  and  $L$ )
- $\tau$  vs.  $\theta_o$  (for fixed  $m$  and  $L$ )
- $\tau$  vs.  $\sqrt{L}$  (for fixed  $m$  and  $\theta_o$ ) use this as the scale for  $\tau$

Comment on how the period depends on each of the three parameters. If any of the graphs should be a straight line, determine the best line, calculate its slope and uncertainty.

### QUESTIONS

- 1) Use the slope of the graph of  $\tau$  vs.  $\sqrt{L}$  to calculate  $g$ . Compare your result with  $980 \text{ cm/sec}^2$ .
- 2) For one of your measurements calculate the angular acceleration ( $\alpha$ ) at the starting angle  $\theta_o$ , at  $\theta = 0$  and at the far end of the swing.

- 3) Estimate the period of a pendulum, the length of which is that of the tallest
- a) tree
  - b) building
  - c) mountain
- in the world. You should not need or use the Guinness Book of World Records for this.