

EXPERIMENT: PROJECTILE MOTION

PRIMARY OBJECTIVE: To follow in detail the motion of an object in two dimensions, and to ascertain that the motion can be analyzed by considering the motion in each dimension separately.

SECONDARY OBJECTIVE: To check for the existence of possible systematic errors by comparing your experimental value for the horizontal acceleration with the expected value of zero, and by comparing your experimental value for the vertical acceleration with the accepted value for g .

APPARATUS:

Recorded video of the projectile motion

THEORY

This lab involves a two-dimensional projectile motion problem. It goes, therefore, one step beyond the one-dimensional studies of forces and acceleration done in the previous labs. The most general projectile motion would involve three dimensions and could be described by referring to the x -, y -, and z components of the velocity and acceleration vectors of the problem. This approach greatly simplifies the understanding of the motion because, for example, the x -component of acceleration affects only the x -component of velocity, etc. Thus the equations describing three-dimensional motion simplify to three sets of equations, each of which we already understand from one-dimensional studies.

Further simplification comes through shrewd choice of the orientation of the coordinate system to be used. If there is only one acceleration vector in the problem, choose one axis to be in that direction. This makes two components of the acceleration equal to zero and simplifies the equation of motion considerably. Proper choice of the second axis can often make $v_z = 0$ as well as $a_z = 0$, and then the problem involves only two dimensions (x and y) since there is no motion at all in the z -direction. This is the situation in the present experiment.

At any time, t , the instantaneous velocity of a projectile is a vector, \mathbf{v} , tangent to the flight path. This velocity vector can be decomposed into a horizontal component, v_x , and a vertical component, v_y . If we neglect air friction, the only force acting on the projectile AFTER it leaves the launcher is that of gravity, directed vertically downward. In this case, there are no forces acting in the x -dimension.

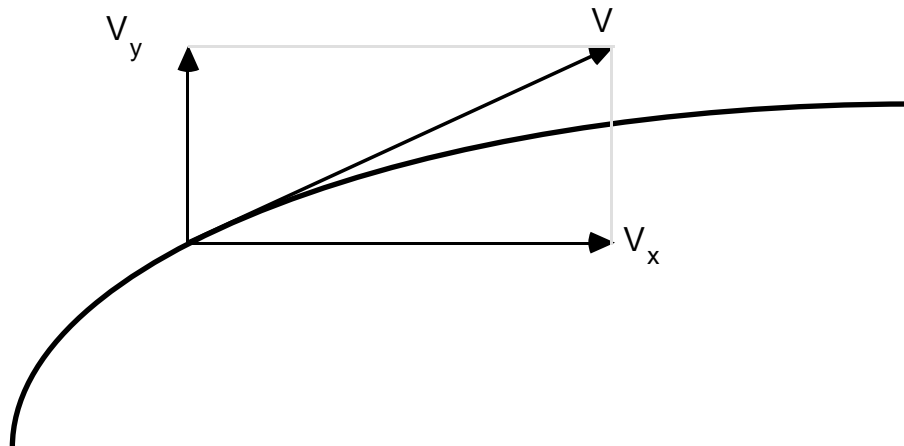


Figure 1

Recall from Newton's Second Law that:

$$\mathbf{F} = m\mathbf{a} \quad (1)$$

This is a vector equation. The vector on the left (\mathbf{F}) can equal the vector on the right ($m\mathbf{a}$) if and only if each component of the left vector equals each component of the right vector. Hence the motion of a projectile can be separated mathematically into independent components. In the x -dimension:

$$F_x = ma_x \quad (2)$$

But if $F_x = 0$, then ma_x must equal zero also, and since we know that m is not equal to zero, we must have:

$$a_x = 0 \quad (3)$$

Similarly, in the y -dimension:

$$F_y = ma_y \quad (4)$$

F_y is the force of gravity, so a_y is expected to be g , the acceleration due to gravity:

$$\begin{aligned} F_y = ma_y = -mg & \quad \text{or} \\ a_y = -g & \quad (5) \end{aligned}$$

where the minus sign indicates that the force (and thus also the acceleration) is directed vertically downward.

According to equations (3) and (5), the two-dimensional projectile motion can be described in terms of two one-dimensional motions: 1) Motion with constant velocity (no acceleration) in the horizontal direction and 2) motion with constant acceleration in the vertical direction. That is,

$$v_x = v_{x0} \quad (6a)$$

$$v_y = v_{y0} - gt, \quad (6b)$$

where v_{x0} and v_{y0} are the initial components of velocity in the horizontal and vertical directions respectively.

PROCEDURE

In this lab you will use a set of recorded video images to calculate the position and speed of the ball. The video images have been prerecorded, because it takes some time to create a good one.

The trajectory of the ball has been recorded with a video camera. The camera takes 30 images (frames) per second, so the time between images is roughly 0.033 sec. These images are captured on our computers and are available for analysis. The video you will be using is in the "Projectile Motion" folder. The software called "Videopoint" will allow you to select a particular frame and measure the position of the ball in that frame. Start the program from the *Videopoint* icon on the desktop. After starting the program, select the "Open Movie" command and open the one stored in "Projectile Motion". You will use only one object for digitization.

After opening the movie, Videopoint shows you the image number of the frame visible in the image viewer. Watch the whole movie, using the control bar at the bottom of image viewer. Then, go to the 1st frame which corresponds to the moment when the ball leaves the launcher and becomes visible.

There is a table created which contains the image number, the time associated with each image and a location for storing x,y coordinates. If you simply click on the position of the ball in an image, Videopoint will store the x,y coordinates in the reserved location. The coordinates will be measured in the "*Origin 1*" coordinate system in units of "pixels". This is not the units we want, so they have to be converted. To do this click on the "ruler" icon in the toolbar. Type in the length of the bar you see in the image (0.5 meters) and measure its starting point and endpoint. In the origin window you will now see a conversion factor from pixels to meters. From now on all your x,y locations will be measured in meters and any that were already stored are converted to meters. Do NOT change the origin of the "*Origin1*" coordinate system i.e. leave it at the lower left corner of the frame.

After you digitized all the ball positions that are visible, make sure that you did not forget any. Click on the "trails" icon and it will show you all the points on your video image.

Make the first set of graphs in Videopoint by clicking the "graph" icon. Graphs to be made: 1) y versus x , y versus time and x versus time. Those graphs are part of your write up and should be discussed in your conclusions. Why are they linear, not linear etc. Save your Videopoint data at this point in your section folder !!!

The table with times of the images and x,y coordinates will be copied to the clipboard so they can be used in the Excel spreadsheet. Only copy the part of the table, that contains data (skip the frames where the ball is invisible). The empty spreadsheet can be found in the "Projectile Motion" folder.

From the transferred coordinates, you will calculate v_x and v_y for each projectile image using the equations:

$$v_x = \frac{\Delta x}{\Delta t}, \quad (7a) \quad v_y = \frac{\Delta y}{\Delta t}, \quad (7b)$$

where $\Delta x, \Delta y$ refers to the distance between the **preceding** and the **following** projectile image, in the x view similarly for Δy in the y view, and Δt is the time interval between

these two images $= t_{i+1} - t_{i-1}$). The formulas are given on the spreadsheet. On the spreadsheet you will calculate the quantity "DeltaT" first. This is the time elapsed between two adjacent images and is simply a check of the time between two images. **Beware: DeltaT is not the same as Δt .**

According to equations 6a and 6b, the graphs of v_x vs. t and v_y vs. t should be straight lines, the slopes of which are zero and $-g$ -respectively. If the instant at which the projectile left the gun can be established from your picture, then the vertical intercepts should be v_{x0} and v_{y0} .

In order to see how linear the two graphs are, make graphs of v_x vs. t and v_y vs. t . Do this by transferring the spreadsheet data for time and speed into **Kaleidagraph** and making one graph containing both of v_x vs. t and v_y vs. t . When the data are all graphed, draw your estimates of the "best straight lines" through the data points or do a fit of the data points. Determine the slopes of your lines and compare the resulting accelerations with those expected.

You will be asked to estimate the accuracy with which you measure an x, y point. To do this, measure a point a few times and look at the scatter of the points. Then, measure the second set of points (ask the other partner to measure) and save it. Since Your spreadsheet is already set up with all the formulas, transfer the second set into it and look at how different the resulting speeds are you obtain for v_x and v_y . Copy these velocities into **Kaleidagraph** and calculate their slopes (do not spend much time on editing these graphs!). Now, You can compare two sets of the slopes with the anticipated values. Is the agreement good? You can also comment on the possibility of systematic errors in Your measurements. HINT: if the results of Your measurements are systematically larger or smaller than the well-known expected value (say, the world average of g), this can witness in favor of some unknown factor, systematically influencing Your measurements. Of course, to be definitely sure about systematic errors we would have to make more than two independent calculations of the slopes (at least, 10 or 100) - if not the limited time of this lab. However, the procedure of pinpointing the systematic errors would be essentially the same as the one we are using.

Comment on these points in your laboratory report.

CHECKLIST:

Your lab report should include:

- 1) A graph of y vs. x of your original data (from Videopoint) with explanation.
- 2) Graphs of y vs. t and x vs. t from Videopoint, with explanations
- 3) One graph showing v_x vs. t and v_y vs. t and fits/calculations of the slopes.
- 4) Comments upon how well you obtained the expected acceleration within the uncertainties. Are any systematic errors present in Your data?
- 5) Estimates of how accurately you measure an x, y point on the video. This can be done by measuring a point more than once and looking at the differences.
- 6) Your conclusions, based on the results of your experiment, as to how successfully the motion of a projectile can be described in terms of two independent components.

The equations describing displacement in the x - and y -dimensions are

$$y = v_{yo} t - \frac{at^2}{2} \quad \text{and} \quad x = v_{xo} t$$

These two equations can be combined by eliminating the variable t , i.e., substituting t from the second equation into the first:

$$t = \frac{x}{v_{xo}}$$
$$\therefore y = v_{yo} \frac{x}{v_{xo}} - \frac{g}{2} \left(\frac{x}{v_{xo}} \right)^2$$

When simplified this can be seen to be the equation of a parabola which opens downward. Is this what you observe in your graph of y vs. x ?