EXPERIMENT: THE SPRING
Subtitle: Hooke's law and Oscillations

OBJECTIVES: 1)To investigate how a spring behaves if it is stretched under influence of an external force. This behavior is described by Hooke's law, which states that the amount of stretching(Δx) is linearly proportional to the external force

2) A stretched spring is also a good example of an oscillator, which oscillates around an equilibrium position (sum of all forces is zero), with a characteristic period (T), which will be determined in this lab.

APPARATUS:

A simple spring with different masses attached to it will be used. In the first part the distance it stretches is measured as a function of the mass attached. In the second part the period of oscillation is measured. Here we use the timing gate units in the following fashion: When the light beam is interrupted the first time, it starts the timer, keeps it going when it is interrupted the second time but stops the timer on the third interruption of the beam, thus measuring the time for one complete oscillation of the mass, its period. Use setting: "PEND".

THEORY

1. **Hooke's Law**

An ideal spring is remarkable in the sense that it is a system where the generated force is linearly dependent on how far it is stretched. This behavior is described by Hooke's law which states that to extend a string by an amount Δx from its previous position, one needs a force F which is determined by $F_s = k\Delta x$. Here k is the so called spring constant and depends on the construction of the spring. This behavior is illustrated in Figure 1. In our case the external force is determined by attaching a mass m to the end of the spring. The force exerted downward on the spring will be $F_g = mg$.

Let's consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring $(k\Delta x)$ is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point where the two forces are equal but pointing in opposite directions:

$$F_s - F_g = 0,$$

$$k\Delta x = mg$$

After that the system spring + mass can stay at this point as long as no external forces are exerted on it. This allows us to determine the spring constant k and this will be done for several values of m and Δx.

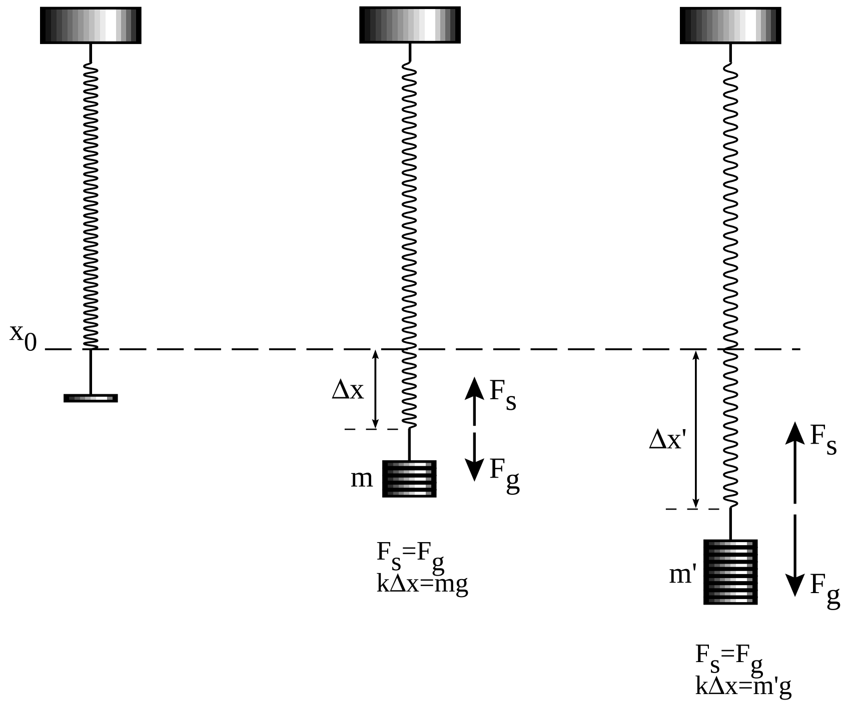

Figure 1: The spring in the equilibrium

2. Oscillation

The position where the mass is at rest is called the equilibrium position ($x = x_0$). The downward force due to gravity $F_g = mg$ and the upward force due to the spring pulling upward $F_s = kx$ cancel each other. This is shown in the first part of figure 2. However, if the string is stretched more by pulling it down and released, the mass will accelerate upward ($a > 0$), because the force due to the spring is larger than gravity pulling down. After release it will pass through the equilibrium point and continue to move upward. Once above the equilibrium position gravity will start to exceed the force pulling upward due to the spring and acceleration will be directed downward. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a) and velocities (v) are illustrated in Figure 2 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period T, which is determined by the spring constant and the attached total mass. This period is the time it takes to complete one oscillation i.e. the time necessary to return to the point where the cycle starts repeating (the points where x, v, a are the same). One complete cycle is shown in Figure 2 and the time of each position is indicated in terms of the period T. The period is given by :

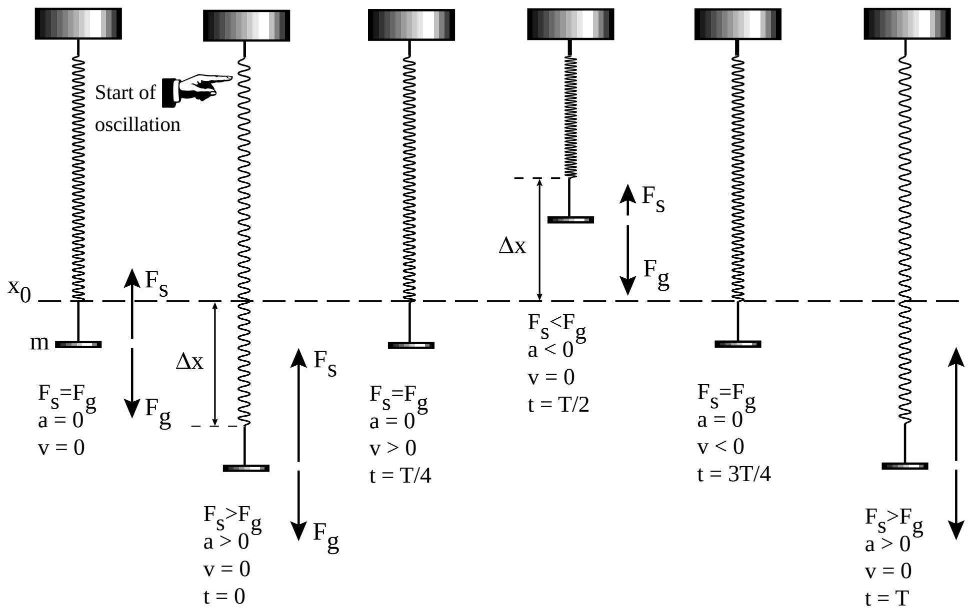

Figure 2: One cycle of an oscillation of the spring.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

By measuring the period for given masses the spring constant can be determined.

PROCEDURE

1. Hooke's law

- Attach the support table for the masses to the spring. This table nominally has a mass m_0=50 grams (check this !). Measure the position of the end of the spring after the table has been attached. This position is x_0. It is important to stretch the string slightly before the first measurement is taken and that is the reason for attaching the support table. Assign an error to your measurement of the distance.

- Start measuring by increasing the mass attached to the spring to 120 grams then increase the mass by increments of 10 grams up to a total of 220 grams and measure the corresponding position of the spring for each mass. This results in a series of measurements m_i and x_i. To calculate the forces due to gravity and the spring calculate: $\Delta x_i = x_i - x_0$ and $\Delta m_i = m_i - m_0$. The corresponding forces for gravity and the spring are: $F_g^i = g\Delta m_i$ and $F_s^i = k\Delta x_i$. If you are using the spreadsheet this will be done automatically for most of the measurements.
- Graph F_g^i vs. Δx_i and from the slope determine k.

2. Period of oscillation.

- For attached masses varying from 120 to 220 grams, in steps of 10 grams, determine the period for each. Do not stretch the spring more than about 2 cm from its equilibrium position.
- For each measurement , using $T = 2\pi\sqrt{\frac{m}{k}}$, determine the spring constant k. Note: in this equation m is the total mass attached to the spring.

QUESTIONS

1) What are the units for the spring constant used in this lab?

2) You determined the spring constant in two independent ways. Do they agree ?

3) Which is the more accurate measurement ?

4) Is Hooke's law correct ?

5) Estimate the spring constant of springs used in the suspension of your car. Hint: estimate how much closer to the ground the car body is, when its trunk is fully loaded. From added mass and lowering of the car estimate the spring constant. Pay attention that the car has at least 4 wheels.

OPTIONAL

Measure the period as a function of the amplitude of the oscillation. To do this start by using a particular mass and stretching the spring to start the oscillation. Now measure the period every 30 seconds, for a total of 10 minutes, without interrupting the oscillation. During this time, the amplitude of the oscillation will decrease. Does the period depend on the amplitude, or on the time expired ?